EQUAL-LOSS SOLUTION FOR MONOTONIC COALITIONAL GAMES*

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A B S T R A C T

A new solution concept to monotonic cooperative games with nontransferable utility is introduced. This proposal, called the coalitional equal-loss solution, is based on the idea that players within a coalition should have equal losses from a point of maximum expectations. The proposal generalizes the rational equal-loss solution defined on the subclass of bargaining problems as well as the Shapley value defined on the subclass of superadditive cooperative games with transferable utility.

KEYWORDS: Coalitional Games; Rational Equal-Loss Solution; Shapley Value.

1. INTRODUCTION

The most general formulation of multi-person cooperative situations has been modeled by means of coalitional games, also known as games without sidepayments or nontransferable utility games. Such games are described by a set of players and a set of feasible outcomes for each subset of players. In these games any coalition may have some weight, and the players may not be able to make sidepayments to each other in such a way that the total utility gains are equal to the total utility losses.

Two subclasses of coalitional games have been extensively studied, and several solutions have been suggested for them. Nash's solution (1950) is the first of such subclasses, called the n-person bargaining problems, in which utility is not necessarily transferable, but solution concepts are thought of as a unanimous compromise among participants, so that when there are more than two agents, intermediate coalitions do not play any role. The second subclass, called n-person transferable utility games, started with the Shapley (1953) value; in these games any coalition may have some weight, although the utility is transferable among players.

Most of the solutions introduced for coalitional games are extensions of, at the same time, both a solution for n-person bargaining problems and a solution for n-person transferable utility games. Specifically, different generalizations of the Nash solution and the Shapley value to coalitional games have been considered by Harsanyi (1959, 1963), Shapley (1969) and Owen (1972); the monotonic solutions for coalitional games, defined by Kalai and Samet (1985) coincide with the weighted Shapley values and with the proportional bargaining solutions introduced by Kalai (1977). Finally, the solution of Kalai and Smorodinsky (1975) for the subclass of bargaining problems and the τ-value proposed by Tijs (1981) for transferable utility games have been extended to coalitional games by Borm et. al. (1992), by means of the compromise value.

The aim of this paper is to propose a new solution concept for monotonic coalitional games by considering the equal-loss principle. This principle was introduced by Yu (1973) for bargaining problems. Other authors have considered this idea to be interesting, and different bargaining solutions involving the equal-loss criterion have been proposed, see Chun (1988), Chun and Peters (1991), and Herrero and Marco (1993). Nevertheless, the equal-loss principle has not be considered in more general coalitional games. Our proposal, *the coalitional equal-loss solution*, is an extension of the rational equal-loss solution, which was introduced for bargaining problems by Herrero and Marco (1993), and generalizes the Shapley value for superadditive transferable utility games.

The paper is organized as follows. Section 2 introduces the notation and describes monotonic coalitional games; Section 3 contains the definition of some bargaining solutions involving the equal-loss principle; its generalization, the coalitional equal-loss solution, is defined in Section 4, and the particular cases of bargaining games and superadditive games with transferable utility are analyzed in Section 5. Finally, Section 6 provides with some remarks on the differences between the coalitional equal-loss solution and other solutions incorporating the idea of agents' maximum expectations in a game.

2. PRELIMINARIES

We start by introducing some notation. Let $N=\{1,2,...,n\}$ be the set of players. A coalition S , is a subset of N and s denotes the cardinal of S. For two coalitions S and T, $S\backslash T$ will be the coalition formed by the players which are in S but not in T. For x, y in $\mathbb{R}^n$, the n-dimensional Euclidean space, $x \geqq_S y$ means $x_i \geq y_i$ for each i in S, $x \geq_S y$ means $x \geqq_S y$ and, for some $i \in S$, $x_i > y_i$, and $x >_S y$ means $x_i > y_i$ for each i in S. When S=N, we omit the subscript N. For each coalition S we denote $\mathbb{R}^S = \{ x \in \mathbb{R}^n \mid x_j = 0 \ \forall j \notin S \}$ and if $x \in \mathbb{R}^n$, x_S will be its projection in $\mathbb{R}^S$. The vector $e \in \mathbb{R}^n$ denotes the vector with $e_i = 1$ for each $i \in N$. For x in $\mathbb{R}^n$, $y = (t,x_{-i})$ is the vector such that $y_j = x_j$ for each $j \neq i$ and $y_i = t$. We use the notation $A \subseteq B$ for the set-inclusion, and $A \subset B$ when A is a proper subset of B.

A coalitional game, or a nontransferable (NTU) game in coalitional form, is an ordered pair (N,V), where V, the characteristic function, is a set-valued function that assigns a subset $V(S) \subset \mathbb{R}^S$ to each coalition $S \subseteq N$.

The disagreement point is the vector d, where $d_i=\max\{t \in \mathbb{R} \mid t \in V(i)\}$; thus $IR(V(S))=\{ x \in V(S) \mid x \geqq d_S\}$ will denote the set of individually rational points.

We call $\Gamma(N)$ the class of monotonic NTU games satisfying, for each coalition $S \subseteq N$, the following conditions:

(1) $V(\emptyset) = \{0\}$.

(2) $V(S)$ is a closed, nonempty set of $\mathbb{R}^S$.

(3) $V(S)$ is comprehensive: if $x,y \in \mathbb{R}^S$, $x \in V(S)$ and $x \geq_S y$, then $y \in V(S)$.

(4) $V(S)$ is bounded from above, which means that no monotonically increasing unbounded sequence of points exists in $V(S)$.

(5) (N,V) is monotonic; that is, $\forall S \subseteq N$ and $\forall\, j \notin S$, if $x \in IR(V(S))$, then there exists x' in $V(S \cup \{j\})$ such that $x' \geqq_S x$ and $x'_j \geq d_j$.

Two subclasses of this class of n-person games have been studied extensively: the class of superadditive games with transferable utility (TU) and the class of bargaining games. We will denote these classes by B_n and Σ_n respectively.

A superadditive TU game on N is a function v that assigns a real number $v(S)$ to each coalition $S \subseteq N$, with $v(\emptyset) = 0$ and $v(S) + v(T) \leq v(S \cup T)$ for any two disjoint coalitions S,T. Such a game v, defines an NTU game (N,V) in $\Gamma(N)$ by means of

$$V(S) = \{\, x \in \mathbb{R}^S \mid \Sigma\, x_i \leq v(S) \,\} \text{ for each } S \subseteq N.$$

According to Nash (1950), an n-person bargaining problem is a pair $(\mathcal{S},d)$, where the set of possible agreements, $\mathcal{S}$, is a nonempty, closed and comprehensive set in $\mathbb{R}^n$, bounded from above; $d \in \mathcal{S}$ represents the disagreement point, and there exists a feasible agreement strictly greater than d. A bargaining problem defines an NTU game (N,V) in $\Gamma(N)$ by means of

$$V(S) = \{\, x \in \mathbb{R}^S \mid x \leqq d_S \,\} \text{ for each } S \neq N \text{ and } V(N) = \mathcal{S}.$$

For each NTU game and for every coalition S, the set of weakly Pareto optimal outcomes is $WPO(V(S))=\{ x \in V(S) \mid y \in \mathbb{R}^S,\ y >_S x \text{ then } y \notin V(S) \}$.

For each game (N,V) in $\Gamma(N)$ the following subgames can be considered: for every coalition $S \subseteq N$, (S,V_S) is the game such that $V_S(T) = V(T)$ for each $T \subseteq S$.

A solution in the class $\Gamma(N)$ is a function $\gamma : \Gamma(N) \longrightarrow \mathbb{R}^n$ which assigns an outcome $\gamma(N,V)$ in V(N) to each problem (N,V) in $\Gamma(N)$.

3. THE EQUAL-LOSS PRINCIPLE IN BARGAINING PROBLEMS

Chun (1988) defined the equal-loss solution in the class of bargaining problems. In general, such a solution is not individually rational for more than two agents. In order to solve this shortcoming, Herrero and Marco (1993) proposed a new solution called the rational equal-loss solution. A solution in Σ_n is a function $\sigma: \Sigma_n \longrightarrow \mathbb{R}^n$ such that $\sigma(S,d) \in S$ for all $(S,d) \in \Sigma_n$. For each problem $(S,d) \in \Sigma_n$ we shall call $IR(S,d)=\{x \in \mathbb{R}^n \mid x \geqq d\}$, and the ideal point, $a(S,d)$, is defined as $a_i(S,d)=\max\{t \mid \exists x \in \mathbb{R}^n \text{ such that } (t, x_{-i}) \in IR(S,d)\}$.

Definition 1 (Chun, 1988)

The equal-loss solution, $E: \Sigma_n \longrightarrow \mathbb{R}^n$, is defined by setting, for all (S,d) in Σ_n, $E(S,d) = t^*e + a(S,d)$ with $t^* = \max\{t \in \mathbb{R} \mid te + a(S,d) \in S\}$.

Herrero and Marco (1993) proposed the following modification of Chun's solution.

Definition 2 (Herrero and Marco, 1993)

The rational equal-loss solution, $ER: \Sigma_n \longrightarrow \mathbb{R}^n$, associates to each problem $(S,d) \in \Sigma_n$ the vector with the following coordinate i

$$ER_i(S,d) = \begin{cases} d_i & \text{if } E_i(S^*,d) < d_i \\ E_i(S^*,d) & \text{if } E_i(S^*,d) \geq d_i \end{cases}$$

where S^* is the comprehensive hull of $IR(S,d)$.

This solution equalizes losses from the ideal point whenever it represents an acceptable agreement for all agents. If it does not, and some players are below their status-quo, the solution accepts smaller losses for these agents keeping them at their disagreement level and equalizing losses for the others.

If in Definitions 1 and 2 we use a vector $\lambda \in \mathbb{R}^n_{++}$ instead of the vector e, we can define the nonsymmetric solutions E_λ and ER_λ, where the losses are not equally allocated, but are arranged according to some positive vector of weights.

Solutions based on loss principles have been extended to bargaining problems with claims, which have been introduced by Chun and Thomson (1992).

An n-person bargaining problem with claims is a triplet (S,d,c) where $S \subseteq \mathbb{R}^n$ is the set of possible agreements, d represents the disagreement point and c_i is the promise made to agent i.

Consider now the following class $\hat{\Sigma}_n$ of problems (S,d,c) where S is a nonempty, closed, bounded from above, and comprehensive set in $\mathbb{R}^n$, and there exists a feasible agreement strictly greater than d. Although c is usually considered to be unfeasible and greater than the disagreement point, we do not impose these conditions.

In this class of games, a solution is a function $\sigma: \hat{\Sigma}_n \longrightarrow \mathbb{R}^n$ such that $\sigma(S,d,c) \in S$ for all $(S,d,c) \in \hat{\Sigma}_n$. We will call $IR(S,d,c) = IR(S,d)$.

The following solutions, defined on $\hat{\Sigma}_n$, will be used:

Definition 3 (Bossert, 1993)

The claim egalitarian solution, $\hat{E}: \hat{\Sigma}_n \longrightarrow \mathbb{R}^n$, is defined by setting the vector $\hat{E}(S,d,c) = t^*e + c$, with $t^* = \max \{t \in \mathbb{R} \mid te + c \in S\}$ for each (S,d,c) in the class $\hat{\Sigma}_n$.

Definition 4 (Bossert, 1993; Marco, 1994)

The extended claim egalitarian solution, $\hat{E}R: \hat{\Sigma}_n \longrightarrow \mathbb{R}^n$, associates to each problem $(S,d,c) \in \hat{\Sigma}_n$ the vector with the following coordinate i

$$\hat{E}R_i(S,d,c) = \begin{cases} d_i & \text{if } \hat{E}_i(S^*,d,c) < d_i \\ E_i(S^*,d) & \text{if } \hat{E}_i(S^*,d,c) \geq d_i \end{cases}$$

where S^* is the comprehensive hull of IR(S,d,c).

4. THE COALITIONAL EQUAL-LOSS SOLUTION

As shown in the previous section, the equal-loss and the rational equal-loss solutions depend on the ideal point. This point can be interpreted as the greatest amount that each player can obtain when the grand coalition forms. The main difference between NTU games and bargaining games is that in the former, intermediate coalitions do not play any role, whereas in the latter, they may have some weight. This means that we can consider ideal payoffs of players in coalitions other than the grand one, and in consequence, we should redefine these ideal points in order to support their interpretation, as the next example shows.

Example 1

Let (N,V) be the NTU game defined from the following TU game $N = \{1,2,3\}$, $v(i) = 0\ \forall i$, $v(1,2) = v(2,3) = 2$, $v(1,3) = 0$ and $v(N) = 4$. For this game, the ideal point for the grand coalition is the vector (4,4,4). However, player 1 cannot expect an outcome greater than 2 because, for any greater amount, players 2 and 3 would decide not to cooperate with him due to the fact that they could obtain greater gains by playing on their own. Thus the maximum expectation value for player 1 in coalition N is 2.

We propose the following definition.

Definition 5

For each game (N,V) in $\Gamma(N)$, we call the vector $A(S,V) \in \mathbb{R}^S$ the *point of maximum expectations for coalition S*, where

$$A_i(S,V) = \max\{x_i \mid x \in M_i\}, \text{ for all } i \in S$$

and $M_i = \{\ x \in V(S) \mid \text{there is not } z \in IR(V(S\backslash\{i\})),\ z \geq_{S\backslash\{i\}} x\ \}$.

Note that for any game in $\Gamma(N)$ the point of maximum expectations of any coalition, $A_i(S,V)$, always exists. It is greater than or equal to the disagreement point, d_S, and in some sense it expresses the greatest amount that player i can obtain in coalition S, since by the definition of M_i the other agents can agree with player i in such an outcome because they cannot be better off by playing on their own.

In the cooperative game of example 1 we have that $A_i(\{i\},V) = 0$ for all $i \in N$, $A_i(\{1,2\},V) = A_k(\{2,3\},V) = 2$ for $i = 1,2$ and $k = 2,3$, $A_i(\{1,3\},V) = 0$ for $i = 1,3$, $A_i(N,V) = 2$ for $i = 1,3$, and $A_2(N,V) = 4$.

In the context of bargaining problems, the point of maximum expectations for any coalition $S \neq N$ is d_S, and it coincides with the ideal point for the grand coalition. In the class of superadditive TU games the point of maximum expectations is the marginal contribution of player i to the coalition S for all $S \subseteq N$, $A_i(S,v) = v(S) - v(S\backslash\{i\})$.

We define the coalitional equal-loss solution for a game (N,V), denoted as EC(N,V), by inductively constructing two functions L and C from the set of coalitions to $\mathbb{R}^n$. Similar procedures can be found in Owen (1982) and Kalai and Samet (1985).

For each game (N,V) in $\Gamma(N)$ let L and C be the following two functions: $L(\emptyset,V) = 0$, $C(\emptyset,V) = 0$, and for each coalition $S \subseteq N$,

$$C(S,V) = A(S,V_S) + \sum_{T \subset S} L(T,V) \text{ and}$$

$$L(S,V) = e_S \max \{ t \in \mathbb{R} \mid C(S,V) + t\, e_S \in V(S) \}.$$

Definition 7

The *coalitional equal-loss solution*, EC: $\Gamma(N) \longrightarrow \mathbb{R}^n$, associates to each problem (N,V) in $\Gamma(N)$, the vector $EC(N,V) = \hat{ER}(V(N),d,C(N,V))$.

The existence of the function L(S,V) is due to the comprehensiveness and boundedness of V(S). It represents the losses for the players in S in the subgame (S,V_S), although in some cases C(S,V) can be in V(S) and then, L(S,V) will be interpreted as gains. EC(N,V) is the rational equal-loss solution to the bargaining problem with claims (V(N),d,C(N,V)), so, in some way, it could be interpreted as the accumulation of successive dividends, losses or gains, shared according to the equal-loss principle.

As in Kalai and Samet (1985), a nonsymmetric coalitional equal-loss solution can be defined when the losses L(S,V) are not equal for all the players but are allocated according to some positive weights. Thus, for $\lambda \in \mathbb{R}^n_{++}$ the solution $EC_\lambda(N,V)$ will be as in Definition 5, although the vector λ is used instead of e in the definition of L(S,V), and the solution $\hat{ER}$ is sustituted by its asymmetric version with weight λ.

5. PARTICULAR CASES

The coalitional equal-loss solution, like the egalitarian, Harsanyi and Shapley solutions, coincides with the Shapley value in the class of superadditive TU games, as the next proposition shows.

Proposition 1

Let (N,V) be the NTU game defined from $(N,v) \in B_n$. Then EC(N,V) = Sh(N,v), where Sh(N,v) denotes the Shapley value of the game (N,v).

In order to prove the above result we will use the following Lemma.

Lemma

For each (N,v) in B_n, $\hat{E}(V(N),d,C(N,V)) \in IR(V(N))$, where (N,V) is the NTU game defined from (N,v).

Proof

Let (N,V) be the game defined from (N,v) in B_n. For each coalition $S \subseteq N$ we have

$$\hat{E}(V(S),d_S,C(S,V)) = C(S,V) + L(S,V),$$

where

$$C(S,V) = A(S,V_S) + \sum_{T \subset S} L(T,V).$$

Then,

$$\hat{E}(V(S),d_S,C(S,V)) = A(S,V_S) + \sum_{T \subseteq S} L(T,V). \qquad [1]$$

We will prove, by induction on the coalition size, that for each coalition $S \subseteq N$, $\hat{E}(V(S),d_S,C(S,V)) \geq d_S$. This inequality is obvious when S has only one player. Now, assume that it holds for all coalitions with k-1 agents. Let S be a coalition with cardinal k, and let j be such that

$$C_j(S,V) - d_j = \min_i (C_i(S,V) - d_i).$$

We define the following vector

$$x = C(S,V) - (C_j(S,V) - d_j)\, e_S,$$

then, $x \geq d_S$, $x_j = d_j$, and if $i \neq j$

$$x_i = C_i(S,V) - C_j(S,V) + d_j = A_i(S,V_S) + \sum_{i \in T \subset S} L_i(T,V) - A_j(S,V_S) - \sum_{j \in T \subset S} L_j(T,V) + d_j.$$

By using [1] together with the form of the maximum expectation point for the kind of games at hand, we get

$$x_i = A_i(S,V_S) - A_j(S,V_S) + \sum_{i \in T \subseteq S\setminus\{j\}} L_i(T,V) - \sum_{j \in T \subseteq S\setminus\{i\}} L_j(T,V) + d_j =$$

$$= A_i(S,V_S) - A_j(S,V_S) + \hat{E}_i(V(S\setminus\{j\}),d_{S\setminus\{j\}},C(S\setminus\{j\},V)) -$$

$$-A_i(S\setminus\{j\},V_{S\setminus\{j\}}) - \hat{E}_j(V(S\setminus\{i\}),d_{S\setminus\{i\}},C(S\setminus\{i\},V)) + A_j(S\setminus\{i\},V_{S\setminus\{i\}}) + d_j =$$

$$= \hat{E}_i(V(S\setminus\{j\}),d_{S\setminus\{j\}},C(S\setminus\{j\},V)) - \hat{E}_j(V(S\setminus\{i\}),d_{S\setminus\{i\}},C(S\setminus\{i\},V)) + d_j.$$

By the induction hypothesis, $\hat{E}_j(V(S\setminus\{i\}),d_{S\setminus\{i\}},C(S\setminus\{i\},V)) \geq d_j$, then

$$x_i \geq \hat{E}_i(V(S\setminus\{j\}),d_{S\setminus\{j\}},C(S\setminus\{j\},V)) \text{ for each } i \text{ in } S,\ i\neq j.$$

Given that $V(S\setminus\{j\})$ is comprehensive, $x_{S\setminus\{j\}} \in V(S\setminus\{j\})$. Since $x_{S\setminus\{j\}} \geqq d_{S\setminus\{j\}}$ and the game is monotonic, there exists x'_j such that $x'_j \geq d_j$ and $(x_{S\setminus\{j\}},x'_j) \in V(S)$. Again by comprehensiveness $x \in V(S)$. Now, from the definition of $L(S,V)$ we get

$$e_S(-C_j(S,V) + d_j) \leq L(S,V),$$

then,

$$x \leq \hat{E}(V(S),d_S,C(S,V)),$$

and therefore

$$\hat{E}(V(S),d\ ,C(S,V)) \geq d.$$ ■

For $(N,v) \in B_n$ and $\lambda \in \mathbb{R}$, we will denote by $(N,\lambda v)$ the TU game defined by $[\lambda v](S) = \lambda(v(S))\ \forall\ S \subseteq N$. Given $(N,v^1),(N,v^2)$ in B_n, we will denote by $(N,\ v^1+ v^2)$ the TU game defined by $[v^1+ v^2](S) = v^1(S) + v^2(S)\ \forall S \subseteq N$.

<u>Proof of proposition 1</u>

Given the above lemma, in order to prove the result, we only need to use the equal-loss solution $\hat{E}$ in the definition of EC. It is well known that any game in B_n can be generated by a linear combination of unanimity games. For each coalition S, the unanimity game for S, v_S, is the TU game such that

$v_S(T) = 1$ for each T that contains S, and

$v_S(T) = 0$ when S is not contained in T.

We will denote by (N,V^S) the NTU game defined from v_S.

In order to calculate the coalitional equal-loss solution for a unanimity game, (N,V^S), consider that for each T such that T does not contain S, $C(T,V^S) = 0$ and $L(T,V^S) = 0$. On the other hand, $C(S,V^S) = e_S$. If $S = N$, $EC(N,V^S) = \frac{1}{s} e_S$. If $S \subset N$, $L(S,V^S) = \frac{1-s}{s} e_S$, and for each coalition T such that $S \subset T$, $C(T,V^S) = e_S + \frac{1-s}{s} e_S = \frac{1}{s} e_S$, so $L(T,V^S) = 0$. In particular, $C(N,V^S) = \frac{1}{s} e_S$, and then $EC(N,V^S) = \frac{1}{s} e_S$.

Let $\lambda,\beta \in \mathbb{R}$, and let S and R be two coalitions in N such that the NTU game $(N,\lambda V^S + \beta V^R)$, defined from the TU game $(N,\lambda v^S + \beta v^R)$, is monotonic.

We will distinguish three cases:

(i) Neither $S \subseteq R$ nor $R \subseteq S$.

If coalition T does not contain S or R, then

$$C(T,\lambda V^S + \beta V^R) = 0, \quad L(T,\lambda V^S + \beta V^R) = 0,$$

$$C(S,\lambda V^S + \beta V^R) = \lambda e_S, \quad L(S,\lambda V^S + \beta V^R) = \lambda \frac{1-s}{s} e_S,$$

$$C(R,\lambda V^S + \beta V^R) = \beta e_R, \text{ and } L(R,\lambda V^S + \beta V^R) = \beta \frac{1-r}{r} e_R.$$

If $S \subset T$, but R is not contained in T, then

$$C(T,\lambda V^S + \beta V^R) = \lambda e_S + \lambda \frac{1-s}{s} e_S = \lambda \frac{1}{s} e_S, \text{ and}$$

$$L(T,\lambda V^S + \beta V^R) = 0.$$

If $R \subset T$, but S is not contained in T, then

$$C(T,\lambda V^S + \beta V^R) = \beta e_R + \beta \frac{1-r}{r} e_R = \beta \frac{1}{r} e_R, \text{ and}$$

$$L(T,\lambda V^S + \beta V^R) = 0.$$

On the other hand,

$$C(S \cup R,\lambda V^S + \beta V^R) = \lambda e_S + \beta e_R + \lambda \frac{1-s}{s} e_S + \beta \frac{1-r}{r} e_R = \lambda \frac{1}{s} e_S + \beta \frac{1}{r} e_R, \text{ and}$$

$$L(S \cup R,\lambda V^S + \beta V^R) = 0.$$

Finally, if $S \cup R \subset T$, then

$$C(T,\lambda V^S + \beta V^R) = \lambda \frac{1}{s} e_S + \beta \frac{1}{r} e_R, \text{ and}$$

$$L(T,\lambda V^S + \beta V^R) = 0.$$

So, in any case, that is, if $S \cup R \subset N$ or $S \cup R = N$, we get

$$C(N,\lambda V^S + \beta V^R) = \lambda \frac{1}{s} e_S + \beta \frac{1}{r} e_R, \text{ and } EC(N,\lambda V^S + \beta V^R) = \lambda \frac{1}{s} e_S + \beta \frac{1}{r} e_R.$$

(ii) $S \subset R$.

If coalition T does not contain S, then

$$C(T,\lambda V^S + \beta V^R) = 0, \quad L(T,\lambda V^S + \beta V^R) = 0,$$

$$C(S,\lambda V^S + \beta V^R) = \lambda e_S, \text{ and } L(S,\lambda V^S + \beta V^R) = \lambda \frac{1-s}{s} e_S.$$

If $S \subset T$, but R is not contained in T, then

$$C(T,\lambda V^S + \beta V^R) = \lambda e_S + \lambda \frac{1-s}{s} e_S = \lambda \frac{1}{s} e_S, \text{ and}$$

$$L(T,\lambda V^S + \beta V^R) = 0.$$

On the other hand,

$$C(R,\lambda V^S + \beta V^R) = \lambda e_S + \beta e_R + \lambda \frac{1-s}{s} e_S = \lambda \frac{1}{s} e_S + \beta e_R, \text{ and}$$

$$L(R,\lambda V^S + \beta V^R) = \beta \frac{1-r}{r} e_R.$$

Finally, if $R \subset T$, then

$$C(T,\lambda V^S + \beta V^R) = \lambda \frac{1}{s} e_S + \beta \frac{1}{r} e_R, \text{ and}$$

$$L(T,\lambda V^S + \beta V^R) = 0.$$

If $R = N$, then

$$C(N,\lambda V^S + \beta V^R) = \lambda \frac{1}{s} e_S + \beta e_R, \text{ and } EC(N,\lambda V^S + \beta V^R) = \lambda \frac{1}{s} e_S + \beta \frac{1}{r} e_R.$$

If $R \subset T$, then

$$C(T,\lambda V^S + \beta V^R) = \lambda \frac{1}{s} e_S + \beta \frac{1}{r} e_R, \text{ and } EC(N,\lambda V^S + \beta V^R) = \lambda \frac{1}{s} e_S + \beta \frac{1}{r} e_R.$$

(iii) $R \subset S$, idem case (ii).

Therefore, in any case, we get that

$$EC(N,\lambda V^S + \beta V^R) = \lambda \frac{1}{s} e_S + \beta \frac{1}{r} e_R = \lambda EC(N,V^S) + \beta EC(N,V^R).$$

This fact can be generalized for more than two unanimity games.

As it is well known $Sh(N,v_S) = \frac{1}{s} e_S$, and both solutions coincide, since the Shapley value is linear.

■

It is straightforward to prove that the coalitional equal-loss solution coincides with the rational equal-loss solution in the class of bargaining problems.

Proposition 2

Let (N,V) the NTU game defined from $(\mathcal{S},d) \in \Sigma_n$. Then $EC(N,V) = ER(\mathcal{S},d)$

Proof

The result is straightforward because $L(S,V) = 0$ for each $S \neq N$ and then $C(N,V) = a(\mathcal{S},N)$. ■

6. SOME COMMENTS ON THE RELATED LITERATURE

Roth (1980) and Shafer (1980) show some examples in which the main solutions in the class of NTU games can yield predictions which are highly counterintuitive. Those controversial examples have been discussed in several papers, (Roth (1980, 1986), Shafer (1980), Harsanyi (1980), Aumann (1985,1986), and Hart (1985)).

In particular Roth proposes the following family of games. The set of players is $N = \{1,2,3\}$, and for each $p \in [0,1/2]$

$$V_p(i) = \{ x \in \mathbb{R} \mid x \leq 0 \} \quad i = 1,2,3$$
$$V_p(1,2) = \{ x \in \mathbb{R}^2 \mid x_1 \leq 1/2,\ x_2 \leq 1/2 \}$$
$$V_p(2,3) = \{ x \in \mathbb{R}^2 \mid x_2 \leq p,\ x_3 \leq 1-p \}$$
$$V_p(1,3) = \{ x \in \mathbb{R}^2 \mid x_1 \leq p,\ x_3 \leq 1-p \}$$
$$V_p(N) = \{ x \in \mathbb{R}^3 \mid x \leqq y \text{ for some } y \text{ in } A \}$$

where A is the convex hull of the vectors (1/2,1/2,0), (p,0,1-p) and (0,p,1-p).

Roth's argument is the following: "for $p < 1/2$, the payoff vector (1/2,1/2,0) is the unique outcome of the game consistent with the hypothesis that the players are rational utility maximizers. This is because, when $p < 1/2$, the outcome (1/2,1/2,0) is strictly preferred by both players 1 and 2 to every other feasible outcome, and because the rules of the game permit players 1 and 2 to achieve this outcome without the cooperation of player 3". This reasoning has been disputed by Aumann who argues that there are instances where (p,0,1-p) or (0,p,1-p) may be the outcome. For example when players use the strategy of accepting the first offer received, he concludes that (1/3,1/3,1/3) is also a reasonable solution, especially when p is close to 1/2.

Of all the solutions proposed for coalitional games, the Egalitarian solution and the coalitional equal-loss solution imply interpersonal comparisons of utility, whereas the others do not. So that, if the aforementioned games arise from a situation where agents do not compare their utilities, the solutions proposed by Harsanyi (1959, 1963), Shapley (1969) and Borm et. al. (1992) would be possible, and the only one that coincides with Roth's criterion is the compromise value, of which differences from the solution proposed in this paper are noted below. In the context where agents compare their utilities, the coalitional equal-loss solution proposes $EC(N,V_p) = (1/2,1/2,0)$ for each $p < 1/2$ and $EC(N,V_{1/2}) = (1/3,1/3,1/3)$, whereas the egalitarian solution introduced by Kalai and Samet (1985) recommends the outcome $(\frac{1}{2} - \frac{p}{3}, \frac{1}{2} - \frac{p}{3}, \frac{2p}{3})$. Note that in both situations, only for the solutions which depend on a point representing the maximal payoff each player may expect to obtain coincide with Roth's criterion.

The idea that in cooperative games, the of maximal expectations should be taken into account when proposing a solution concept was introduced, for bargaining problems, by Yu (1973). Later, other authors considered it for this class of problems (Kalai and Smorodinsky (1975), Chun (1988), Chun and Peters (1991) and Herrero and Marco (1993)).

Tijs (1981) defined the utopia payoffs vector for TU games, and used it to propose and characterize the τ-value in the subclass of quasi-balanced TU games, Tijs (1987). Borm et. al. (1992) generalized this utopia payoffs vector to coalitional games and extended the τ-value to the subclass of admissible coalitional games by means of the compromise value. Another definition of ideal payoffs for TU games was introduced by Milnor (1952) and is called the vector of maximum aspirations. Bergantiños and Massó (1994) introduced a new solution concept for all essential TU games, the χ-value, taking this vector into account.

In order to remark on some differences between the coalitional equal-loss solution and these values, we need additional notation and some definitions that will be restricted to the class of our interest.

Definition 8 (Borm et. al., 1992)
Given (N,V) in Γ(N), the *utopia payoff to player i* is $K_i(V) = A_i(N,V)$.

This point represents the greatest amount that each player can obtain in the grand coalition.

Definition 9 (Borm et. al., 1992)
Given (N,V) in Γ(N), the *minimal right of player i* is $k_i(V) = \max_{S:\, i \in S} \rho_i^S(V)$,
where $\rho_i^S(V) = \sup\{t \in \mathbb{R} \mid \exists a \in \mathbb{R}^S,\ (t, a_{-i}) \in V(S) \text{ and } a >_{S\setminus\{i\}} K_{S\setminus\{i\}}(V)\}$.

Considering a coalition S such that $i \in S$, the formation of such a coalition is attractive for player $j \in S\setminus\{i\}$ if he gets more than the utopia payoffs $K_j(V)$. Thus player i can claim the remainder, $\rho_i^S(V)$.

Moreover, player i can choose one, from the possible coalition to which he belongs, where this remainder is maximal.

An NTU game, (N,V), is called *monotonic* and *compromise admissible* if $(N,V) \in \Gamma(N)$ and the utopia vector, K(V), and the minimal right vector k(V) satisfy the following three properties:

(i) $k_i(v)$ and $K_i(V)$ are real numbers $\forall i \in N$

(ii) $k(V) \leq K(V)$

(iii) $k(V) \in V(N)$, and there is not $z \in V(N)$ such that $z > K(V)$.

In fact, property (i) is always satisfied in monotonic games, but it is not redundant if the requirement of monotonic game is eliminated.

By *MC*(N) we denote the class of all monotonic and compromise admissible NTU games.

Definition 10 (Borm et. al., 1992)

For $(N,V) \in MC(N)$, the *compromise value* T(N,V) is defined by

$$T(N,V) = \lambda_V K(V) + (1-\lambda_V)k(V),$$

where

$$\lambda_V = \max\{\lambda \in [0, 1] \mid \lambda K(V) + (1-\lambda)k(V) \in V(N)\}.$$

T(N,V) is the unique vector on the line segment between k(V) and K(V) which lies in V(N) and is closest to the utopia vector K(V). Therefore, according to the compromise value, players will receive from their minimal right, as many payoffs as possible that are proportional to the differences between their utopia and minimal right payoffs.

Definition 11 (Bergantiños and Massó, 1994)

For $(N,v) \in B_n$ and for each $i \in N$, the *player i's maximun aspiration* in the game (N,v), $M_i^{\chi}(v)$, is

$$M_i^{\chi}(v) = \max_{\substack{S \subseteq N \\ i \in S}} \{ v(S) - v(S \setminus \{i\}) \}$$

Note that the utopia payoffs to player i (Tijs (1981)) coincide with his marginal contribution to the grand coalition, whereas player i's maximum aspiration is his maximal marginal contribution of all coalitions.

Definition 12 (Bergantiños and Massó, 1994)

For $(N,v) \in B_n$ and for each $i \in N$, *player i's minimun aspiration* in the game (N,v), $m_i^{\chi}(v)$, is

$$m_i^{\chi}(v) = \max_{\substack{S \subseteq N \\ i \in S}} \{ v(S) - \sum_{j \in S \setminus \{i\}} M_j^{\chi}(v) \}$$

The minimum aspiration for player i, is the maximal remainder he can obtain after conceding to the other players their maximum aspirations, so in order to define a lower vector, these authors follow the Tijs idea but use a different ideal expectation vector.

Definition 13 (Bergantiños and Massó, 1994)

Given $(N,v) \in B_n$, the *χ-value* of (N,v), $\chi(N,v)$, is

$$\chi(N,v) = \alpha M^{\chi}(v) + (1-\alpha) m^{\chi}(v),$$

where

$$\alpha = \max\{\alpha \in [0, 1] \mid \alpha M(V) + (1-\alpha) m(V) \in V(N)\}.$$

$\chi(N,v)$ is the unique efficient vector in the segment having $m^{\chi}(v)$ and $M^{\chi}(v)$ as extreme points. Therefore, according to the χ-value, players will receive, as many payoffs as possible, from their minimum aspiration, that are proportional to their differences between their maximum and minimum aspirations.

Next, we note some differences between the coalitional equal-loss solution, the compromise value and the χ-value.

The coalitional equal-loss solution has been introduced for monotonic NTU games. The compromise value exists for all compromise admissible NTU games which are not necessarily monotonic, but this class of games does not have a natural justification, Bergantiños and Massó (1994) present a superadditive TU game that is not compromise admissible. Finally, the χ-value exists for all essential TU games, that is, for any TU game, (N,v), such that $\sum_{i \in N} v(\{i\}) \leq v(N)$, although this value has not been extended to some subclasses of NTU games.

The general justice criteria behind these solutions are clear enough. The compromise value and the χ-value incorporate the proportionality principle, although with different weights. The former considers the differences between utopia and minimal right payoffs and the latter the differences between maximum and minimum aspirations. Therefore, both the compromise value and the χ-value allocate gains and losses according to the same principle. The coalitional equal-loss solution supports the equality criterion, as does the Kalai-Samet solution, but the first one focuses on losses whereas the latter considers gains to be more important than losses. Actually whether gains or losses are the focus of attention depends on the psychology of the situation, and, if this is not clear, solutions which behave without distinction would be appealing.

The coalitional equal-loss solution, the compromise value and the χ-value take into account the idea of players' ideal payoffs when proposing an allocation, but the way they do so is quite different. The coalitional equal-loss solution takes into account the ideal payoff of each player in all possible coalitions, that is, all coalitions play a role. Player i's utopia payoff used in defining the compromise value is his maximal contribution to the grand coalition, so coalitions formed by n-2 agents do not play any role. Player i's maximum aspiration considered in the χ-value, takes into account the maximal marginal contribution of this player, so it observes all coalitions containing player i, but in the end only some of them will have some weight. These differences are shown in the following example.

Example 2

Let (N,v) be a TU game where $N = \{1,2,3,4\}$, $v(\{i\}) = 0 \ \forall\, i \in N$, $v(\{1,2\}) = v(\{1,4\}) = v(\{3,4\}) = 0{,}5$, $v(\{1,3\}) = v(\{2,3\})\ v(\{2,4\}) = 1$, $v(\{i,j,k\}) = 1{,}25 \ \forall\, i,j,k \in N$ and $v(N) = 2$. Neither the compromise value nor the χ-value take into account the asymmetries of v, but the coalitional equal-loss solution does, since

$K_i(V) = 0{,}75\ \forall i \in N$, $k_i(V) = 0{,}25\ \forall i \in N$, so $T(N,V) = (0{,}5,\ 0{,}5,\ 0{,}5,\ 0{,}5)$; $M_i(V) = 1\ \forall i \in N$, $m_i(V) = 0\ \forall i \in N$, so $\chi(N,V) = (0{,}5,\ 0{,}5,\ 0{,}5,\ 0{,}5)$ and $EC(N,V) = (11/24,\ 13/24,\ 13/24,\ 11/24)$.

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