THE DIFFERENTIATION TRIANGLE*

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ABSTRACT

The paper formalizes the observation that submarkets for high-quality and low-quality variants are markedly different from each other. We study a simple model where variants of low quality cannot be horizontally differentiated, whereas customers disagree about the value of variants in the high-quality range. We determine the outcome under price competition in the *differentiation triangle* with sequential entry when each firm can develop the vertical product line or decide to follow a niche strategy, i.e. to develop only one variant.

KEYWORDS: Product Differentiation; Product Design; Multi-Product Competition; Two-Dimensional Taste Heterogeneity.

1 Introduction

Consider a generic good, e.g. a car, and one specific variant of the good, e.g. a blue BMW 318. In the eyes of the customers the BMW can be distinguished from other cars by a wide range of characteristics. Some of them refer to the "objective" quality of the car. Probably all customers would agree that a BMW is of a higher quality than a VW Beetle. Other characteristics depend more on the personal taste of the customers. Given particular prices (say equal prices), this may result in some customers preferring the BMW over a particular Mercedes and others preferring the Mercedes. In general, variants of a good have several characteristics, some of which may be thought of giving rise to horizontal, others to vertical product differentiation. So in comparing two cars, price, quality, and horizontal characteristics determine the choice of customers.

Having identified the determinants of customers' choices, what can we say about their interdependence? Recent empirical work on the car market (Berry, Levinsohn, and Pakes, 1995, Goldberg, 1995, and Feenstra and Levinsohn, 1995) has established that price elasticities (both own and cross) in the high-quality segment are lower than the ones in the low-quality segment. The car market is an example of a market where horizontally differentiated variants of a certain quality compete with each other as well as with variants of another quality. One explanation for the quality-dependence of price elasticities is that customers' evaluations for cars are more dispersed in the high-quality range than in the low-quality range. Hence low quality does not allow for much horizontal differentiation. To put it bluntly, a "no-name" variant can only be sold if its price is not higher than the price of competing "no-name" variants.

The importance of asymmetric customer behavior, as described above, was already observed as early as Katz (1984). Katz made a first attempt to analyze this issue, but his exploratory paper left a number of questions open. It is the purpose of our paper to model the asymmetry by a heterogeneous population of customers and to analyze its impact on the product choice in a market which firms enter sequentially. By doing so we also follow two useful suggestions for further research made by Gould (1984) and Moorthy (1984) who commented on Katz (for further discussion see Section 2).[1]

In many market settings a firm can choose between a niche strategy (produce either a high or a low-quality variant) and a multi-product strategy in which it develops a high-quality and

[1] We are grateful to Frank Verboven for pointing this work out to us. Moorthy (1984) suggests that one expects less qualities than customer types. This makes it impossible to have perfect self-selection of customer types. Gould (1984) comments on Katz that "it would be interesting to see if an extension of the models shows whether firms can use product-line planning as a strategy to discourage entry to their industry".

a low-quality variant.[2] A decision in favor of one or the other can be influenced by strategic and non-strategic (cost or demand related) reasons. Since we fix the parameters of market demand we do not consider demand effects. Cost effects do play a role and can be a dominant factor: the presence of strong economies of scope favors a multi-product strategy. There can also be strategic reasons to produce both variants. An early mover may be able to deter entry by occupying locations in the product space. Since each configuration of the product choice generates different prices, the intensity of price competition is also a factor which should be taken into account. As an implication, multi-products can be chosen in absence of economies of scope, while a single product can be chosen in the presence of economies of scope. It is the purpose of this paper to identify the trade-off between various cost and strategic considerations and to determine the equilibrium outcomes for each parameter configuration. We analyze a four-stage game with perfect information in which, at the first three stages, two incumbents and one potential entrant sequentially decide upon the specification of the variants which they develop (product choice) and then, in the fourth stage, they compete in prices.[3] Our main interest is the equilibrium profile of the product choice, given that firms can only locate at corner points in the quality-variety space (one low-quality variant and two horizontally differentiated high-quality variants), and the impact of the various product choices on price competition. The differentiation *triangle* is used as a representation tool.

We provide a model and a solution for an empirically relevant situation that has been neglected in the theoretical literature. In particular, the asymmetry in the two-dimensional product differentiation has not been studied in a model with entry.[4]

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[2] An interesting phenomenon, which suggests that some firms follow the second strategy, is that some producers of an established brand offer the identical physical specification of their high-quality variant with a different packaging at a much lower price not using the brand name, i.e., they differentiate vertically and exploit strong economies of scope because they can use the same production process.

[3] Sequential entry in our static model is supposed to capture the entry process in a growing market. It can also capture the idea that firms played a timing game, i.e., the first mover has won an R&D race.

[4] Previous literature on multi-product firms concentrated on entry deterrence by a multi-product monopolist in a one-dimensional model (see e.g. Eaton and Lipsey, 1989, and the references therein), on multi-product firms competing on several markets against each other (Bulow, Geanakoplos, and Klemperer, 1985, and Lal and Matutes, 1989), or on multi-product duopoly in a one-dimensional model (e.g. Champsaur and Rochet, 1989). There also exists some work on multi-product firms using the multinomial logit model, see Anderson, de Palma and Thisse (1992) and the references therein. Such a (nested) approach has been fruitful in empirical research, see Berry, Levinsohn, and Pakes (1995) and Goldberg (1995) on the car market. Other important contributions include Brander and Eaton (1984), Shaked and Sutton (1990) and Dobson and Waterson (1996). In addition, there exists

In our model we obtain the following results:
(1) Incumbent 1 chooses the most profitable strategy which can either be to develop the vertical product line and leaving incumbent 2 with a niche strategy or to follow a niche strategy himself.
(2) Given that incumbent 1 wants to follow a niche strategy, incumbent 2 either produces the vertical product line in order to deter entry or she develops one variant which implies accommodating entry by a third firm. If the second mover produces the product line she deters entry by brand proliferation.
Consequently, different firms choose niche strategies for different reasons. Incumbent 1 is guided by comparing the profitability of the niche strategy (in the environment in which he will find himself) with the profitability of the roduct line. In case incumbent 1 chooses a niche, a niche strategy of incumbent 2 implies that there is entry by a third firm.
The fact that the number of variants per firm (one or two) is determined endogenously, contrasts the literature on multi-product firms motivated by cost and demand considerations (see e.g. Bulow, Geanakoplos, and Klemperer, 1985). But strategic factors may dominate these cost factors. In our differentiation triangle we can analyze the trade-off between strategic and cost effects which results in firms choosing a product line or a niche. Our results cannot be replicated in a two-dimensional product differentiation model in which dimensions are independent and all corner points occupied.
Finally, we want to stress that our model does not intend to be a description of the car market or indeed any other *specific market.* In the car market some firms produce low-quality cars, some produce high-quality cars and some produce both. Also variants are differentiated in more than two dimensions. Still, firms are faced with strategic and cost considerations that are similar to the ones studied in our model. We want to single out these issues leaving other issues undiscussed.

2 The Model

Consider a market in which firms have the option to produce more than one variant of a good. Each variant is described by its quality and its horizontal characteristic. Variants are positioned at the corner points of a symmetric triangle which can be interpreted as the assumption of maximal differentiation. This allows us to represent variants and customers in

work on single-product firms which compete in a more than one-dimensional space (e.g. Neven and Thisse, 1990, Tabuchi, 1994, Vandenbosch and Weinberg, 1995, and Degryse, 1996).

the same figure although they are not defined in the same space.[5]

Firms.

The assumption on the product space implies that firm $j = 1, 2, 3$ can choose its products from the set $V_j = \{\emptyset, \{Lj\}, \{Hj\}, \{Lj, Hj\}\}$, where L is low quality and H is high quality while we assume that a firm cannot produce two high-quality variants. One interpretation for this assumption in our multi-stage game with sequential entry is that incumbent 1 only has a limited first-mover advantage: he can choose one variant or the vertical product line but is not able to choose more than one high-quality variant. Our interpretation is that a firm only has the resources to develop one high-quality variant at once because developing an alternative up-market variant requires additional human resources in R&D whereas downgrading a product does not. The assumption implies that there cannot arise a scenario in which one firm produces all three variants.[6]

Let $R_j \in V_j$ be the realized choice of firm j. It sets prices p_i, $i \in R_j$ such that it maximizes

$$\pi_j = \sum_{i \in R_j} [p_i - cq_i]\lambda_i - K_{R_j}$$

where $\lambda_i \in [0, 1]$ is the market share of variant i. For each variant of quality q_i there are unit costs cq_i.

[5]Degryse (1996) also assumed maximum differentiation in a two-dimensional product space. In simpler models of product differentiation firms do not always choose maximum differentiation in case of potential entry (e.g. Bonanno, 1987, and Canoy and Peitz, 1995b). However, for sufficiently small entry costs the location is not affected by entry consideration. However, the specification of linear transportation costs would lead to existence problems under endogenous location. See our conclusion on locational choice. A justification for maximum differentiation are indivisibilities of characteristics. It may be technically impossible to develop variants which are not at corner points. Lancaster (1979, p. 12) wrote: "Characteristics of products cannot always be varied continuously, and there are some characteristics that are inherently discontinuous. The specifications of many products may be defined partly by the presence or absence of features..." Also legal restrictions might lead to a fixed degree of differentiation (patent laws). For further justifications see Katz (1984).

[6]In our context it seems reasonable to assume that there are technological constraints that prevent one firm to become a monopolist. We opted for one particular interpretation. Another possible argument is that anti-trust authorities or regulators forbid a monopoly in the submarket of high-quality variants. Clearly, there exist markets in which the market structure resembles more one where a single-product low-quality producer competes against a multi-product high-quality producer. The analysis can easily be redone to study this alternative case. In the model, a horizontal product line is less interesting than the vertical product line because it always leads to equal equilibrium prices in the high-quality range and the comparative statics seem less interesting.

In the triangle a variant with quality $q = 0$ cannot be horizontally differentiated from a variant with the same quality. This captures the idea that in the low-quality segment of the market variants are very much the same in the eyes of a customer whereas in the high-quality segment horizontal differentiation makes a variant more valuable to some customers and less valuable to others. Hence the horizontal characteristic depends upon the quality level.
Variant i is described by two numbers $q_i \in [0,1]$ and $l_i \in [-\frac{1}{2}, \frac{1}{2}]$ where q_i denotes the quality of the variant and l_i the horizontal characteristic. Hence, variant L is described by $(0, l_i)$, $H1$ by $(1, -\frac{1}{2})$, and $H2$ by $(1, \frac{1}{2})$.

Customers.

Each customer buys one unit of one variant and nothing of the other variants. His indirect 'utility'[7] is given by

$$\begin{aligned} v_i &= r + \theta q_i + \theta(\frac{1}{2} - |\delta - l_i|)q_i - p_i \\ &= r + \theta(\frac{3}{2} - |\delta - l_i|)q_i - p_i \end{aligned}$$

where r is the willingness-to-pay for one unit of a variant of zero quality. Each type of customer is described by its taste parameter for quality θ and its taste parameter for horizontal specification δ. The taste parameter for quality is uniformly distributed over $[0, 1]$.[8]
By the assumption of a uniform distribution we do not put more weight, in terms of demand, to the high-quality range than to the low-quality range. We see our assumption as a benchmark case, i.e., in many markets one might even want to give more weight to the low-quality range. Among the customers those with a higher preference for quality are more sensitive to the horizontal specification.
We assume that customers with preference for quality θ are uniformly distributed over $[-\frac{1}{2}, \frac{1}{2}]$. Customers do not perceive horizontal differences to be important if they do not care much for quality, i.e., θ small. See Figure 1: customer A with type $(0, \frac{1}{2})$ is indifferent between $H1$ and $H2$ if $p_{H1} - p_{H2} = 0$. Positioned in the middle, she is not willing to pay more for one than for the other. She is indifferent between the low-quality variant L and the high-quality variant $H1$ if $p_{H1} - p_L = \frac{1}{2}$. Now consider the horizontal aspect of product differentiation. Customer

[7] To prove that the behavior is consistent with utility maximization under a budget constraint one has to introduce an outside option. Our specification of indirect utility is similar to Katz (1984).

[8] This contrasts Katz (1984) who assumes as many qualities in the market as there are different taste parameters. In his model with a finite number of types, submarkets of different qualities are linked but remain separate submarkets. The idea of one market which can be distinguished by its horizontal and its vertical aspect seems to us better captured by the present formulation.

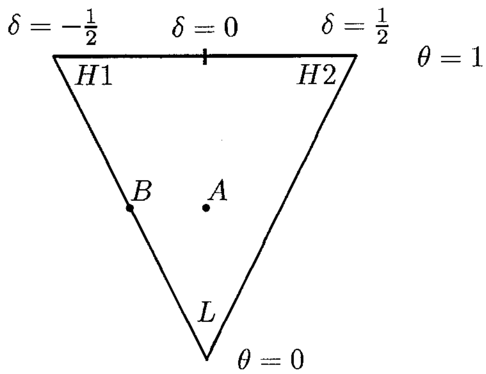

Figure 1: The Differentiation Triangle.

B with $(\theta\delta, \theta) = (-\frac{1}{4}, \frac{1}{2})$ is willing to pay an extra $\frac{1}{2}$ for quality. She is prepared to pay an extra $\frac{1}{4}$ in order to obtain her favorable variant $H1$. In other words, the horizontal distance to the middle is a measure of how much a customer wants to pay extra in order to obtain her favorable variant. Equally so, customer B is further away from $H2$ so that she is willing to pay $\frac{1}{4}$ less for brand $H2$. Therefore, B is indifferent between L and $H2$ if $p_{H2} - p_L = \frac{1}{4}$ and between $H1$ and L if $p_{H1} - p_L = \frac{3}{4}$. To determine when a customer wants to buy $H1$ rather than $H2$, only the horizontal position is of importance. According to the argument from above, it follows that B is indifferent between $H1$ and $H2$ if $p_{H1} - p_{H2} = \frac{1}{2}$. Customer B in Figure 1, when represented in the unit square, has coordinates $(\delta, \theta) = (-1/2, 1/2)$.
The description of customer behavior implies that the curves, which define the customers who are indifferent between two particular variants, are linear in the triangle (see Figure 2). They are given by the following equations:

$$p_{H2} - p_{H1} = 2\theta\delta \tag{1}$$

$$p_{H2} - p_L = \theta(1 + \delta) \tag{2}$$

$$p_{H1} - p_L = \theta(1 - \delta) \tag{3}$$

We will come back to these equations when we calculate the market shares for various specifications of the model.

The Oligopoly Game.

We will look at the following oligopoly situation. Firm 1 is the first to enter the market with the low-quality variant L, with high-quality variant $H1$, with both L and $H1$, or not at all. Next, firm 2 can enter the market with L, $H2$, L and $H2$ or not at all. In the third stage,

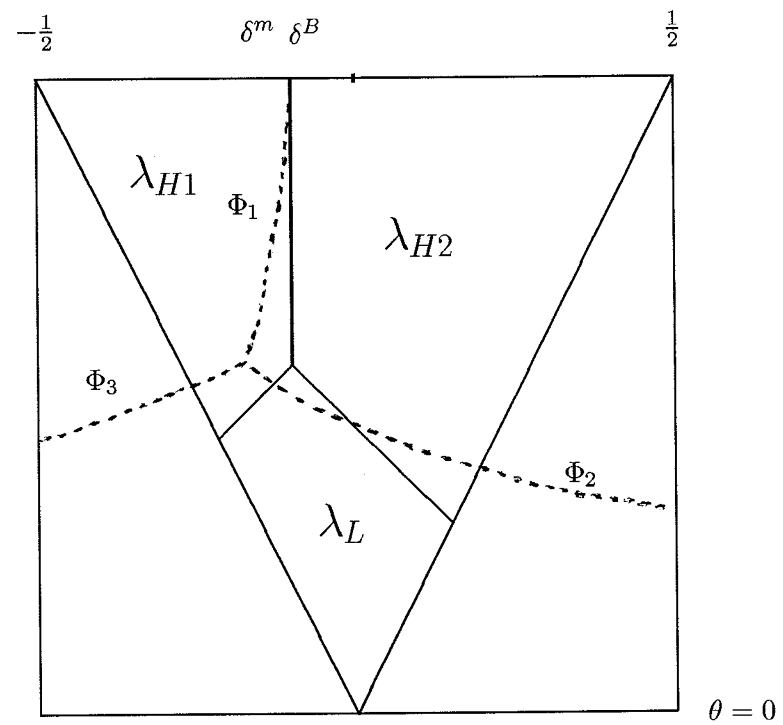

Figure 2: The Representation of the Share Functions.

there is a potential entrant waiting to enter the market at corner points, with a full vertical product line, or not at all.[9] In the fourth stage firms compete in prices. At this stage the fixed costs are sunk. The game structure can be summarized as follows:

Stage 1: Product choice of incumbent 1. Incumbent 1 develops $\{L\}$, $\{H1\}$, $\{L, H1\}$, or $\emptyset$ and incurs the associated fixed cost.

Stage 2: Product choice of incumbent 2. Incumbent 2 develops $\{L\}$, $\{H2\}$, $\{L, H2\}$, or $\emptyset$ and incurs the associated fixed cost.

Stage 3: Product choice of the potential entrant. The new firm develops $\{L\}$, $\{H1\}$, $\{H2\}$, $\{L, H1\}$, $\{L, H2\}$, or $\emptyset$ and incurs the associated fixed cost.[10]

Stage 4: Firms set prices simultaneously.

The fixed costs are assumed to satisfy $K_{\{L,H1\}} = K_{\{L,H2\}} \geq K_{\{H1\}} = K_{\{H2\}} \geq K_{\{L\}} \geq K_{\emptyset} = 0$ and $K_{\{L\}} + K_{\{H1\}} \geq K_{\{L,H1\}}$. This implies that there are weak economies of scope. Given the game structure the appropriate equilibrium concept is the subgame perfect Nash equilibrium. We consider only pure strategies.

At this point it is appropriate to compare our multi-stage game to the game analyzed by Katz (1984). As already mentioned, Katz was the first to introduce asymmetric customer behavior in a model of product differentiation. In his model prices and product choice are determined simultaneously. Due to the absence of fixed cost in Katz's model there may be fierce price competition in the low end of the market with several firms producing the vertical product line. In the presence of fixed costs this cannot happen. A model with fixed costs gives rise to asymmetric equilibria in a completely symmetric setting. In order to avoid existence problems one would need to study a two-stage game in which firms first choose products and then compete in prices. Our paper stresses the importance of product lines as entry deterring devices. For this we study a model with sequential product choice. This avoids the multiplicity of equilibria which usually arises in the above mentioned two-stage game and relates the product choice to incumbency. In addition, the introduction of fixed

[9]Including potential entry by a new firm in stage 3 helps to exclude situations in which no firm produces the low-quality variant in equilibrium. It is possible that fixed costs are such that in absence of potential entry, firms want to produce the high-quality variants only, but in the presence of additional potential entry one of them produces the low-quality variant as well in order to deter further entry. We did not incorporate exit into the model. We refer to Canoy and Peitz (1995b) in response to the critique by Judd (1985) which says that allowing for exit leads to single-product firms.

[10]Firms 1,2, and 3 essentially have the same choice set. As a matter of labeling, firm 1 chooses $H1$ if it chooses in the high-quality range and analogously for firm 2. Firm 3 can take the label which is not taken.

costs allows us to further analyze the trade-off between strategic and cost effects.

3 Results in Price Subgames

The objective is to determine the subgame perfect equilibria of the game. When entering stage 4 the firms have already developed their variants. The costs of this development are sunk. In this section we compute the price equilibria of the scenarios in which all corner points are occupied and which are candidates for a subgame perfect equilibrium. We will show in the next section that all other scenarios cannot occur in a subgame perfect equilibrium (see Result 1 below).

S_1: $\{H1, L\}, \{H2\}, \emptyset$,
S_2: $\{H1\}, \{H2, L\}, \emptyset$,
S_3: $\{H1\}, \{H2\}, \{L\}$,
S_4: $\{L\}, \{H2\}, \{H1\}$.

Each scenario contains the product choice of the firm in sequence 1,2,3. A remark on notation: we denote profits with the superscript of the scenario under consideration. Subscript 1 stands for incumbent 1, subscript 2 for incumbent 2 and firm 3, the potential entrant, has subscript 3.

3.1 Market Shares

For a price combination p_L, p_{H1}, p_{H2} such that all market shares are positive, only customers of one particular type are indifferent between the three variants which are offered. If one knows where in the triangle the marginal type is located one can calculate the market shares for the variants in the market.

The customers of type (δ^m, θ^m) are indifferent between the three variants. They are located at:

$$\theta^m = \frac{1}{2}(p_{H1} + p_{H2}) - p_L$$

$$\delta^m = \frac{\frac{1}{2}(p_{H2} - p_{H1})}{\frac{1}{2}(p_{H1} + p_{H2}) - p_L}$$

It will be more convenient to work with the unit square instead of the triangle because customers are uniformly distributed over the unit square and market shares can be expressed as areas in the unit square. The indifference curves (1) - (3) are non-linear in the (δ, θ)-space (see Figure 2).[11] Figure 2 represents the indifference curves when the marginal customer

[11] The assumption that customers are uniformly distributed over the triangle would facilitate the computations, but it would not distribute the mass of customers uniformly over the interval of possible

(δ^m, θ^m) is $(-0.2, 0.45)$.

Since we want to work in the (δ, θ)-space we solve the indifference curves for θ and define the following functions:

$$\Phi_1(p_{H1}, p_{H2}) \equiv \frac{p_{H2} - p_{H1}}{2\delta} \tag{4}$$

$$\Phi_2(p_{H2}, p_L) \equiv \frac{p_{H2} - p_L}{1+\delta} \tag{5}$$

$$\Phi_3(p_{H1}, p_L) \equiv \frac{p_{H1} - p_L}{1-\delta} \tag{6}$$

For (δ^m, θ^m) in the interior of the support of the customer density ($\delta^m \in (-\frac{1}{2}, \frac{1}{2})$ and $\theta^m \in (0,1)$), we calculate the market shares of the variants.

We define δ^B (B for bound) as the δ such that $\Phi_1 = 1$ which implies $\delta^B = (p_{H2} - p_{H1})/2$. The $H1$-share is equal to the sum of the integrals of $1 - \Phi_1$ and $1 - \Phi_3$ between the appropriate bounds. Analogously, we obtain the $H2$-share.

$$\begin{aligned}
\lambda_{H1} &= \int_{-\frac{1}{2}}^{\delta^m} (1 - \Phi_3) d\delta + \int_{\delta^m}^{\delta^B} (1 - \Phi_1) d\delta \\
\lambda_{H2} &= \int_{\delta^m}^{\frac{1}{2}} (1 - \Phi_2) d\delta - \int_{\delta^m}^{\delta^B} (1 - \Phi_1) d\delta \\
\lambda_L &= 1 - \lambda_{H1} - \lambda_{H2}
\end{aligned} \tag{7}$$

The evaluated shares become

$$\lambda_{H1}|(\delta^m \neq 0) = \frac{1}{2} + \log[\frac{2}{3}(1 - \delta^m)](p_{H1} - p_L) + \delta^B[1 + \log\frac{\delta^m}{\delta^B}] \tag{8}$$

$$\lambda_{H2}|(\delta^m \neq 0) = \frac{1}{2} + \log[\frac{2}{3}(1 + \delta^m)](p_{H2} - p_L) - \delta^B[1 + \log\frac{\delta^m}{\delta^B}] \tag{9}$$

$$\lambda_{H1}|(\delta^m = 0) = \frac{1}{2} + \log\frac{2}{3}(p_{H1} - p_L) \tag{10}$$

$$\lambda_{H2}|(\delta^m = 0) = \frac{1}{2} + \log\frac{2}{3}(p_{H2} - p_L) \tag{11}$$

3.2 Price Equilibria

In this subsection we compute the price equilibria under various scenarios.

Scenario S_1

We start with scenario S_1 where incumbent 1 produces variants of high and low quality and incumbent 2 chooses the other high-quality variant. We do not need to analyze scenario S_2

qualities. We do not want to introduce such an asymmetry which favors variants in the high-quality segment.

Scenario	p_{H1}	p_{H2}	p_L	λ_{H1} (in %)	λ_{H2} (in %)	λ_L (in %)	Π_{I1}	Π_{I2}	Π_E
S_1	0.343	0.302	0.207	40.29	50.67	9.04	0.157	0.153	-
S_2	0.302	0.343	0.207	50.67	40.29	9.04	0.153	0.157	-
S_3	0.332	0.332	0.166	43.27	43.27	13.46	0.144	0.144	0.022
S_4	0.332	0.332	0.166	43.27	43.27	13.46	0.022	0.144	0.144

Table 1: Price Equilibria of Scenarios S_1-S_4.

separately because the results are the same except for changing the indices. In the price subgame both firms maximize profits Π (at stage 4 fixed costs are sunk). Profits before deduction of the fixed costs will be denoted by Π, whereas π is profit after deduction of fixed costs.

$$\begin{aligned} \Pi_1(p_{H1}, p_{H2}, p_L) &= (p_{H1} - c)\lambda_{H1}(p_{H1}, p_{H2}, p_L) + p_L\lambda_L(p_{H1}, p_{H2}, p_L) \\ \Pi_2(p_{H1}, p_{H2}, p_L) &= (p_{H2} - c)\lambda_{H2}(p_{H1}, p_{H2}, p_L) \end{aligned}$$

Most interesting results already occur if we restrict attention to the case where $c = 0$, so that we concentrate on $c = 0$. Note that setting $c = 0$ is not a matter of "normalization" because $c > 0$ reflects the relative advantage of low-quality variants versus high-quality varaints in variable costs. For details see Section 5.2. For $c = 0$, the cost difference between high- and low-quality variants lies only in the investment costs and not in the variable production costs. Appendix 1 contains the first-order conditions of profit maximization. Appendix 2 outlines how we determine the equilibrium. Equilibrium values for the variables are presented in the first and second line of Table 1. As expected, the price of the high-quality variant of the firm developing the product line is above the price of its single-product competitor. Total output of the single-product firm is higher than the output of the multi-product firm.

The following changes occur when the marginal cost c is strictly positive:

(1) Absolute price differences of the high-quality variants are increasing in c. The intuition is: the higher c is, the smaller the market shares for the high-quality variants. By increasing its mark-up in the high-quality segment, firm 1 can exploit its low-quality variant, i.e., by raising the price for its high-quality variant at a high c it loses less customers than it would do at a low c. Notice that relative price differences do not matter, as can be seen from equations (4) to (6). For $c > 0.05$, the single-product firm serves less than half the market.

(2) The profit of firm 2 is first increasing and then decreasing in c. For small c, the low-quality variant does not play a big role, while the share of $H1$ is decreasing faster than the share

of $H2$. Competition between high- and low-quality variants is relaxed and firm 2's profit increases. As c increases further, firm 2's disadvantage of not producing the low-quality variant becomes more important so that its profits eventually fall. Its profits are maximal at c around 0.4.

(3) The profit of firm 1 is increasing in c. As c increases, firm 1 can exploit its comparative advantage of producing the low-quality variant more and more. A high c gives firm 1 a lot of monopoly power on the customers who are not interested in quality. Markups for both products are increasing in c.

(4) For $c \geq 1.25$, θ^m greater than 1, i.e., the high-quality variants are no longer competing with each other.[12] At this cost the high-quality variant of the multi-product firm is purchased by around 2 % of the customers (the other high-quality variant has a market share of 28 %). It is the introduction of a *low-quality* variant which separates the high-quality variants from each other. With a vertical product line it means that the high-quality variant of the multi-product firm is used to imperfectly price-discriminate between the customers who buy from this firm. Equilibrium profits are slightly higher than profits which would occur if the multi-product-firm was only offering its low-quality variant and one high-quality corner was left empty.

Scenario S_3

Under this scenario there will be three single-product firms. The price game is essentially the same in scenario S_4 apart from firms choosing different corner points.

The potential entrant chooses the low-quality variant in stage 3. When firms compete in prices they maximize their profits which are given by

$$\begin{aligned} \Pi_{H1}(p_{H1}, p_{H2}, p_L) &= (p_{H1} - c)\lambda_{H1}(p_{H1}, p_{H2}, p_L) \\ \Pi_{H2}(p_{H1}, p_{H2}, p_L) &= (p_{H2} - c)\lambda_{H2}(p_{H1}, p_{H2}, p_L) \\ \Pi_L(p_{H1}, p_{H2}, p_L) &= p_L\lambda_L(p_{H1}, p_{H2}, p_L) \end{aligned}$$

where the shares are as in equations (7) to (11). On the solution of the first-order conditions we again refer to Appendices 1 and 2. The solution for $c = 0$ is reported in Table 1. The equilibrium is symmetric, i.e., $p_{H1} = p_{H2}$.

Looking at $c > 0$ the following can be observed: Profits of firm 3 increase in c. Firm 1 and firm 2's profits also increase in c when c is small. This effect occurs because price competition between high- and low-quality variants is relaxed: The profit gain from relaxed price

[12]For $c < 1.25$ there is a set of marginal customers for each pair of variants, whereas for $c \geq 1.25$ there are no marginal customers between $H1$ and $H2$. Computations for this case and the duopoly below are also found in Canoy and Peitz (1995a).

competition dominates the increase in costs for the high-quality firms when c is small.

4 Perfect Equilibrium in the Sequential Entry-then-Price Game

We want to find out which of the subgames can be reached in a subgame perfect equilibrium. First we argue that only four scenarios are candidates for a subgame perfect equilibrium when all corner points are occupied. Then we argue that for fixed costs sufficiently small all corner points will be occupied. The main result shows how fixed costs affect the product choice. We only look at the case where $c = 0$. For $c > 0$ and adequate fixed costs, it still holds that only one of the four scenarios can occur in the unique subgame perfect equilibrium of the extensive game (see Section 5.2).

Result 1.
Only one of the following four scenarios can occur in any subgame perfect equilibrium such that all corner points are occupied:
S_1 $\{H1, L\}, \{H2\}, \emptyset$,
S_2 $\{H1\}, \{H2, L\}, \emptyset$,
S_3 $\{H1\}, \{H2\}, \{L\}$,
S_4 $\{L\}, \{H2\}, \{H1\}$.

Proof. Since firms have the same cost structure *ex ante*, all other combinations can be ruled out because of the fixed sequence of moves. This is trivial for all scenarios but $\{H1\}, \{L\}, \{H2\}$. This scenario cannot occur because incumbent 1 prefers high quality to low quality in a single-product oligopoly but so must incumbent 2. The choice of incumbent 2 is not optimal because she could have chosen $\{H2\}$ leaving $\{L\}$ to the entrant. □

The numerical solutions give us a clear picture of the equilibrium strategies used by the firms. No qualitative result seems to be sensitive to the chosen values of the parameters. Therefore, we feel comfortable to present the outcomes as *results.*
With Result 2 we state that for fixed costs sufficiently small the monopoly and single-product duopoly solutions are not 'stable' under potential entry. Hence we exclude the monopoly and single-product duopoly outcomes as a subgame perfect equilibrium of the total game.

Result 2.
There exist upper bounds for fixed costs such that for fixed costs smaller than these bounds all corner points will be occupied in subgame perfect equilibrium. Hence, neither a monopoly nor a single-product duopoly will arise.

Proof. The upper bounds follow from Table 1. If e.g. $K_{\{H1\}} < 0.144$ and $K_{\{L\}} < 0.022$ incumbents will at least occupy two corner points and the potential entrant will occupy any single corner which has not been occupied by the incumbents. □

A more detailed analysis on the bounds of the fixed costs is found in Canoy and Peitz (1995a). There we also analyze the single-product duopolies. Result 2 is not expressed in terms of the *strength* of the economies of scope although it is relevant: if the economies of scope are very strong, e.g. $K_{\{Hj,L\}} = K_{\{Hj\}}$, it is always profitable to develop the vertical product line if it is profitable to enter the market at all. But note that, for a large range of parameter values with fixed costs and fixed costs difference $K_{\{H1\}} - K_{\{L\}}$ high, a single-product duopoly would arise in the subgame perfect equilibrium. Assuming that fixed costs are sufficiently small also puts bounds on the fixed costs differences.
We state our Main Result in terms of the parameters $K_{\{H1\}}$, $K_{\{L\}}$, and $K_{\{H1,L\}}$. There exists a unique outcome which is either S_1 when incumbent 1 develops the vertical product line or S_4 when incumbent 1 develops the low-quality variant only, or incumbent 1 develops the high-quality variant in S_2 or S_3. In S_2 incumbent 2 deters entry by developing the vertical product line and in S_3 she accommodates entry.

Main Result.
Let fixed costs be sufficiently small, such that all variants are produced in equilibrium. There exists a unique subgame perfect equilibrium which is characterized by a scenario and the associated price equilibrium.
(A) If $K_{\{H1,L\}} - K_{\{H1\}} \leq 0.013$ then
scenario S_1 occurs if $K_{\{H1,L\}} - K_{\{H1\}} \leq 0.04$ and
scenario S_2 occurs if $0.04 < K_{\{H1,L\}} - K_{\{H1\}} \leq 0.013$.
(B) If $K_{\{H1,L\}} - K_{\{H1\}} > 0.013$ then
scenario S_3 occurs if $K_{\{H1\}} - K_{\{L\}} \leq 0.122$ and
scenario S_4 occurs if $K_{\{H1\}} - K_{\{L\}} > 0.122$.

Proof. In each case incumbent 1 has three choices: $\{H1, L\}$, $\{H1\}$ and $\{L\}$. We have to look at profits in the equilibrium of the subgame of stage 4 (see Table 1).

Case (A). Incumbent 2's best response to $\{H1\}$ is $\{H2, L\}$. This implies that $\{H2, L\}$ gives a higher profit than the outcome with three single-product firms that would occur if incumbent 2 responded with $\{L\}$ or $\{H2\}$. Because of symmetry between incumbent 1 and incumbent 2 this implies that the single-product oligopoly cannot occur. Incumbent 1, as the first mover, simply compares profits under scenario S_1 and S_2 and acts accordingly. If scenario S_1 gives him the highest profit he will produce $\{H1, L\}$. If scenario S_2 gives him the highest profit he will leave $\{H2, L\}$ to incumbent 2.
Case S_3. Because of the symmetry of the fixed costs incumbent 1 will never develop $\{H1, L\}$. In addition, he will not develop $\{L\}$. Hence incumbent 1 develops $\{H1\}$ and scenario S_3 emerges.
Case S_4. Since incumbent 2 develops $\{L\}$ in response to $\{H1\}$, the product choice $\{H1\}$, $\{L\}$, $\{H2\}$ will occur if incumbent 1 develops $\{H1\}$. If incumbent 1 develops $\{L\}$ in stage 1 scenario S_4 emerges. Because of symmetry $\{H1\}, \{L\}, \{H2\}$ cannot occur (see Result 1). Also scenario S_1 cannot occur. Consequently, scenario S_4 emerges in the subgame perfect equilibrium of the total game. □

As will become apparent in Subsection 5.2, there exists a non-empty set of parameters for each scenario such that this scenario occurs in the unique subgame perfect equilibrium. In the next section we discuss the Main Result in detail and consider some extensions.

5 Discussion and Extensions

In this section we want to discuss our results in more detail. First, we provide an intuition for the Main Result. Second, we analyze the effect of cost parameters on the market structure where we allow for positive marginal costs of production of the high-quality variants.

5.1 Discussion of the Main Result

The intuition for the Main Result is the following. It depends on the cost structure whether incumbent 1 wants to produce the vertical product line himself or whether he wants to choose only one variant. The incumbency advantage is a real first-mover advantage here. By developing the variants in stage 1 incumbent 1 can guarantee himself profits at least as high as incumbents 2's profits. Incumbent 1 chooses his strategy by using backward induction. If it is more profitable to develop the vertical product line incumbent 1 develops $\{H1, L\}$. If it is more profitable to develop only the low-quality variant he develops $\{L\}$. If neither is the case, it depends on incumbent 2. If incumbent 2 finds it in her interest to deter entry by brand proliferation she develops $\{H2, L\}$. But it is also possible that entry by the third firm is accommodated in the equilibrium. To explain this, one should distinguish between cost and strategic effects. The cost effect favors entry deterrence because incumbent 2 can exploit economies of scope. There are several consequences which are due to strategic effects. Compare $\{H1\}, \{H2, L\}$ (scenario S_2) with $\{H1\}, \{H2\}, \{L\}$ (scenario S_3). From Table 1 it can be observed that firm 2 is more aggressive in S_3 for the high-quality variant, while firm 3 is more aggressive in S_3 than firm 2 in S_2 for the low-quality variant. These consequences are in favor of scenario S_2. However, firm 1 is more aggressive in S_2 than in S_3 ($p_{H1} = 0.302$ in S_2 and $p_{H1} = 0.332$ in S_3). It turns out to be possible that the latter effect outweighs the other effects. For this to hold, economies of scope must be quite weak (so that the cost effect becomes relatively unimportant). Note that it is profitable for firm 1 to be more aggressive in S_2 than in S_3 because it faces only one competitor who in turn reacts less aggressive than two single-product competitors.

Summarizing, even when we restrict marginal costs being independent of quality, four effects have to be considered: there are two types of cost effects (levels of fixed costs and economies of scope) and two types of strategic effects (strategic pricing and strategic product choice which possibly leads to entry deterrence). By analyzing the interaction of these effects we formalize what is often called a *niche strategy*. Whether or not a particular firm chooses a niche strategy depends on the identity of the firm and on the relevant trade-off between cost and strategic effects. Incumbent 1 can go for a niche strategy for two reasons, either because it is a profitable niche and entry accommodation is better than deterrence or because it is a profitable niche and incumbent 2 deters entry. Incumbent 2 chooses the niche either because entry deterrence is not worthwhile compared to entry accommodation when incumbent 1 also has chosen a niche or because it is the only option left in the market. When the entrant enters, he enters in the remaining niche which is the only choice left.

5.2 Technology and Market Structure

Finally, we want to focus on the impact of the cost structure on market structure and outcome. Apart form fixed cost effects we allow for marginal costs which increase in quality. Figure 3 shows that, for $c = 0$, each of the four scenarios can occur when there are no economies of scope, i.e., $K_{\{H1,L\}} = K_{\{H1\}} + K_{\{L\}}$. For instance, if the difference in fixed cost is sufficiently large scenario S_4 occurs in which incumbent 1 only produces $\{L\}$.

If there are economies of scope the picture looks similar to Figure 3 with the bounds between scenarios S_1 and S_2 and between S_2 and S_3 moving up and the area of scenario S_4 becoming

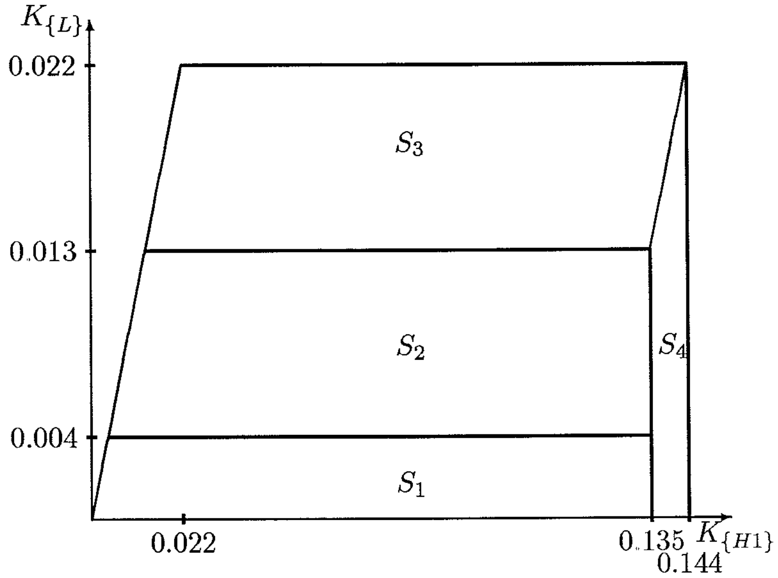

Figure 3: Equilibrium Scenarios when $c = 0$ and $K_{\{H1,L\}} = K_{\{H1\}} + K_{\{L\}}$.

smaller. For $K_{\{H1,L\}} = K_{\{H1\}}$ either scenario S_1 or S_4 will occur. Consequently, if economies of scope are sufficiently large scenarios S_2 and S_3 will disappear.

Let us assume now that there are no economies of scope, i.e., $K_{\{H1,L\}} = K_{\{H1\}} + K_{\{L\}}$ and the choice of each variant entails the same fixed cost $K \equiv K_{\{H1\}} = K_{\{L\}}$. The variable cost of quality c and the magnitude of the fixed costs K are the only free parameters. The unique outcome of the game is represented by Figure 4. To the right of the boundary for scenario S_3 the assumption on fixed costs is not satisfied. The figure shows that despite the absence of economies of scope the single-product outcome is hardly on. Only in a small area scenario S_3 occurs and scenario S_4 does not occur at all.

For $c = 0$ it is relatively unfavorable to produce two variants, since price competition is relatively fierce. Single-product oligopolies cannot occur for $c > 0.1$ because the profits of the firm with the vertical product line in the duopoly are greater than the sum of profits of one high-quality firm and the low-quality firm in the single-product oligopoly. In other words, the difference in profits in choosing $\{H1, L\}$ and choosing $\{H1\}$ increases faster in c than Π_L in scenario S_3. Since we assumed weak economies of scope neither S_3 nor S_4 can emerge for $c > 0.1$.

When parameters are such that S_4 emerges at $c = 0$ we obtain a surprising comparative statics result. It shows that strategic effects can dominate cost effects.

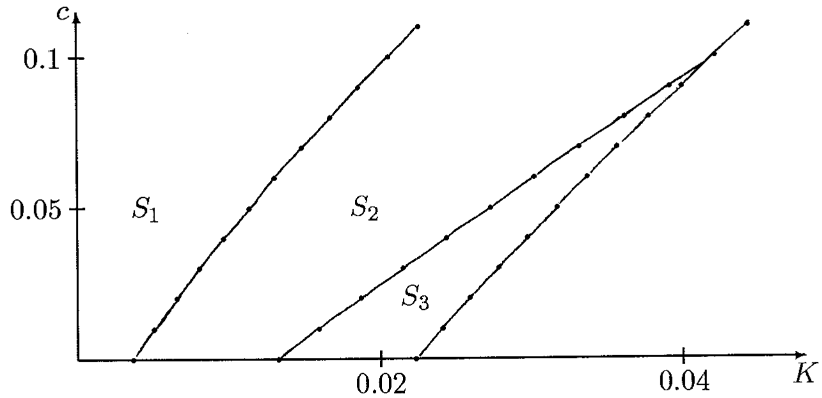

Figure 4: Equilibrium Scenarios when $K \equiv K_{\{H1\}} = K_{\{L\}}$ and $K_{\{H1,L\}} = K_{\{H1\}} + K_{\{L\}}$

Result 3.
Increasing c, while keeping the other parameters fixed can lead to a shift in strategy for incumbent 1 from low quality to multi-products despite the fact that high quality has become more expensive.

Sketch of the proof. It follows from Figure 3, that there are parameter constellations such that S_4 emerges at $c = 0$. Assume that under the parameter constellation, results 1 and 2 hold for some $c > 0.1$. For $c > 0.1$, single-product oligopolies do not emerge and each incumbent produces a high-quality variant. Since under the parameter constellation low-quality is very profitable, incumbent 1 also produces the low-quality variant. For instance, $K_{\{L\}} = 0.015$, $K_{\{H1\}} = 0.14$, $K_{\{H1,L\}} = 0.154$ is such a parameter constellation which satisfies the assumption for $c < 0.25$. □

For $c \geq 0.5$, also scenario S_2 cannot emerge. Consequently, for sufficiently large marginal costs of quality scenario S_1 is the unique outcome of the game for all admissible parameter constellations. This shows again that setting $c = 0$ is a restrictive assumption.

6 Conclusion

The paper was motivated by the observation that customers' evaluation in the high-quality range of a product is often more dispersed than in the low-quality range. The example of the car market suited well to exemplify our point. One can think of other examples such as machines, office equipment and particular food submarkets which are dominated in the high-quality end by few well-established brand names. These markets all share the feature that the low-quality variants are often purchased by people who just want a low-priced product of some generic (low) quality. For the high-quality variants, on the other hand, customers are much more sensitive to the specific characteristics of the variant they buy. Some of these people highly value Bosch versus Black&Decker drilling machines, Parker versus Sheaffer pens, Coke versus Pepsi, or Danone versus Yoplait yoghurts.

In addition to the above observation, there is the observation that firms can - and often do - develop both high- and low-quality variants of a good. The following questions emerged from this:

(1) Do firms develop a product line or do they follow a niche strategy?

(2) If a firm develops a product line what is the motivation for such a strategy?

(3) Which prices will be realized in equilibrium under various scenarios?

In the paper we gave the following answers to these questions:

On question 1: incumbent 1 develops the vertical product line if it is profitable to do so (which is more favorable if the economies of scope are very strong). He then achieves a higher profit than incumbent 2, who follows a niche strategy. If the low-quality variant is very favorable, he only produces the low-quality variant and a three-firm oligopoly emerges. Otherwise, incumbent 1 follows a niche strategy in the high-quality segment, while incumbent 2 either develops the vertical product line or the other high-quality variant, i.e., she accommodates entry.

On question 2: for incumbent 1 the motivation is different from that of incumbent 2. Incumbent 1 develops a low-quality in addition to his high-quality variant, simply because that gives him a higher profit than by letting an additional firm produce the low-quality variant. Since he has a first-mover advantage in the product choice, he chooses his most profitable set of products. On the other hand, incumbent 2 observes incumbent 1's product choice and develops the vertical product line when incumbent 1 followed a niche strategy and entry deterrence is profitable. Hence brand proliferation is an entry deterring strategy for incumbent 2 but not for incumbent 1.

On question 3: the firm that develops the vertical product line always sets a higher price for its high-quality variant than the firm who follows a niche strategy. The intuition is that a firm does not want to compete too fiercely with its own low-quality variant.

These results show that incorporating multi-product possibilities in a two-dimensional product differentiation model, does not only capture empirically relevant issues but also gives us some qualitative insights into the choice of product lines versus niches in an imperfectly competitive environment with entry: we can determine conditions under which strategic effects dominate cost effects. This contrasts the literature which ignores strategic effects and it also contrasts the literature on two-dimensional product differentiation with single-product firms. Our analysis can be extended in several ways. In Canoy and Peitz (1995a) we analyze the model of the unit square and a generalized version of the triangle in which the relative importance of horizontal with respect to vertical product differentiation is varied. The former is important to study separately. The latter allows for comparative statics results on the role of the extent of horizontal differentiation.

In this paper we did not investigate the locational choices of the firms in the characteristics space. We simply assumed maximum differentiation and singled out the strategic effects of the product choice. It seems worthwhile to try to model the differentiation triangle with the possibility to locate a variant in a two-dimensional product space with the possibility of continuous changes of the product choice. The specification with quadratic disutility in distance for the horizontal characteristic seems to be a promising candidate.

To allow for some horizontal product differentiation for low-quality variants one should analyze a trapezoid (BrE trapezium) in which also low-quality variants are to some extent horizontally differentiated. For given high-end horizontal product differentiation β and for given fixed costs, one would like to determine the critical low-end differentiation α which separates say $\{H1, L\}, \{H2\}$ from $\{H1, L1\}, \{H2, L2\}$ as the equilibrium product choice. Alternatively, one may want to add some noise into the utility function so that price competition in one point does not completely destroy profits. This can result in more than one firm producing the vertical product line in spite of the asymmetry of the different horizontal markets, and entry deterrence is made more difficult.

On the empirical side we would like to see further work which evaluates the impact of quality aspects of goods on variety. For instance, knowledge whether higher income makes consumers less price sensitive to variety would allow for comparative statics results which would be important for the prediction of future market structures.

Appendix 1: First-Order Conditions of Profit Maximization.

(1) $\{H1, L\}, \{H2\}$.

The first-order conditions for the case $\theta^m \in (0,1)$, $\delta^m \in (-\frac{1}{2}, \frac{1}{2})$ are

$$
\begin{aligned}
\lambda_L + p_L \log[\frac{4}{9}(1+\delta^m)(1-\delta^m)] + (p_{H1} - c)\log\left[\frac{3}{2}\frac{1}{1-\delta^m}\right] &= 0 \\
\lambda_{H1} + (p_{H1} - c)\log\left[\frac{2}{3}(1-\delta^m)\left(\frac{p_{H1}+p_{H2}-2p_L}{2}\right)^{\frac{1}{2}}\right] & \\
+p_L \log\left[\frac{3}{2}\frac{1}{1-\delta^m}\right] &= 0 \\
\lambda_{H2} + (p_{H2} - c)\log\left[\frac{2}{3}(1+\delta^m)\left(\frac{p_{H1}+p_{H2}-2p_L}{2}\right)^{\frac{1}{2}}\right] &= 0
\end{aligned}
$$

Analogously for $\{H1\}, \{H2, L\}$.

(2) $\{H1\}, \{H2\}, \{L\}$ and $\{L\}, \{H2\}, \{H1\}$.

The first-order conditions for the case $\theta^m \in (0,1)$, $\delta^m \in (-\frac{1}{2}, \frac{1}{2})$ are

$$
\begin{aligned}
\lambda_L + p_L \log[\frac{4}{9}(1+\delta^m)(1-\delta^m)] &= 0 \\
\lambda_{H1} + (p_{H1} - c)\log\left[\frac{2}{3}(1-\delta^m)\left(\frac{p_{H1}+p_{H2}-2p_L}{2}\right)^{\frac{1}{2}}\right] &= 0 \\
\lambda_{H2} + (p_{H2} - c)\log\left[\frac{2}{3}(1+\delta^m)\left(\frac{p_{H1}+p_{H2}-2p_L}{2}\right)^{\frac{1}{2}}\right] &= 0
\end{aligned}
$$

Appendix 2: Description of the Numerical Methods. Even our "simple" model turns out to be too complicated to obtain algebraic solutions. This feature is endemic: if we want to incorporate the empirically relevant asymmetry between high- and low-quality variants, we are faced with non-linear shares. As a consequence, we had to use numerical methods to obtain a solution to the first-order conditions. Of course, uniqueness cannot be formally proved with such a method.

Unique equilibrium candidate. The equation systems were solved with the aid of the software package *Mathematica* using Newton's method and a variant of the secant method with *FindRoot.* We started the algorithms with different initial values at different marginal cost c and we always found exactly one admissible solution. Programs are available from the authors upon request.

A remark on the three-firm cases: in order to avoid numerical problems we introduced a small cost asymmetry between the two high-quality variants. This enabled us to use the first-order conditions from Appendix 1. We then made this cost difference turn towards zero to obtain the results for our model. The results were the same as solving for the first-order conditions while assuming $p_{H1} = p_{H2}$.

Equilibrium Existence. We checked the second-order conditions at the equilibrium candidate showing that we have found a local maximum. This means in particular that in scenario S_1

$$\left| \begin{array}{cc} \frac{\partial^2 \Pi_1}{\partial p_{H1}^2} & \frac{\partial^2 \Pi_1}{\partial p_{H1} \partial p_L} \\ \frac{\partial^2 \Pi_1}{\partial p_L \partial p_{H1}} & \frac{\partial^2 \Pi_1}{\partial p_L^2} \end{array} \right|_{p=p^*} < 0.$$

We checked that we have found a global maximum for each firm given the prices of the competitors. Note that market shares in equations (7)-(11) are only valid on the set of prices $\{(p_{H1}, p_{H2}, p_L) \in \Re_+^3 | \delta^m \in (-\frac{1}{2}, \frac{1}{2}), \theta^m \in (0,1)\}$. Outside this set we had to use different share functions, which are easily computed. In a given scenario the profit function of a firm only depends on the price(s) of this firm when the price(s) of the competitor(s) are fixed at its(their) equilibrium value(s). By an abuse of notation $\delta^m, \delta^B, \Phi_1, \Phi_2, \Phi_3$ are also only functions in the price(s) of this firm.

Consider scenario S_1 with $c = 0$. The profit function for incumbent 1, who produces two variants, is

$$\Pi_1 = \begin{cases} p_{H1} \int_{-\frac{1}{2}}^{\delta^B} (1 - \Phi_1) d\delta & 1.302 > p_{H1} > 0.302 \text{ and } p_L \geq 0.302 \\ \frac{1}{2} p_{H1} & p_L \geq p_{H1} = 0.302 \\ p_{H1} \left(1 - \int_{\delta^B}^{\frac{1}{2}} (1 - \Phi_1) d\delta\right) & p_L \geq p_{H1} \text{ and } 0 < p_{H1} < 0.302 \\ p_L \int_{-\frac{1}{2}}^{\frac{1}{2}} \Phi_2 d\delta & p_{H1} \geq 1.302 \text{ and } 0 < p_l < 0.302 \\ p_L \int_{-\frac{1}{2}}^{\frac{1}{2}} \Phi_2 d\delta + p_{H1} \int_{-\frac{1}{2}}^{\delta^B} (1 - \Phi_1) d\delta & p_{H1} < 1.302, p_L > 0, \text{ and } \frac{1}{4} p_{H1} \geq p_L + 0.226 \\ p_L \left(\int_{-\frac{1}{2}}^{\delta^m} \Phi_3 d\delta + \int_{\delta^m}^{\frac{1}{2}} \Phi_2 d\delta \right) + p_{H1} \left(\int_{-\frac{1}{2}}^{\delta^m} (1 - \Phi_3) d\delta - \int_{\delta^B}^{\delta^m} (1 - \Phi_1) d\delta \right) & p_L < p_{H1}, \frac{1}{4} p_{H1} < p_L + 0.226, \text{ and } \frac{3}{4} p_{H1} > \frac{1}{2} p_L + 0.075 \\ p_L \int_{-\frac{1}{2}}^{\frac{1}{2}} \Phi_3 d\delta + p_{H1} \left(\int_{-\frac{1}{2}}^{\frac{1}{2}} (1 - \Phi_3) d\delta - \int_{\delta^B}^{\frac{1}{2}} (1 - \Phi_1) d\delta \right) & p_L < p_{H1} < 0.302 \text{ and } \frac{3}{4} p_{H1} \leq \frac{1}{2} p_L + 0.075 \\ 0 & p_{H1} = 0 \text{ or } (p_{H1} \geq 1.302 \text{ and } (p_L = 0 \text{ or } p_L \geq 0.302)) \end{cases}$$

Incumbent 2 has the following profit function:

$$\Pi_2 = \begin{cases} p_{H2}\left(1 - \int_{-\frac{1}{2}}^{\delta^B}(1-\Phi_1)d\delta\right) & 0 < p_{H2} \leq 0.207 \\ p_{H2}\left(-\int_{-\frac{1}{2}}^{\delta^B}(1-\Phi_1)d\delta - \int_{-\frac{1}{2}}^{\frac{1}{2}}(1-\Phi_2)d\delta\right) & 0.207 < p_{H2} \leq 0.252 \\ p_{H2}\left(-\int_{\delta^m}^{\delta^B}(1-\Phi_1)d\delta - \int_{\delta^m}^{\frac{1}{2}}\Phi_2 d\delta\right) & 0.252 < p_{H2} < 0.615 \\ p_{H2}\left(\int_{\delta^B}^{\frac{1}{2}}(1-\Phi_1)d\delta\right) & 0.615 \leq 1.343 \\ 0 & p_{H2} \geq 1.343 \text{ or } p_{H2} = 0 \end{cases}$$

In scenario S_3, $\delta^m = 0$. The profit function of the low-quality producer has a market share according to the functions in the text because $\theta \in [0,1)$ for $p_L \geq 0$. So we only need to write down the profit functions of the high-quality producers. Since for the equilibrium candidate $p_{H1} = p_{H2}$, the situation is symmetric for the two high-quality producers and we only need to consider $I2$. It turns out that one only has to modify the piecewise defined profit function for $I2$ under S_1. Incumbent 2 has the following profit function

$$\Pi_2 = \begin{cases} p_{H2}\left(1 - \int_{-\frac{1}{2}}^{\delta^B}(1-\Phi_1)d\delta\right) & 0 < p_{H2} \leq 0.166 \\ p_{H2}\left(-\int_{-\frac{1}{2}}^{\delta^B}(1-\Phi_1)d\delta - \int_{-\frac{1}{2}}^{\frac{1}{2}}(1-\Phi_2)d\delta\right) & 0.166 < p_{H2} \leq 0.221 \\ p_{H2}\left(-\int_{\delta^m}^{\delta^B}(1-\Phi_1)d\delta - \int_{\delta^m}^{\frac{1}{2}}\Phi_2 d\delta\right) & 0.221 < p_{H2} < 0.664 \\ p_{H2}\left(\int_{\delta^B}^{\frac{1}{2}}(1-\Phi_1)d\delta\right) & 0.664 \leq 1.332 \\ 0 & p_{H2} \geq 1.332 \text{ or } p_{H2} = 0 \end{cases}$$

We did not write down profit functions which hold for all c because for high c they are of a different form. Our numerical analysis has shown that profits which follow from deviations from the equilibrium candidate, are always dominated. Hence, we have found an equilibrium.

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