

**MODELLING CONDITIONAL HETEROSKEDASTICITY:
APPLICATION TO STOCK RETURN INDEX "IBEX-35"**

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A B S T R A C T

This paper compares alternative time-varying volatility models for daily stock-returns using data from Spanish equity index IBEX-35. Specifically, we have estimated a parametric family of models of generalized autoregressive heteroskedasticity (which nests the most popular symmetric and asymmetric GARCH models), a semiparametric GARCH model, the stochastic volatility model SV(1), the Poisson jump diffusion process and finally, a non-parametric model. We obtain that those models which use conditional standard deviation produce better fits than all other GARCH models. We also compare all models using a standard efficiency test (which compares within sample predictive power) and conclude that general GARCH models (specifically the TGARCH(1,1) model) perform better than all others.

KEYWORDS: Stock Returns; Conditional Heteroskedasticity; GARCH Models.

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1. INTRODUCTION.

In the last decade there has been an increasing interest in modelling the predictable component of volatility, or conditional variance h_t , for a given time series ε_t . This concern with conditional variance is due to the fact that, in many financial models, the risk premium is a function of it. This paper compares alternative statistical models for daily stock return conditional variance proposed in recent literature. The focus is on the Spanish equity index IBEX-35.

As we know, the return can be decomposed into anticipated (conditional mean) and unanticipated returns. It has become commonplace to model unanticipated returns using a white noise η_t and a term which reflects conditional standard deviation $h_t^{1/2}$. The pioneer models were Engle's (1982) autoregressive conditional heteroskedasticity (ARCH) model and Bollerslev's (1986) Generalised ARCH model (GARCH). ARCH and GARCH models with normal disturbances cannot capture some important characteristics of the data. The most interesting features not addressed by these Gaussian models are leptokurtosis and leverage or asymmetric effect (Black 1976). In order to handle with the former, some authors have proposed various alternative distributions for the white noise (Kon 1984, Kim and Kon 1994), whereas others adopt a semiparametric approach and leave it unspecified (Engle and Gonzalez-Rivera 1991, Linton 1994). In order to handle with asymmetries, various generalisations of GARCH models have been proposed (see Hentschel 1995 and references therein); most of these models are nested on the General

Family of GARCH Models proposed by Hentschel (1995), what makes it possible to test all other models using standard testing techniques.

In recent years, some other alternative approaches to model conditionally heteroskedastic time series have appeared. One of the most fruitful ones is the stochastic volatility (SV) model, first proposed by Taylor (1986), which introduces a stochastic term in h_t , which is no longer the conditional variance. Another interesting model is the Poisson Jump Diffusion (PJD) model, which introduces a stochastic term in ε_t , in order to capture additional extraneous shocks (see, for example, Kim and Kon 1994). Both the SV and PJD models are discrete time approximations of continuous time models used in financial literature. Finally, we have also considered in this paper a purely nonparametric method, which is based on the fact that in most models h_t is a conditional expectation (see, for example, Pagan and Ullah 1988); thanks to it, it is easy to propose estimates of this quantity which do not require any functional specification.

The remainder of the paper is structured as follows: in Section 2 we describe the data set we analyse; in Section 3 we analyse the dynamic structure in mean of our series; in Section 4 we first introduce and estimate the conditional heteroskedasticity models ARCH, GARCH, the general family GARCH(1,1); then we describe and estimate several alternative models for conditional heteroskedasticity; in Section 5 we present the results of a standard efficiency which examines the "in-sample" predictive power of all models. Finally, Section 6 concludes.

2. DESCRIPTION OF DATA.

The IBEX-35 is a stock-exchange index that includes the shares having more liquidity in Madrid Stock Market. Since November 1991, this is one of the official indexes in Madrid Stock Market, though it has been computed since the beginning of 1987. The IBEX-35 was constructed with the aim of getting a difficult-to-manipulate stock-market indicator which was computed in a continuous way.

Our data set consists of the IBEX-35 daily closing prices from January 1987 to June 1995. Prior to our formal analysis, we took logarithmic differences of the daily closing price series, and then multiplied it by 100. That is, our daily index return is computed as $R_t = 100 \times \ln(S_t/S_{t-1})$, where S_t is the closing price at day t .

Before analysing the dynamic structure of our series, we first analyse whether there exists any "day of the week" effect in the series. Recent empirical studies (Peña 1995) show that before Computer Assisted Trading System (CATS) was introduced at Madrid Exchange Market (December, 1989) the "day of the week" effect was not negligible. Specifically the "dummy" variable corresponding to Monday is significant, showing the presence of "weekend effect". Nevertheless, after CATS implantation this effect vanished. In order to test the veracity of this hypothesis, we have divided our sample into two periods:

Period 1: January 1987/December 1989 (734 observations).

Period 2: April 1990/June 1995 (1311 observations).

In each period, we estimated by OLS regression models with R_t as dependent variable and five "dummy" variables, one for each day of the week, as independent variables (days after holidays were recorded as Mondays). The results of these OLS regressions were:

$$\text{Period 1: } R_t = .388 D_{1t} - .085 D_{2t} - .161 D_{3t} - .049 D_{4t} + .015 D_{5t}, \bar{R}^2 = 0.022$$

(3.45) (-.84) (-1.60) (-.54) (.18)

$$\text{Period 2: } R_t = -.088 D_{1t} - .072 D_{2t} - .068 D_{3t} + .099 D_{4t} + .102 D_{5t}, \bar{R}^2 = 0.022$$

(-1.08) (1.00) (-1.07) (1.33) (1.63)

(heteroskedasticity-consistent t-statistics are shown into brackets). These results confirm the evidence shown in Peña (1995): during the period 1990-1995 there is no "day of the week" effect but this is not the case for the period 1987-1989. For this reason, we have decided to consider for the remainder of the article only Period 2.

3. MODELLING DYNAMIC STRUCTURE IN MEAN.

First of all we must analyse whether the IBEX-35 return series presents some kind of mean dependence. Several different autoregressive moving-average (ARMA) models have been adjusted to our 1311 observations. We estimated all models by maximum likelihood (ML) assuming a normal distribution for the disturbances (these estimations were carried out using TSP). In order to compare all estimated models, we report the Schwarz Information Criterion (SIC), defined as $SIC = \ln(L_{ML}) - [q \times \ln(T)]/2$, where L_{ML} stands for the likelihood function of the model evaluated in the ML estimator and q is the number of parameters in the model. According to this criterion, the model with highest SIC is the preferred one; observe that the second term in SIC is a penalty for models with a high number of parameters (q). Table 1 presents SIC values for each estimated ARMA model.

TABLE 1

COMPARISON OF VARIOUS ARMA(p,q) MODELS

Model	AR(1)	AR(2)	MA(1)	MA(2)	ARMA(1,1)
SIC	-198.52	-202.08	-198.66	-202.12	-202.07

These models do not include a constant term.

We do not report results for models estimated with constant term because, in all cases, the constant term was not significantly different from 0 ($\alpha=0.05$). In models MA(2) and ARMA(1,1) some parameter was not statistically

significant ($\alpha=0.05$). In all models the Ljung-Box statistic for residuals with 20 lags was close to 19, with p-value well above 0.05. However, as expected, there is evidence of autocorrelation in squared residuals. According to these results we decided to choose the AR(1) model, for which the parameter estimate is 0.103 with a t-ratio of 3.762.

In the remainder of the work we use the residuals of these estimations, hereafter denoted as ε_t . We have split this series into two periods: from April 1990 to August 1994 ("in-sample" period: 1103 observations) and from September 1994 to June 1995 ("out-of-sample" period: 208 observations). Finally, the first 6 observations from the "in-sample" period are only used as initial values in the various estimation processes. Thus, the series ε_t which we consider in the remainder of the paper consists of 1097 observations of the residuals of an AR(1) model.

4. MODELLING DYNAMIC STRUCTURE IN VARIANCE.

Our main interest is to examine the performance of alternative models for conditionally heteroskedastic time series using the observations ε_t described in previous section. All models we will use here consist of two equations: one of them specifies a decomposition for the observations ε_t and the other one is the volatility equation. In Table 2 we summarize these equations for the models we use in this paper. For a detailed description of these models see, for example, Hamilton (1994) or references cited below.

TABLE 2

MODELS FOR CONDITIONALLY HETEROSKEDASTIC TIME SERIES

Model	Equation for ε_t	Volatility Equation
GARCH(p,q)	$\varepsilon_t = h_t^{1/2} \eta_t$, η_t i.i.d. $E[\eta_t]=0$, $E[\eta_t^2]=1$	$h_t = \omega + \sum_{j=1}^q \alpha_j \varepsilon_{t-j}^2 + \sum_{i=1}^p \beta_i h_{t-i}$
Family-G(1,1)	$\varepsilon_t = h_t^{1/2} \eta_t$, η_t i.i.d. $E[\eta_t]=0$, $E[\eta_t^2]=1$	$g(h_t, \lambda) = \omega_G + \alpha_G h_{t-1}^{\lambda/2} f(\eta_{t-1})^\mu + \beta_G g(h_{t-1}, \lambda)$; $f(\eta) \equiv \eta - b - c(\eta - b)$; $g(h, \lambda) \equiv \begin{cases} (h^{\lambda/2} - 1)/\lambda & \text{if } \lambda \neq 0 \\ \ln(h^{1/2}) & \text{if } \lambda = 0 \end{cases}$
SV(1)	$\varepsilon_t = h_t^{1/2} \eta_t$, η_t i.i.d. $E[\eta_t]=0$, $E[\eta_t^2]=1$	$\ln(h_t) = \omega + \beta \ln(h_{t-1}) + \psi_t$, ψ_t ind. η_s ψ_t i.i.d., $E[\psi_t]=0$, $E[\psi_t^2]=\sigma_\psi^2$

Poisson-JD	$\varepsilon_t = h_t^{1/2} \eta_t + X_t, \eta_t \text{ i.i.d.}$ $h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$ $X_t = \sum_{i=1}^N Y_{ti}, Y_{ti} \sim N(\mu_Y, \sigma_Y^2)$ $Y_{ti} \text{ ind. } Y_{sj} \text{ if } t \neq s \text{ or } i \neq j$ $N_t \sim \text{Poisson}(\lambda), N_t \text{ i.i.d.}$
Non-Parametric	$\varepsilon_t = h_t^{1/2} \eta_t, \eta_t \text{ i.i.d.}$ $h_t = \text{Var}(\varepsilon_t I_{t-1})$ $E[\eta_t] = 0, E[\eta_t^2] = 1$ $I_{t-1} \equiv \text{information set at } t-1$

4.1. Generalised autoregressive conditional heteroskedasticity (GARCH).

Models of generalised autoregressive conditional heteroskedasticity (GARCH) specify present conditional variances as a function of past conditional variances and past squared observations. They were first proposed by Bollerslev (1986), who generalised the simpler autoregressive conditional heteroskedasticity (ARCH) models previously formulated by Engle (1982). Certain restrictions on parameters are necessary in order to ensure positiveness and stationarity of conditional variance (see, i.e., Nelson and Cao 1992). In recent years these models have been extensively used and must be considered in the first place as a benchmark.

Estimation of GARCH models can be easily carried out by constrained maximum likelihood (ML) assuming a specific distribution for η_t . The log-likelihood function to be maximized in a GARCH model is

$$\ln[L(\varepsilon_1, \dots, \varepsilon_T)] = - (1/2) \sum_{t=1}^T \ln(h_t) - \sum_{t=1}^T \ln[f(\varepsilon_t / h_t^{1/2})], \quad (1)$$

where f is the density function of η_t . The starting values of the conditional variance sequence $h_0, \dots, h_{-(p-1)}$ have been considered as parameters. First we estimated different GARCH specifications assuming normality of η_t and imposing non-negativity restrictions on all parameters in the conditional variance equation. The estimation process was carried out using the CML subroutine of GAUSS. In order to compare the goodness-of-fit of these estimations we used the Schwarz Information Criterion (SIC), whose values are reported in Table 3.

TABLE 3
COMPARISON OF VARIOUS GARCH(p,q) MODELS

Model	ARCH(1)	ARCH(2)	ARCH(3)	ARCH(4)	ARCH(5)	ARCH(6)
SIC	-1717.1	-1707.6	-1701.1	-1704.6	-1693.2	-1696.7
Model	G(1,1)	G(1,2)	G(1,3)	G(2,1)	G(3,1)	
SIC	-1691.0	-1694.5	-1697.9	-1693.1	-1700.1	

According to SIC, the GARCH(1,1) model is the preferred one. It is also worth noting that in all models where $q > 3$ the constrained likelihood function is maximized when some of the parameters are exactly equal to 0.

As non-normality is known to be a typical feature of financial time series, we have also estimated the GARCH(1,1) model considering two other distributions: a t distribution with ν degrees of freedom and the

Generalised Error Distribution (GED) with parameter ν . Both have been normalized to have zero mean and unit variance. Their corresponding density functions can be seen, for example, in Hamilton (1994), p. 662 and p. 668, respectively. The t distribution has thicker tails than the normal one and is, thus, more appropriate when modelling financial time series. The GED distribution is used because its density function is flexible enough to capture important discrepancies with respect to normality, but nests the normal distribution as a special case when $\nu=2$; in this distribution ν is a positive parameter governing the thickness of the tails. In Table 4 we report the estimates we obtained with their standard errors into brackets (computed using the Berndt *et al.* (1974) (BHHH) procedure), and SIC.

TABLE 4
ESTIMATION OF GARCH(1,1) MODELS

Distrib.	ω	α_1	β_1	h_0	ν	SIC
Normal	.139 (.0302)	.118 (.0206)	.783 (.0387)	1.605 (2.560)	-	-1691.0
t	.082 (.0275)	.161 (.0351)	.785 (.0415)	2.596 (3.964)	6.760 (.8156)	-1631.4
GED	.108 (.0366)	.146 (.0368)	.779 (.0514)	2.546 (5.043)	1.320 (.0460)	-1649.8

Standard errors into brackets.

In order to compare these models, it is also convenient to compute several diagnostics measures based on standardized residuals, constructed as $\hat{\eta}_t = \varepsilon_t / \hat{h}_t^{1/2}$. Specifically, in Table 5 we report skewness (SK) and excess

kurtosis (EK) statistics of $\hat{\eta}_t$ with their asymptotic standard errors into brackets, Bera-Jarque normality test-statistic (BJ) with its asymptotic p-value in square brackets and Box-Ljung statistics with 20 lags for $\hat{\eta}_t$ and $\hat{\eta}_t^2$ ($Q(20)$ and $Q(20)^*$, respectively) with their asymptotic p-values in square brackets.

TABLE 5
DIAGNOSIS OF GARCH(1,1) MODELS

Distrib.	SK	EK	BJ	$Q(20)$	$Q(20)^*$
Normal	-0.8321 (.0740)	10.2427 (.1479)	4904.1 [.0000]	13.83 [.7398]	2.35 [1.000]
t	-1.1498 (.0739)	18.3263 (.1768)	-	13.56 [.7570]	2.08 [1.000]
GED	-1.0042 (.0739)	15.8781 (.1824)	-	13.65 [.7518]	2.10 [1.000]

Standard Errors into brackets; p-values into square brackets.

Results on Tables 4 and 5 show that the normal distribution is by no means appropriate: the BJ test based on skewness and excess kurtosis statistics clearly rejects the null hypothesis of normality; and if we test $H_0: \nu=2$ vs $H_1: \nu \neq 2$ when the GED is used, the asymptotic p-value we obtain is also virtually 0. Between the other two specifications, the t distribution is the preferred one according to SIC.

Some authors have suggested that the problem of non-normality in η_t may be handled leaving that distribution unspecified and using nonparametric

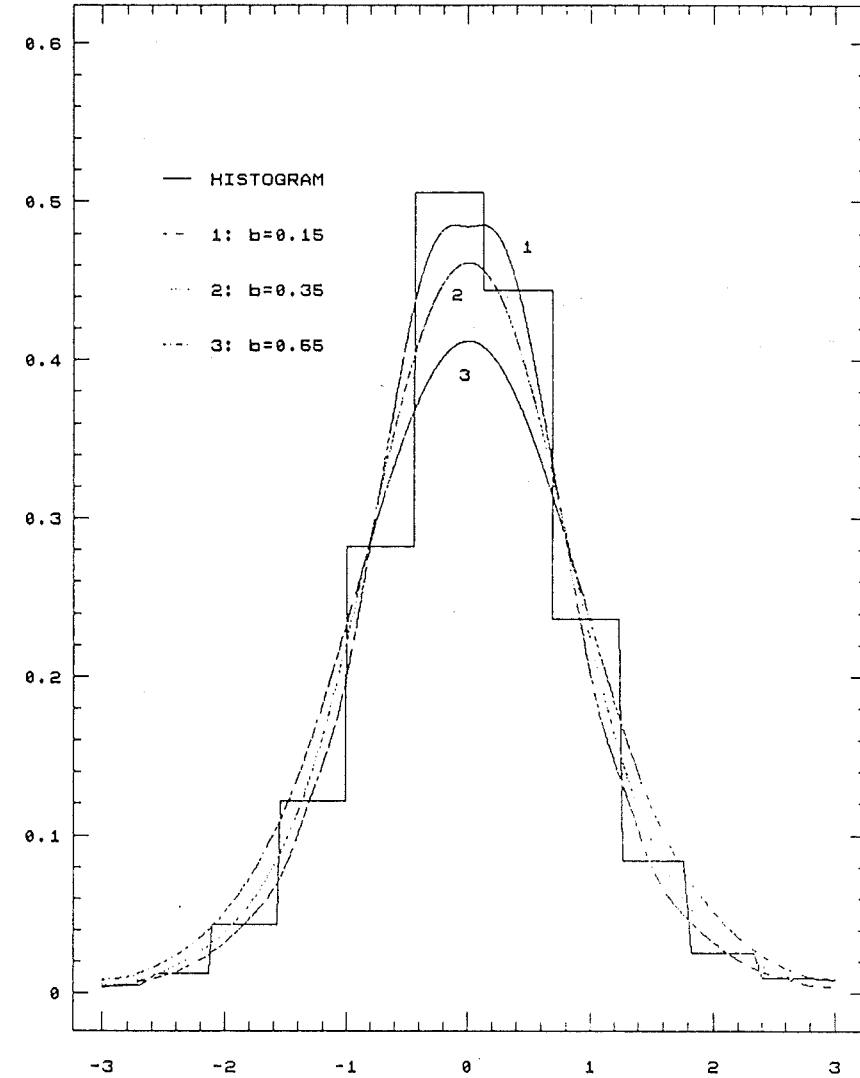
methods to estimate it. Engle and Gonzalez-Rivera (1991) suggested this procedure and Linton (1994) studied it in detail proving that it is possible to obtain an asymptotically efficient estimate of a reparameterized ARCH(q) model and his estimate may be also used in a reparameterized GARCH(p,q) model. Specifically, the semiparametric estimate applies to a GARCH(p,q) in which the equations for ε_t and volatility are, respectively:

$$\varepsilon_t = (h_t^*)^{1/2} \eta_t^*, \quad \eta_t^* \text{ i.i.d, } E[\eta_t^*] = 0, \quad (2)$$

$$h_t^* = 1 + \sum_{j=1}^q \gamma_j \varepsilon_{t-j}^2 + \sum_{i=1}^p \delta_i h_{t-i}^*. \quad (3)$$

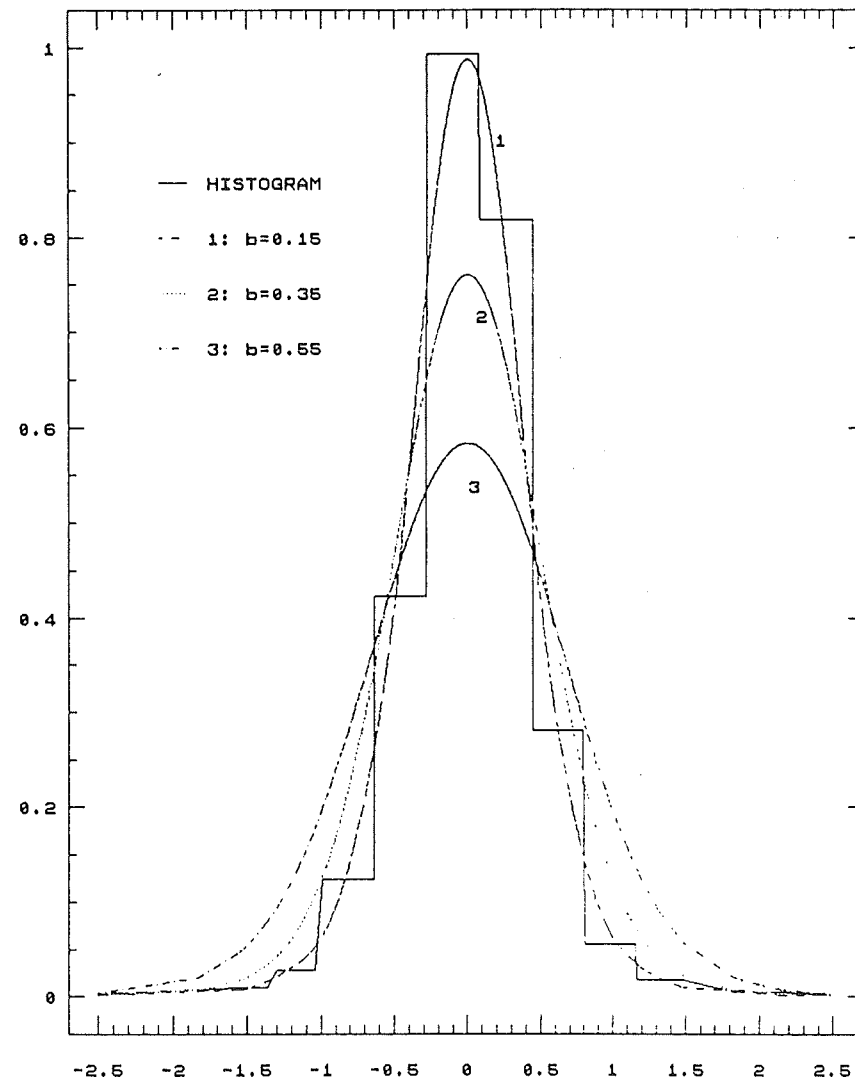
Observe that if we denote $\sigma_\eta^2 \equiv E[\eta_t^*]$ and rewrite (2) and (3) with $\eta_t^* = \sigma_\eta \eta_t$, $h_t^* = h_t / \sigma_\eta^2$, $\sigma_\eta^2 = \omega$, $\gamma_j = \alpha_j / \sigma_\eta^2$ ($1 \leq j \leq q$) and $\delta_i = \beta_i / \sigma_\eta^2$ ($1 \leq i \leq p$) then we obtain a GARCH(p,q) model. This reparameterization is necessary because the restriction $E[\eta_t^2] = 1$ in the standard GARCH(p,q) model is not appropriate for semiparametric estimates. Following Linton (1994) we have estimated an ARCH(3) and a GARCH(1,1) model using FORTRAN (Linton only describes the semiparametric ARCH model, but the generalisation to GARCH models is straightforward). In order to estimate semiparametrically these models, initial estimates $(\tilde{\gamma}, \tilde{\delta}, \tilde{\sigma}_\eta^2)$, a smoothing value b and three trimming values c, d, e must be selected. The initial estimate was obtained by ML estimation assuming a normal distribution for η_t (this estimator is root- n -consistent even though η_t does not follow a normal distribution). We have selected the smoothing value by previous inspection of various nonparametric estimates of the density of η_t , which were obtained using as data the standardized observations $\varepsilon_t / \tilde{h}_t$. These estimates are depicted in Figures 1 and 2.

FIGURE 1



Nonparametric Density Estimate of Standardised Residual (ARCH(3))

FIGURE 2



Nonparametric Density Estimate of Standardised Residuals (GARCH(1.1))

These figures suggest that $b=0.15$ may be an appropriate smoothing value. Anyway, we also report all results for $b=0.35$ and $b=0.55$. As trimming values we selected $c=e=1000$, $d=0.0001$; we also computed estimates with other logical values of c , d and e but results were entirely similar. Finally, following Linton's (1994) advice, we did not use sample splitting or discretization. The estimates we obtained are reported in Table 6.

TABLE 6
SEMIPARAMETRIC ESTIMATION OF ARCH AND GARCH MODELS

Model	ARC(3)	ARC(3)	ARC(3)	ARC(3)	G(1,1)	G(1,1)	G(1,1)	G(1,1)
	Init.	b=.15	b=.35	b=.55	Init.	b=.15	b=.35	b=.55
γ_1	.184 (.100)	.181 (.022)	.206 (.050)	.250 (.064)	.847 (.562)	.849 (.064)	.662 (.073)	.941 (.087)
γ_2	.142 (.081)	.168 (.021)	.187 (.047)	.199 (.061)	-	-	-	-
γ_3	.188 (.132)	.179 (.023)	.213 (.050)	.179 (.065)	-	-	-	-
δ_1	-	-	-	-	.783 (.042)	.690 (.073)	.729 (.052)	.667 (.062)

Standard errors into brackets.

We observe that semiparametric estimates do not differ from parametric ones, though standard errors decrease dramatically in most cases. In Section 5 we compare these estimates using a standard regression efficiency test.

4.2. General Family of GARCH(1,1) models.

In recent years, there have been numerous refinements of GARCH models. These new models have been formulated trying to explain different empirical regularities of data which GARCH models fail to capture, chiefly:

- 1) -Asymmetric behaviour of responses, also known as "leverage effect": negative surprises seem to increase volatility more than positive surprises;
- 2) -Leptokurtosis: fatter tails than those allowed by GARCH models, specially when estimated with normal disturbances;
- 3) -Nonlinearities: the assumption of linear dependence between h_t and $h_{t-j}, \varepsilon_{t-1}^2$ ($i>0, j>0$) seems too restrictive in many situations.

Though most of these generalisations of GARCH models do not display obvious links to one another they all can be embedded in the family of GARCH models proposed by Hentschel (1995). In Table 2 we specify the two equations which describe the family of GARCH(1,1) models (hereafter referred to as Family-G(1,1)). This model contains seven parameters $(\omega_G, \alpha_G, \beta_G, \lambda, \mu, c, b)$ in the variance equation. If we impose restrictions on parameters and reparameterize the model, it is possible to obtain the following models: GARCH(1,1) (if $\lambda=\mu=2, b=c=0$ and we rewrite the variance equation of the Family-G(1,1) with $\omega_G=(\omega+\beta-1)/2, \alpha_G=\alpha/2, \beta_G=\beta$ then we obtain the variance equation of the GARCH(1,1) model); Exponential GARCH (EGARCH) model of Nelson (1991) (restrictions $\lambda=b=0, \mu=1$ on Family-G(1,1)); Threshold GARCH (TGARCH) model of Zakoian (1994) ($\lambda=\mu=1, b=0, |c|\leq 1$); Absolute value GARCH (AGARCH) model of Taylor (1986) and Schwert (1989) ($\lambda=\mu=1, |c|\leq 1$); Nonlinear

asymmetric GARCH (NAGARCH) model of Engle and Ng (1993) ($\lambda=\mu=2, c=0$); GARCH model of Glosten *et al.* (1993) (GJR-GARCH) ($\lambda=\mu=2, b=0$); Nonlinear GARCH (NGARCH) model, natural extension of the Nonlinear ARCH model of Higgins and Bera (1992) ($\lambda=\mu, b=c=0$); and Asymmetric power ARCH (APARCH) model of Ding *et al.* (1993) ($\lambda=\mu, b=0, |c|\leq 1$). To sum up, eight well-known generalisations of ARCH models are nested in the Family-G(1,1). The Quadratic GARCH (Q-GARCH) model of Sentana (1995) is the only popular model which is not nested in the Family-G(1,1).

The main advantage of the Family-G(1,1) is that it is easy to check the validity of most generalisations of GARCH models by simply performing standard likelihood-ratio, Wald or LM tests. As before, we have estimated all models using two different distributions for η_t : t and GED. In Tables 7 and 8 we report the estimates and the BHHH covariance matrix of estimates. In the optimization process the value h_0 was considered as a parameter. With the estimates contained in these tables it is straightforward to carry out Wald tests to analyse the validity of all restricted models. Instead, in Table 9 we prefer to report the results for likelihood ratio tests, with the corresponding asymptotic p-values in square brackets.

TABLE 7

ESTIMATION OF FAMILY-G(1,1) - t Distribution

ω_G	α_G	β_G	λ	μ	b	c	ν
-.1088	.1751	.8278	1.0362	.7367	-.2963	.5504	7.0119
.0024	-.0031	-.0030	.0496	.0142	.0010	-.0015	.0052
	.0045	.0029	-.0573	-.0188	-.0028	.0038	-.0068
		.0280	-.2655	-.0402	-.0031	.0084	.0092
			2.7668	.4857	.0375	-.0972	-.0457
				.1215	.0148	-.0289	.0012
					.0159	-.0137	-.0122
						.0261	-.0235
							.7508

Estimates (first row) and BHHH covariance matrix.

TABLE 8

ESTIMATION OF FAMILY-G(1,1) - GED Distribution

ω_G	α_G	β_G	λ	μ	b	c	ν
-.0640	.0918	.8178	1.6637	1.2544	.0124	.3584	1.3433
.0006	-.0009	-.0003	.0196	.0082	.0006	-.0005	.0006
	.0020	.0011	-.0042	-.0199	-.0070	.0062	-.0011
		.0190	-.2459	-.0537	-.0120	.0168	-.0003
			3.7643	.9244	.1843	-.2452	.0156
				.3316	.1019	-.1183	.0082
					.0926	-.0814	.0036
						.0948	-.0008
							.0026

Estimates (first row) and BHHH covariance matrix.

TABLE 9

Likelihood Ratio Test and SIC for Models nested on Family-G(1,1)

H_0	t Distribution		GED Distribution	
	LR Statistic	SIC	LR Statistic	SIC
GARCH	16.778 [.002]	-1631.4	14.214 [.007]	-1649.8
EGARCH	4.808 [.186]	-1628.9	2.128 [.546]	-1647.3
TGARCH	1.886 [.596]	-1627.5	.384 [.944]	-1646.4
AGARCH	1.136 [.567]	-1630.6	.367 [.832]	-1649.9
NAGARCH	8.278 [.041]	-1630.7	4.062 [.255]	-1648.3
GJR	9.740 [.021]	-1631.4	5.506 [.138]	-1649.0
NGARCH	10.366 [.016]	-1635.2	10.901 [.012]	-1655.2
APARCH	1.834 [.400]	-1634.5	.118 [.943]	-1653.3

Asymptotic p-values in square brackets.

From Tables 7, 8 and 9 we can draw various interesting conclusions. First of all, the t distribution produces better fits than the GED distribution (observe the values of SIC). Moreover, those models with $h_t^{1/2}$ in volatility equation (namely TGARCH, AGARCH) perform better than those with h_t (namely GARCH, NAGARCH, GJR); and TGARCH also performs better than EGARCH. In fact, when the t distribution is used, all models which model variance are rejected ($\alpha=0.05$). The reason why this happens is because the estimate of λ is far from 2 (see Table 7). The NGARCH model is also rejected because the hypothesis $c=0$ is not plausible. Among the other three models, the TGARCH model is preferable to the APARCH model because restriction $\lambda=\mu=1$ can be accepted in both cases. It is more difficult to discriminate between TGARCH

and AGARCH when the t distribution is assumed. The former is nested on the latter with $b=0$; but if we test this individual restriction using the general model, the asymptotic p -value we obtain is only slightly greater than 0.01. However, if we examine the values of SIC we observe that the gain of the the additional parameter in the AGARCH model is not sufficient, in terms of likelihood, to justify its presence. When the GED distribution is used, most conclusions continue to hold, though in this case the models which use variance are not rejected and we can accept that parameter b is 0 (usual significance levels) -thus, the TGARCH model results clearly preferable to AGARCH model.

4.3. Stochastic Volatility Models.

In all preceding models, h_t was known, except for parameters, at time $t-1$. In a stochastic volatility (SV) model, h_t is modelled using an unobserved variable. In this paper we estimate the SV(1) model described in Table 2 (see Ruiz 1994 for more details). Before estimating the model, following Ruiz (1994), we make a reparameterization in order to use the Kalman filter estimation techniques. If we denote $\xi_t \equiv \ln(\eta_t^2) - E[\ln(\eta_t^2)]$, $\epsilon_t^* \equiv \ln(\epsilon_t^2) - E[\ln(\eta_t^2)] - E[\ln(h_t)]$ and $h_t^* \equiv \ln(h_t) - E[\ln(h_t)]$, then we can rewrite the SV(1) equations as:

$$\epsilon_t^* = h_t^* + \xi_t, \quad \xi_t \text{ i.i.d.}, \quad E[\xi_t] = 0, \quad E[\xi_t^2] = \pi/2, \quad (4)$$

$$h_t^* = \beta h_{t-1}^* + \psi_t, \quad \psi_t \text{ i.i.d.}, \quad E[\psi_t] = 0, \quad E[\psi_t^2] = \sigma_\psi^2. \quad (5)$$

Using the fact that $E[\ln(\eta_t^2)] + E[\ln(h_t)] = E[\ln(\epsilon_t^2)]$, it is possible to obtain an estimate $\hat{\mu}_*$ of this quantity and construct approximate observations for $\epsilon_t^* \approx \ln(\epsilon_t^2) - \hat{\mu}_*$. And using these observations we can obtain $\hat{\beta}$ and $\hat{\sigma}_\psi^2$ estimating by quasi maximum likelihood β and σ_ψ^2 (for this estimation we use the Kalman filter error/prediction decomposition; see Hamilton 1994, pp. 372-389). Assuming that $\ln(h_t)$ is stationary it is also possible to obtain a consistent estimate of ω taking into account that then

$$E[\ln(\epsilon_t^2)] = E[\ln(\eta_t^2)] + E[\ln(h_t)] = E[\ln(\eta_t^2)] + \omega/(1-\beta) \quad (6)$$

In this model $\hat{\beta}$, $\hat{\mu}_*$ are consistent estimates of β , $E[\ln(\epsilon_t^2)]$, respectively and $E[\ln(\eta_t^2)] = \varphi(1/2) - \ln(1/2) \approx -1.27$ (see, for example, Abramovitz and Stegun 1970; φ is the digamma function). Therefore (6) allows us to obtain a consistent estimate of ω . The estimation results are (standard errors are shown into brackets):

$$\hat{\omega} = 0.005; \quad \hat{\beta} = 0.972; \quad \hat{\sigma}_\psi^2 = 0.023. \\ (0.012) \quad (0.013)$$

4.4. Poisson Jump Diffusion Model.

In the Poisson Jump Diffusion (PJD) process, a new random variable X_t is introduced in the equation for ϵ_t (see Table 2). With the construction described in Table 2, $\{X_t\}$ proves to be a sequence of i.i.d. random variables having mean $\lambda\mu_Y$ and variance $\lambda(\sigma_Y^2 - \mu_Y^2)$, see Parzen (1972). This variable is introduced in order to capture extraneous shocks which affect

the mean of the observed series. Parameter λ controls how frequent these shocks are. (See, for instance, Kim and Kon 1994 for more details).

In this model, η_t follows a $N(0,1)$ distribution; hence, the log-likelihood function can be written as:

$$\ln[f(\varepsilon_t | I_{t-1})] = \sum_{j=0}^{\infty} \Pr(N_t=j) \times f(\varepsilon_t | I_{t-1}, N_t=j); \quad (7)$$

All terms in (7) can be easily obtained because ε_t conditional to I_{t-1} and $N_t=j$ follows a normal distribution with mean $j\mu_Y$ and variance $h_t + j\sigma_Y^2$. In practice, the sum in (7) must be truncated in the estimation process. Following Ball and Torous (1985) criterion, we have only considered the first 11 terms in the summation. The estimation results are (standard errors are shown into brackets):

$$\begin{aligned} \hat{\omega} &= .062; \hat{\beta} = .789; \hat{\alpha} = .147; \hat{\lambda} = .010; \hat{\mu}_Y = -.477; \hat{\sigma}_Y^2 = 17.899; \hat{h}_0 = 1.817. \\ (.024) \quad (.036) \quad (.036) \quad (.005) \quad (.787) \quad (7.605) \quad (1.167) \end{aligned}$$

The value of SIC for this model is -1631.1143. If we compare this value with those in Table 9, we observe that some generalised GARCH models (specifically, TGARCH, EGARCH, AGARCH and AGARCH with t distribution for η_t) seem preferable to this PJD model.

4.5. Nonparametric Estimation of Conditional Variance.

An alternative approach to model conditional variance consists of simply not specifying any functional form for it and use nonparametric estimates of it.

This approach may be specially useful if the researcher is not interested in parameters relating present conditional variances to past ones, but in analysing the series ε_t with predictive purposes or the like.

The nonparametric estimate of h_t is based on the fact that if $E[\varepsilon_t | I_{t-1}] = 0$, then the conditional variance is simply a conditional expectation: $h_t \equiv \text{Var}(\varepsilon_t | I_{t-1}) = E[\varepsilon_t^2 | I_{t-1}]$. This conditional expectation can be estimated using any standard nonparametric regression estimator. The researcher must only decide what variables should be included in I_{t-1} and the type of nonparametric regression estimator. As the convergence rate of nonparametric estimates decreases as the number of regressors increases, it is convenient not to include too many variables in I_{t-1} . As our study is purely univariate, we have decided to consider $I_{t-1} = \{\varepsilon_{t-1}, \varepsilon_{t-2}\}$ and use as nonparametric estimate $\hat{m}_{2t} = \sum_{j=1, j \neq t}^T w_{jt} \varepsilon_j^2$, with kernel Nadaraya-Watson weights, that is, for $j \neq t$:

$$w_{jt} = \frac{k[(\varepsilon_{t-1} - \varepsilon_{j-1})/b] \times k[(\varepsilon_{t-2} - \varepsilon_{j-2})/b]}{\sum_{s=1, s \neq t}^T k[(\varepsilon_{t-1} - \varepsilon_{s-1})/b] \times k[(\varepsilon_{t-2} - \varepsilon_{s-2})/b]}, \quad (8)$$

if the denominator is not 0, or 0 otherwise. Observe that this is a "leave-one-out" estimate because ε_t is not used to estimate $E[\varepsilon_t^2 | I_{t-1}]$. We have used the Epanechnikov kernel $k(u) = 0.75(1-u^2)$ for $|u| \leq 1$ or 0 otherwise. We have not tried to obtain an optimal smoothing value or "bandwidth" b ; instead we have previously depicted the estimates we obtained for various bandwidths and finally present here the results for three

different smoothing values b which cover all possible cases. In Figures 3, 4 and 5 we show the nonparametric estimates of conditional variance we obtained with $b=0.4$, $b=0.8$ and $b=1.2$, respectively. The performance of these different estimates will be compared in Section 5.

FIGURE 3

Nonparametric Estimate of Conditional Variance, $b=0.4$

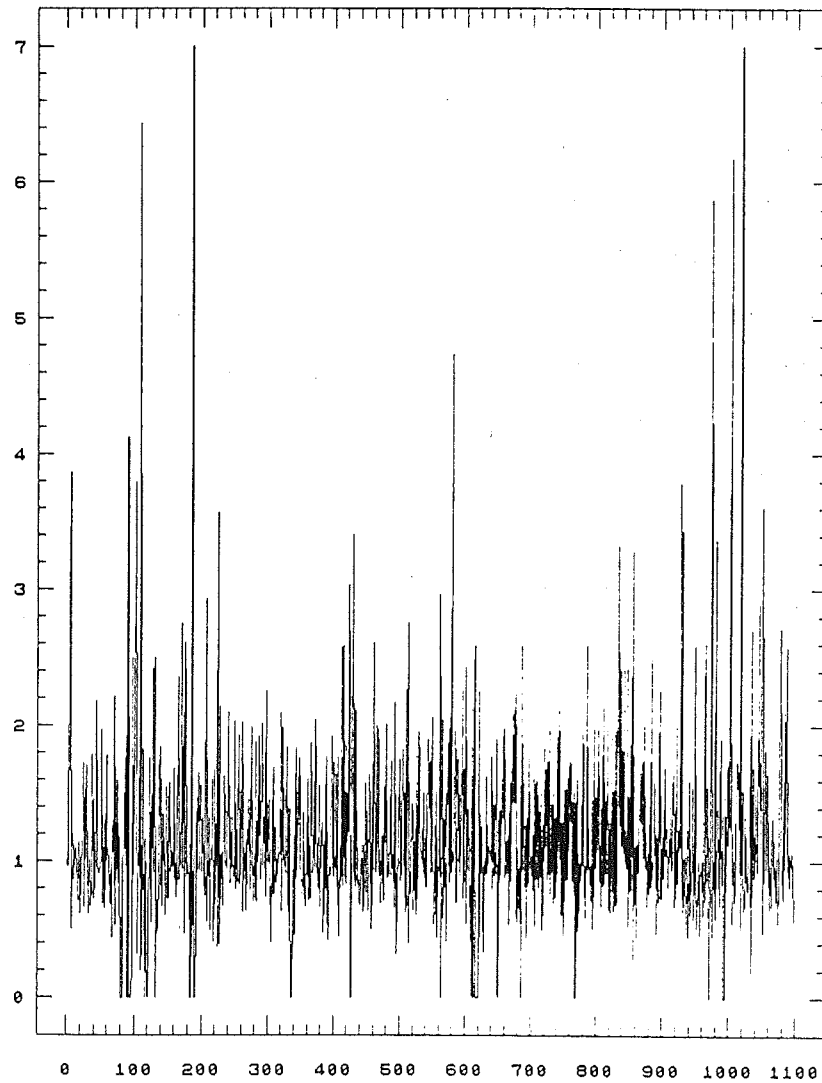
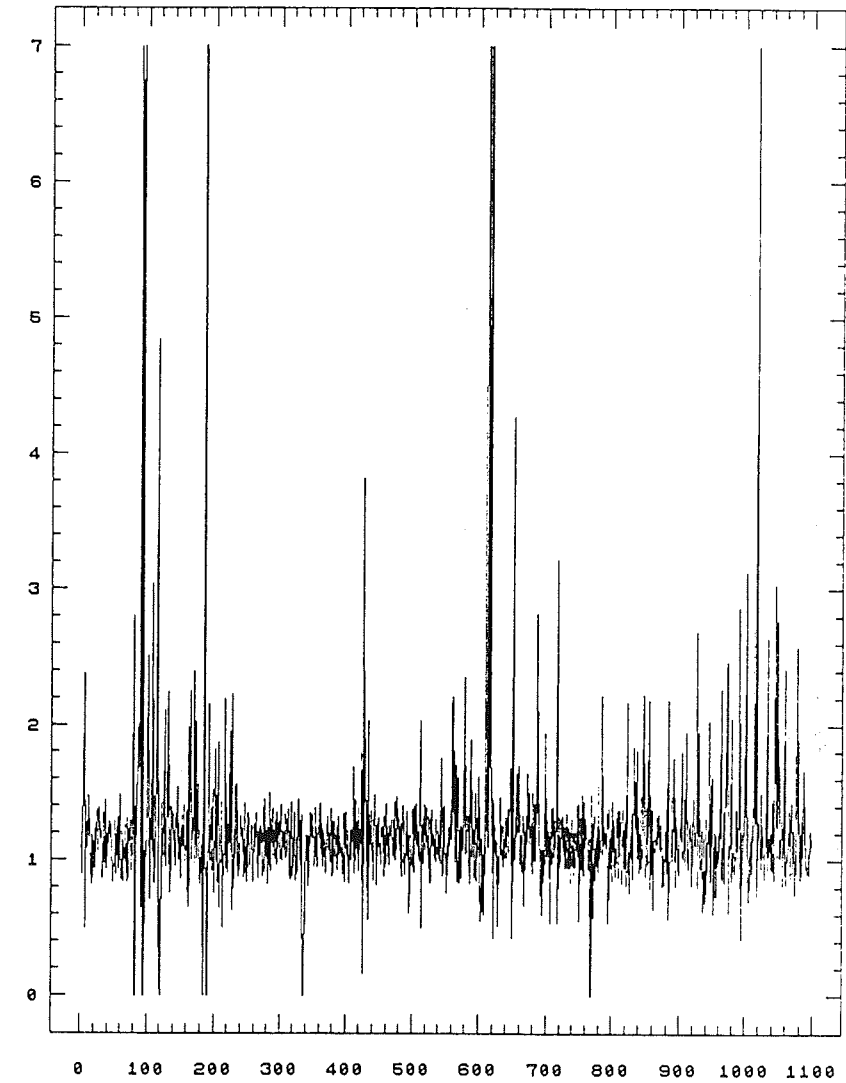
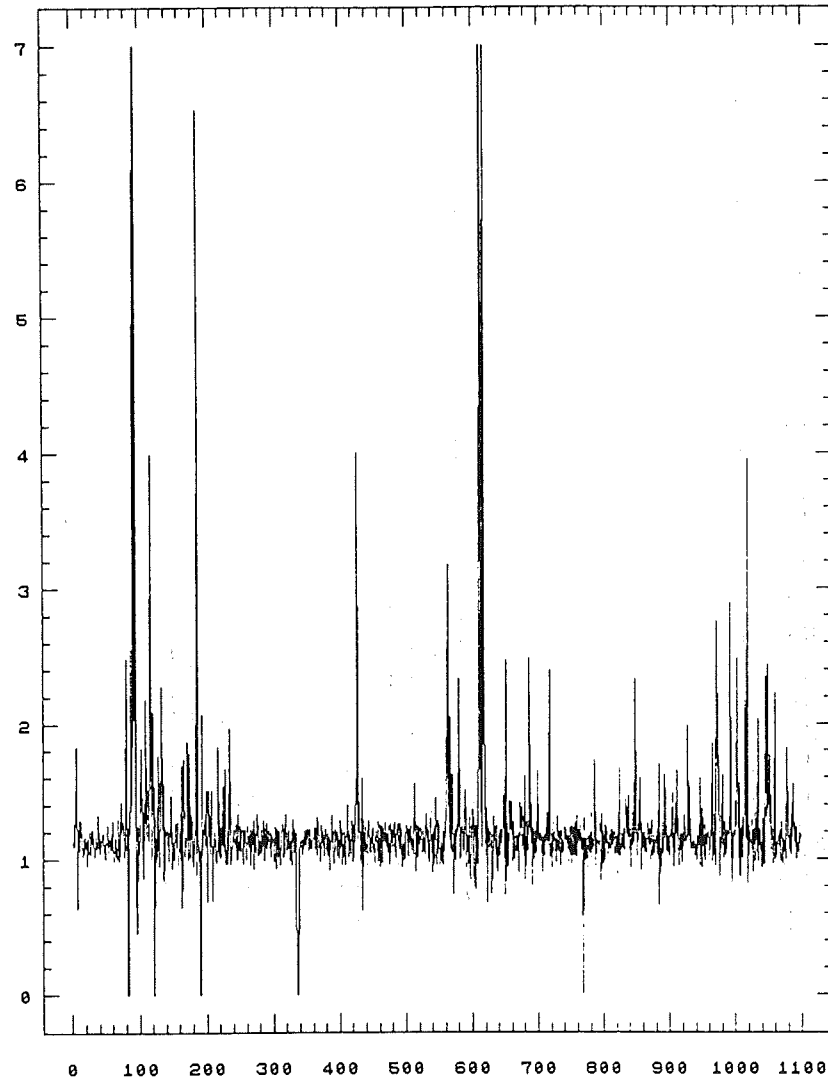


FIGURE 4



Nonparametric Estimate of Conditional Variance, $b=0.8$

FIGURE 5



Nonparametric Estimate of Conditional Variance, $b=1.2$

5. COMPARISON OF "IN-SAMPLE" PREDICTIVE POWER.

We will compare the goodness of fit of the different models carrying out a standard regression efficiency test (see e.g., Pagan and Schwert 1990). The idea behind the regression which is used in this test is the following: if $h_t = E[\epsilon_t^2 | I_{t-1}]$, then $\epsilon_t^2 = \alpha + \beta h_t + v_t$, with $\alpha=0$, $\beta=1$ and v_t is a white noise. As h_t is unknown, we replace h_t by \hat{h}_t and estimate by OLS the equation $\epsilon_t^2 = a + b\hat{h}_t + \text{Error}$; if \hat{h}_t is indeed a good approximation of h_t , then \hat{a} and \hat{b} in this regression should be close to 0 and 1, respectively. Moreover, $h_t = E[\epsilon_t^2 | I_{t-1}]$ also implies that the coefficient of determination R^2 in the regression model $\epsilon_t^2 = \alpha + \beta h_t + v_t$ is higher than the R^2 which would be obtained in any regression model with independent variables within I_{t-1} . Therefore, it is also logical to expect that if \hat{h}_t is a good estimate of h_t then this property of maximum R^2 continues to hold.

We have estimated by OLS the regression equation $\epsilon_t^2 = a + b\hat{h}_t + \text{Error}$ using as observations of $(\hat{h}_t)_{t=1}^{1097}$ the estimates of conditional variance obtained with each one of the models described in previous section. In Table 10 we analyse those models which do not allow for asymmetric behaviour of responses and in Table 11 we analyse those models which do allow for asymmetries. We report the estimates of a and b with its standard errors, the coefficient of determination R^2 and the Ljung-Box statistic of residuals computed with 20 lags ($Q(20)$).

A number of interesting conclusions is derived from Tables 10 and 11.

a) The semiparametric estimates of GARCH models perform better than the parametric estimates, though differences are not high. And this happens even though the assumed distribution for η_t is not normal (observe that the semiparametric GARCH(1,1) models produce higher coefficients of determination than the parametric ones).

b) All generalisations of GARCH models which we have considered in this table perform similarly and they all outperform the traditional GARCH(1,1) model. Surprisingly, the only general model estimated with GED distribution produces a better fit than all models estimated with t distribution. Among these, the AGARCH model performs better, but differences are not remarkable.

TABLE 10

"IN-SAMPLE" PREDICTIVE POWER, Symmetric Models

Model	Distr.	\hat{a}	\hat{b}	R^2	Q(20)
ARCH(3)	Normal	.398 (.185)	.709 (.318)	.0434	54.12
ARCH(3)	Semip. $b=0.15$.409 (.183)	.693 (.098)	.0439	54.33
ARCH(3)	Semip. $b=0.35$.492 (.175)	.599 (.085)	.0430	55.20
GARCH(1,1)	Normal	.116 (.200)	.919 (.117)	.0537	44.15
GARCH(1,1)	Semip. $b=0.15$.240 (.182)	1.188 (.143)	.0591	36.81
GARCH(1,1)	Semip. $b=0.35$.038 (.204)	1.384 (.170)	.0570	39.78
GARCH(1,1)	t	.447 (.168)	.669 (.085)	.0536	44.28
GARCH(1,1)	GED	.341 (.178)	.750 (.095)	.0540	43.83
SV(1)	Normal	-.799 (.274)	1.954 (.220)	.0679	80.59
PJD	Normal	.487 (.165)	.719 (.091)	.0533	44.66

Standard errors into brackets.

c) SV(1) model produces better fits than ARCH and GARCH models, but results obtained with it are not entirely satisfactory because it is not accepted (usual significance levels) that $a=0$ or $b=1$ in this regression; PJD model produces similar results to GARCH(1,1) model.

d) Among the nonparametric estimates, the one with highest smoothing value is the one which performs better (the results for $b=0.4$, not included here, were worse than those obtained for $b=0.8$ and $b=1.20$). Anyhow, all of them perform worse than parametric and semiparametric models.

TABLE 11

"IN-SAMPLE" PREDICTIVE POWER, Asymmetric Models

Model	Distr.	\hat{a}	\hat{b}	R^2	Q(20)
EGARCH(1,1)	t	.028 (.190)	1.024 (.113)	.0700	43.55
TGARCH(1,1)	t	.040 (.196)	1.003 (.116)	.0640	49.26
AGARCH(1,1)	t	.026 (.190)	1.014 (.111)	.0709	47.81
APARCH(1,1)	t	.044 (.193)	.999 (.114)	.0655	48.96
Fam. -G(1,1)	t	.025 (.194)	1.017 (.115)	.0664	48.36
Fam. -G(1,1)	GED	-.084 (.199)	1.084 (.118)	.0713	48.01
Nonparam.	$b=0.80$.831 (.166)	.457 (.093)	.0217	115.88
Nonparam.	$b=1.20$.251 (.223)	.929 (.152)	.0330	94.19

Standard errors into brackets.

e) The hypothesis that the error term in $\varepsilon_t^2 = a + b\hat{h}_t + \text{Error}$ is a white noise process is rejected in all cases. This feature of this regression test has already been observed by other authors (see, for instance, Pagan and Schwert 1990) and is possibly due to the fact that in our regression we replace the true value h_t by an estimate of it.

6. CONCLUSIONS.

This paper intends to present, in a unified way, most of the models for conditionally heteroskedastic time series which have appeared in recent years. We compare their behaviour using the Spanish stock return index IBEX-35.

ARCH and GARCH models use simple equations for conditional heteroskedasticity. But in fact their simplicity makes them unable to capture the empirical regularities observed in most financial time series. Among the various generalisations of GARCH models which have recently appeared in the literature, those models which use conditional standard deviation (specifically TGARCH and AGARCH) have performed better than the rest. Moreover, the leptokurtosis observed in financial time series makes models with t distribution more appropriate. However, the results obtained for the semiparametric GARCH models suggest that semiparametric TGARCH or AGARCH models may outperform parametric TGARCH or GARCH models with t distribution.

We have also analysed here three models which do not fall into the family of GARCH models: Stochastic Volatility (SV), Poisson Jump Diffusion (PJD) and Nonparametric. The former two ones produce worse results than General GARCH models, but possibly because we have only studied here simple versions. It is probable that generalisations of these models may result comparable to the more complicated TGARCH or AGARCH.

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