

**THE EXTENDED CLAIM-EGALITARIAN SOLUTION  
ACROSS CARDINALITIES\***

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THE EXTENDED CLAIM-EGALITARIAN SOLUTION  
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A B S T R A C T

We analyze the behavior of the *extended claim-egalitarian solution* for bargaining problems with claims, in situations involving variations in the number of agents. An axiomatic characterization of this solution is presented.

**Keywords:** Equal-Loss Principle; Bargaining Problems with Claims; Variable Number of Agents.

## 1.- INTRODUCTION

Chun & Thomson (1992) introduced the bargaining problem with claims by adding an unfeasible point representing agents' claims to Nash's (1950) formulation of bargaining problem. The bankruptcy problem is a clear situation covered by this model. Moreover, they showed that the classical axiomatic theory of bargaining, which implies the formulation of "reasonable" properties for solutions and the characterization of them by these conditions, can be adapted to analyze these kind of situations.

Different criteria have been used when proposing solutions for bargaining problems, but the consideration of principles based on losses from some reference point is particularly appealing when the reference point represents claims. In the words of Aumann & Maschler (1985):

"In bankruptcy problems, for example, the creditors will in the end receive checks as a result of the court proceedings. Nevertheless, they are worse off than before making the loan, and they may well conceive of the transaction as a loss rather than an award"

On other hand, one of the recent developments of axiomatic theory of bargaining consists of evaluating the solutions dependence on the number of agents. This approach has been systematically analyzed, for bargaining problems, by Thomson & Lensberg (1989) and has been considered by Chun & Thomson (1992) for bargaining problems with claims.

In the present paper we focus our attention on a solution proposed for bargaining problems with claims by Bossert (1993), and subsequently

studied by Marco (1994), under the name of *extended claim-egalitarian solution*. We analyze its behavior when the number of agents involved in a problem varies.

This solution divides losses from the claims point equally among all agents, subject to no agent losing more than that which corresponds to the disagreement point.

This is a straightforward adaptation for the domain at hand of the *rational equal-loss solution*, proposed by Herrero & Marco (1993) for the classic bargaining problem, and it can be viewed as a "dual" of the *constrained equal award solution* presented by Aumann & Maschler (1985), for the "transferable utility case".

Section 2 contains some preliminaries and definitions. Section 3 is devoted to the behavior of the *extended claim-egalitarian solution* when the number of agents changes. Section 4 presents a characterization of the aforementioned solution using axioms concerning variations in the number of agents.

## 2.- PRELIMINARIES

There is a infinite universe  $I = \{1,2,.. \}$  of "potential agents, although only a finite number of them are present at a given time. Let  $\mathcal{P}$  the class of finite subsets of  $I$ . Elements of  $\mathcal{P}$  are denoted by  $P, Q, \dots$ . For  $P \in \mathcal{P}$ , let  $|P|$  be the cardinal of  $P$ . Given  $P \in \mathcal{P}$ ,  $\mathbb{R}^P$  is the Cartesian product of  $|P|$  copies of  $\mathbb{R}$  indexed by the members of  $P$ .

The group of agents  $P$  can face a bargaining problem with claims, or simply a problem, which is a triple  $(S, d, c)$ , where  $S \subset \mathbb{R}^P$  is the feasible set in utility levels,  $d \in \mathbb{R}^P$  is the disagreement point, and the coordinates of  $c \in \mathbb{R}^P \setminus S$  represent the agents' claims. Claims are not mutually compatible. The interpretation of  $(S, d, c)$  is as follows: the agents can achieve any point of  $S$  if they unanimously agree on it. Otherwise, they end up at the disagreement point  $d$ .

Let  $\Sigma^P$  be the class of bargaining problems with claims  $(S, d, c)$  such that:

- (i)  $S \subset \mathbb{R}^P$ ,  $S \neq \emptyset$ , convex and closed
- (ii)  $\exists x \in S$  with  $x \gg d^{(1)}$
- (iii)  $S$  is comprehensive:  $\forall x \in S, \forall y \in \mathbb{R}^P$ , if  $y < x$ . then  $y \in S$
- (iv)  $\exists p \in \mathbb{R}_{++}^P, r \in \mathbb{R} \mid \forall x \in S \text{ } px \leq r$
- (v)  $c \notin S, c > d$

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<sup>1</sup> Vector inequalities  $\geq, >, \gg$

Every element of  $\Sigma^P$  is a bargaining problem with claims that the group of agents  $P$  may face. Let  $\Sigma = \bigcup_{P \in \mathcal{P}} \Sigma^P$  denotes the class of all bargaining problems with claims for some admissible group of agents.

A **solution** is a function  $F$ , defined on  $\Sigma$ , that associates to every  $P \in \mathcal{P}$  and every  $(S,d,c) \in \Sigma^P$  a unique element of  $S$ , interpreted as a compromise recommended for that problem.

For any  $P \in \mathcal{P}$  and  $(S,d,c) \in \Sigma^P$ , we shall denote the set of individually rational points by  $IR(S,d)$ ,  $IR(S,d) = \{x \in S \mid x \geq d\}$ .  $PO(S)$  will denote the set of Pareto optimal points,  $PO(S) = \{x \in S \mid \text{if } y > x, \text{ then } y \notin S\}$ , and  $WPO(S)$  the set of weakly Pareto optimal points,  $WPO(S) = \{x \in S \mid \text{if } y \gg x, \text{ then } y \notin S\}$ . We will use  $\bar{S}$  to denote the comprehensive hull of the set  $IR(S,d)$ .

In the axiomatic approach to the bargaining problem, properties on how the division should be carried out are formulated and rules that satisfy these properties are sought. The following axioms, which will be used in the remainder of the paper, are standard in the literature.

**(PO) Pareto optimality:** For all  $P \in \mathcal{P}$ , for all  $(S,d,c) \in \Sigma^P$ ,  $F(S,d,c) \in PO(S)$ .

**(WPO) Weak Pareto optimality:** For all  $P \in \mathcal{P}$ , for all  $(S,d,c) \in \Sigma^P$ ,  $F(S,d,c) \in WPO(S)$ .

**(SY) Symmetry:** For all  $P \in \mathfrak{P}$ , for all  $(S,d,c) \in \Sigma^P$ , if for all permutation  $\pi: \{1, \dots, |P|\} \rightarrow \{1, \dots, |P|\}$ ,  $S = \pi(S)$ ,  $d = \pi(d)$  and  $c = \pi(c)$ , then  $F_i(S,d,c) = F_j(S,d,c)$  for all  $i, j \in P$ .

**(AN) Anonymity:** For all  $P, Q$  in  $\mathfrak{P}$  with  $|P| = |Q|$ , for all  $(S,d,c) \in \Sigma^P$ , for all  $(S',d',c') \in \Sigma^Q$  and for all one-to-one function  $\gamma: P \rightarrow Q$ , if  $S' = \{x' \in \mathbb{R}^Q \mid \exists x \in S \text{ with } x'_{\gamma(i)} = x_i \text{ for all } i \in P\}$ ,  $d'_{\gamma(i)} = d_i$ , and  $c'_{\gamma(i)} = c_i$  for all  $i \in P$ , then  $F_{\gamma(i)}(S',d',c') = F_i(S,d,c)$  for all  $i \in P$ .

**(T.INV) Translation invariance:** For all  $P \in \mathfrak{P}$ , for all  $(S,d,c) \in \Sigma^P$ , for all  $t \in \mathbb{R}^P$ ,  $F(\{S+t\}, d+t, c+t) = F(S,d,c) + t$

**(SC.INV) Scale invariance:** For all  $P \in \mathfrak{P}$ , for all  $(S,d,c) \in \Sigma^P$ , for all  $\lambda: \mathbb{R}^n \rightarrow \mathbb{R}^n$  such that for all  $x \in \mathbb{R}^n$   $\lambda_i(x) = a_i x + b_i$  with  $a_i \in \mathbb{R}_{++}$  and  $b_i \in \mathbb{R}$ ,  $F(\lambda(S), \lambda(d), \lambda(c)) = \lambda(F(S,d,c))$ .

**(CONT) Continuity:** For all  $P \in \mathfrak{P}$ , for all sequences  $\{(S^\nu, d^\nu, c^\nu)\}$  of problems of  $\Sigma^P$  and for all  $(S,d,c) \in \Sigma^P$ , if  $S^\nu \rightarrow S$ ,  $d^\nu \rightarrow d$  and  $c^\nu \rightarrow c$ , then  $F(S^\nu, d^\nu, c^\nu) \rightarrow F(S,d,c)$  (convergence is evaluated in the Hausdorff topology).

**(IR) Individual rationality:** For all  $P \in \mathfrak{P}$ , for all  $(S,d,c) \in \Sigma^P$ ,  $F(S,d,c) \geq d$ .

**(ST.IR) Strong individual rationality:** For all  $P \in \mathfrak{P}$ , for all  $(S,d,c) \in \Sigma^P$ ,  $F(S,d,c) \gg d$ .



From now on, given  $A \subseteq \mathbb{R}^P$ ,  $\text{Com}\{A\}$  will denote the comprehensive hull of set  $A$ ,  $\text{ComCo}\{A\}$  the comprehensive and convex hull of set  $A$ , and  $\text{r.int}\{A\}$  the relative interior of  $A$ .

**(INIR)** *Independence of non-individually rational alternatives:* For all  $P \in \mathcal{P}$  and for all  $(S, d, c), (S', d', c') \in \Sigma^P$ , if  $(d, c) = (d', c')$  and  $S' = \text{Com}\{x \in S \mid x \geq d\}$ , then  $F(S', d', c') = F(S, d, c)$ .

**(IUA)** *Independence of unclaimed alternatives;* For all  $P \in \mathcal{P}$  and for all  $(S, d, c), (S', d', c') \in \Sigma^P$ , if  $(d, c) = (d', c')$  and  $S' = \{x \in S \mid x \leq c\}$ , then  $F(S', d', c') = F(S, d, c)$ .

*Pareto optimality* requires no domination of the alternative selected by any other feasible alternative. *Weak Pareto optimality* only asks that there not be strict domination. *Symmetry* says that if all agents are identical, they should receive the same payoffs. *Anonymity* says that the names of the agents are irrelevant when proposing the solution outcome. *Translation invariance* requires that the choice of origin of the utility functions does not matter. *Scale invariance* requires that the solution be independent of the choice of origins and scales of the utility functions representing the agents' preferences. *Continuity* says that "small" variations in a problem do not cause "large" changes in the solution outcome. *Individual rationality* says that the solution outcome always weakly dominates the disagreement point. *Strong individual rationality* asks the solution outcome to provide strict gains from the disagreement point to all agents. *Independence of non-individually rational alternatives* says that the solution outcome is unaffected by the non-individually

rational alternatives. Finally, *Independence of unclaimed alternatives* requires that the feasible alternatives that are not dominated by the claims point be ignored.

Bossert (1993) introduced the equal-loss principle in bargaining problems with claims for the fixed population case by means of the two following solutions.

**Definition 1** [Bossert (1993)]:

For all  $P \in \mathcal{P}$ , and for all  $(S, d, c) \in \Sigma^P$ , the *claim-egalitarian solution*,  $E$ , selects the weakly Pareto optimal point  $y$  in  $S$  such that  $|c_i - y_i| = |c_j - y_j|$  for all  $i, j \in P$ .

The next definition will be used later on. It can be checked that it is equivalent to the definition of the *extended claim-egalitarian solution* that appears in Bossert (1993).

**Definition 2** [Marco (1994)]:

For all  $P \in \mathcal{P}$ , and for all  $(S, d, c) \in \Sigma^P$ , the *extended claim-egalitarian solution*,  $E^*$ , selects the following alternative:

$$E_i^*(S, d, c) = \begin{cases} d_i & \text{if } E_i(\bar{S}, d, c) < d_i \\ E_i(\bar{S}, d, c) & \text{if } E_i(\bar{S}, d, c) \geq d_i \end{cases}$$

### 3.- AXIOMS ACROSS CARDINALITIES

In this section, by using different axioms which have been introduced in the literature, we analyze how the *extended claim-egalitarian solution* responds to variations in the number of agents. Some conditions are requirements of solidarity. Among those, the most important one is *population monotonicity*. Others are stability requirements, *consistency* being the main one.

The *population monotonicity* principle has been applied to several contexts and has played an important role in axiomatic analysis (see Thomson (1994)). It was introduced for classic bargaining problems by Thomson (1983) and reformulated for the domain at hand by Chun & Thomson (1992). We show that the *extended claim-egalitarian solution* verifies *population monotonicity*, and note that it is incompatible with two standard conditions which are presented above.

In order to formulate this property, we need additional notation. For  $P \in \mathfrak{P}$ , let  $e_P$  be the  $|P|$ -dimensional vector with all coordinates equal to 1. Given  $P, Q \in \mathfrak{P}$  such that  $P \subset Q$ , and  $y \in \mathbb{R}^Q$ , let  $y_P$  the projection of  $y$  onto  $\mathbb{R}^P$ , and given  $(S, d, c) \in \Sigma^Q$ , let  $t_P^y(S) = \{x \in \mathbb{R}^P \mid (x, y_{Q \setminus P}) \in S\}$ .

*Population monotonicity* expresses a form of solidarity among agents: the arrival of new claimants, when resources remain fixed, does not benefit anybody, that is, the cost of an increase in the number of agents with the same opportunities is shared among all initially present agents.

**(POP.MON) Population monotonicity:** For all  $P, Q \in \mathfrak{P}$ , for all  $(S, d, c) \in \Sigma^P$  and for all  $(S', d', c') \in \Sigma^Q$ , if  $P \subseteq Q$ ,  $(d'_P, c'_P) = (d, c)$  and  $t_P^{d'}(S') = S$ , then  $F_P(S', d', c') \leq F(S, d, c)$ .

**Proposition 1:**  $E^*$  satisfies *population monotonicity*.

**Proof:** Let  $P, Q \in \mathfrak{P}$ ,  $P \subseteq Q$ . Given  $(T, d', c') \in \Sigma^Q$ , let  $(S, d, c) \in \Sigma^P$  with  $S = t_P^{d'}(T)$  and  $(d, c) = (d'_P, c'_P)$ . Now we take  $(\bar{S}, d, c) \in \Sigma^P$  and  $(\bar{T}, d', c') \in \Sigma^Q$ , so  $\bar{S} = t_P^{d'}(\bar{T})$  and since  $c'_P = c$ ,  $c'_P - E_P(\bar{T}, d', c')$  is proportional to  $c - E(\bar{S}, d, c)$ . In addition  $E_P(\bar{T}, d', c') \in \bar{S}$  and since  $E(\bar{S}, d, c) \in \text{WPO}(\bar{S})$ , it follows that  $E_P(\bar{T}, d', c') \leq E(\bar{S}, d, c)$ . Now, taking into account that  $d'_P = d$ , from definition 2 we conclude  $E^*(T, d', c') \leq E^*(S, d, c)$ . ■

The next proposition shows that *population monotonicity* cannot be satisfied on the domain  $\Sigma$  in conjunction with *Pareto optimality* and *anonymity*. Note that this result can be straightforwardly adapted to the classic bargaining problem, where other incompatibility results related to this one have been reported (see corollary 3.2 in Thomson & Lensberg (1989)).

**Proposition 2:** There is no solution on  $\Sigma$  satisfying *Pareto optimality*, *anonymity* and *population monotonicity*.

**Proof (Fig.1):** Let  $F$  be a solution satisfying *PO*, and *AN*. Let  $Q = \{1, 2, 3\}$ ,  $S' = \text{ComCo}\{(2, 1, 0), (0, 1, 2), (1, 2, 1)\}$ ,  $d' = (0, 0, 0)$  and  $c' = 2e_Q$ . Note that  $(S', d', c') \in \Sigma^Q$ . By *AN* agents 1 and 3 must be equally treated. Since there is a unique Pareto optimal point with  $x_1 = x_3$ , that is the alternative

$(1,2,1)$ ,  $F(S',d',c') = (1,2,1)$ . Consider  $P = \{1,2\}$ , then  $t_P^{d'}(S') = \text{ComCo}(\{(2,1), (1,2)\})$ ,  $d'_P = (0,0)$  and  $c'_P = 2e_P$ . By PO and AN,  $F(t_P^{d'}(S'),d'_P,c'_P) = (3/2)e_P$  in contradiction to POP. MON because  $F_2(t_P^{d'}(S'),d'_P,c'_P) < F_2(S',d',c')$ . ■

Next we consider two properties related to *population monotonicity*, introduced for classical bargaining theory by Thomson (forthcoming).

The first axiom says that the feasible alternatives of a problem at which one of the agents is better off than he would be at the solution outcomes of any of the subproblems obtained after the departure of some of the agents, are irrelevant. Notice that by assuming a commitment to *population monotonicity*, these alternatives would be out of reach.

**(TR.IND) Truncation independence:** For all  $Q \in \mathcal{P}$ , for all  $(T,d,c)$ ,  $(T',d',c') \in \Sigma^Q$ , if  $T' = \{x \in T \mid \text{for all } P \subset Q \ x_P \leq F(t_P^d(T),d_P,c_P)\}$  and  $(d',c') = (d,c)$ , then  $F(T',d',c') = F(T,d,c)$ .

**Proposition 3:**  $E^*$  verifies *truncation independence*.

**Proof:** Let  $Q \in \mathcal{P}$ , and  $(T,d,c) \in \Sigma^Q$  with  $E^*(T,d,c) \gg d$ , otherwise TR.IND does not apply. Let  $(T',d',c') \in \Sigma^Q$  be such that  $T' = \{x \in T \mid \text{for all } P \subset Q \ x_P \leq F(t_P^d(T),d_P,c_P)\}$  and  $(d',c') = (d,c)$ . Since  $E^*(T,d,c) \gg d$ ,  $E^*(T,d,c) = E(\bar{T},d,c)$ , and because  $c' = c$ ,  $c' - E(\bar{T}',d',c')$  is proportional to  $c - E^*(T,d,c)$ . In addition  $E^*(T,d,c) \in \bar{T}' = T'$  since  $E^*$  satisfies POP.MON (proposition 1). Moreover  $\bar{T}' \subseteq \bar{T}$  and  $E(\bar{T},d,c) \in \text{WPO}(\bar{T})$ , so it follows that

$E(\bar{T}', d', c') = E(\bar{T}, d, c) = E^*(T, d, c)$ . Therefore  $E(\bar{T}', d', c') \gg d = d'$ , so  
 $E^*(T', d', c') = E(\bar{T}', d', c') = E(\bar{T}, d, c) = E^*(T, d, c)$ . ■

Notice that *truncation independence* only applies when the solution outcome for the original problem is strictly greater than the disagreement point, so *strong individual rationality* and *truncation independence* imply *population monotonicity*.

The second axiom, *sequential invariance*, was introduced for bargaining problems with claims by Chun & Thomson (1992). Suppose that a problem is obtained on some agents departing from another one. This property says that the following is equivalent: to take into account the agreement achieved when all agents were present, thus dividing among the final agents the additional available gains, or to consider the initial agreement as invalid and therefore solve the final by starting all over again.

**(SEQ.INV) Sequential Invariance:** For all  $P, Q \in \mathfrak{P}$ , for all  $(S, d, c) \in \Sigma^P$  and  $(S', d', c') \in \Sigma^Q$ , if  $P \subseteq Q$ ,  $(d'_P, c'_P) = (d, c)$ ,  $t_P^{d'}(S') = S$  and  $(S, F_P(S', d', c'), c) \in \Sigma^P$ , then  $F(S, d, c) = F(S, F_P(S', d', c'), c)$ .

In order to prove that the *extended claim-egalitarian solution* verifies *sequential invariance* we need a previous result which is contained in the following proposition. We omit its proof because it is closely related to proposition 1 in Herrero & Marco (1993).

**Proposition 4:** For all  $P \in \mathfrak{P}$ , for all  $(S, d, c) \in \Sigma^P$ ,  $E^*$  minimizes the function  $\max\{|c_i - x_i|\}$  within the set  $IR(S, d)$ .

**Proposition 5:**  $E^*$  satisfies *sequential invariance*

**Proof:** let  $P, Q \in \mathcal{P}$  with  $P \subset Q$ . Given  $(S', d', c') \in \Sigma^Q$ , let  $S = t_P^{d'}(S')$  and  $(d, c) = (d'_P, c'_P)$ , note that  $(S, d, c) \in \Sigma^P$ . We assume that  $(S, E_P^*(S', d', c'), c') \in \Sigma^P$ . Suppose that  $WPO(S') = PO(S')$ . By definition 2,  $E_P^*(S', d', c') \geq d'_P = d$ , so  $IR(S, E_P^*(S', d', c')) \subseteq IR(S, d)$ . Furthermore, given that  $E^*$  verifies POP.MON (proposition 1), we have  $E^*(S, d, c) \geq E_P^*(S', d', c')$ , so that  $E^*(S, d, c) \in IR(S, E_P^*(S', d', c'))$ .

Let  $x^* = E^*(S, d, c)$ . By proposition 4, and bearing in mind that  $S'$  is strictly convex, we obtain:

$$\max_{i \in P} \{ |c_i - x_i^*| \} < \max_{i \in P} \{ |c_i - x_i| \} \quad \forall x \in IR(S, d)$$

This inequality is verified for any subset of  $IR(S, d)$  containing  $x^*$ , in particular for all  $x \in IR(S, E_P^*(S', d', c'))$ . Now, again by proposition 4 and the strict convexity of  $S'$ , we get  $x^* = E^*(S, E_P^*(S', d', c'), c)$ .

For an arbitrary element  $(S', d', c') \in \Sigma^Q$  we obtain the desired conclusion by applying CONT. ■

It is obvious to check that *continuity*, *strong individual rationality* and *sequential invariance* together imply *population monotonicity*.

Now, we consider stability requirements.

The *consistency* principle has been adapted to diverse areas because it is an appeal stability requirement. See Thomson (1990) for an overview. In bargaining theory it was originally introduced by Harsanyi (1959), although he only asked for no renegotiation in subproblems facing two agents. *Consistency* says that no member of a subgroup of initial agents, facing the subproblem that arises when the agents outside the subgroup receive the utility levels corresponding to the solution outcome, would ever renegotiate.

**(CONS) Consistency:** For all  $P, Q \in \mathcal{P}$ , for all  $(S, d, c) \in \Sigma^P$  and for all  $(S', d', c') \in \Sigma^Q$ , if  $P \subset Q$ ,  $(d'_P, c'_P) = (d, c)$  and  $t_P^x(S') = S$  where  $x = F(S', d', c')$ , then  $F(S, d, c) = x_P$ .

**Proposition 6:**  $E^*$  does not satisfy *consistency*.

**Proof (Fig.2):** Let  $Q = \{1, 2, 3\}$ ,  $T = \text{ComCo}\{(1, 2, 2)\}$ ,  $c = 3e_Q$  and  $d = 0$ , then  $(T, d, c) \in \Sigma^Q$ . Note that  $E^*(T, d, c) = e_Q$  and therefore  $t_P^{e_Q}(T) = \text{ComCo}\{2e_P\}$ , where  $P = \{2, 3\}$ , so  $E^*(t_P^{e_Q}(T), d_P, c_P) = 2e_P \neq e_P$ . ■

The following property, which is a slightly weaker version of *consistency*, was introduced for bargaining problems by Thomson & Lensberg (1989). We show that the *extended claim-egalitarian solution* verifies it.

*Weak consistency* says that renegotiations by subgroups of initial agents, keeping the rest at levels specified by the solution outcome, would benefit all agents in the subgroup, instead of being equal for them. Notice that this requirement together with *weak Pareto optimality* implies



*consistency* on the subdomain of problems whose weak Pareto optimal and Pareto optimal boundaries coincide.

**(W.CONS)** *Weak consistency*: For all  $P, Q \in \mathcal{P}$ , for all  $(S, d, c) \in \Sigma^P$  and for all  $(S', d', c') \in \Sigma^Q$ , if  $P \subset Q$ ,  $(d_P, c_P) = (d, c)$  and  $t_P^x(S') = S$  where  $x = F(S', d', c')$ , then  $F(S, d, c) \geq x_P$ .

**Proposition 7:**  $E^*$  satisfies weak consistency.

**Proof:** Let  $P, Q \in \mathcal{P}$  with  $P \subset Q$  and  $(T, d, c) \in \Sigma^Q$  such that  $(S, d_P, c_P) \in \Sigma^P$  where  $S = t_P^x(T)$  and  $x = E^*(T, d, c)$ . Let  $Q_1 = \{i \in Q \mid E_i^*(T, d, c) > d_i\}$  and  $Q_2 = Q \setminus Q_1$ . By definition 2,  $E_t^*(T, d, c) = d_t$  for all  $t \in Q_2$ ,  $c_i - x_i = c_j - x_j$  for  $i, j \in Q_1$  and  $c_i - x_i \geq c_t - x_t$  for all  $i \in Q_2$ . Now suppose that  $E^*$  does not verify W.CONS, then there is  $k \in P$  such that  $E_k^*(S, d_P, c_P) < x_k$ . Since  $E^*$  verifies IR,  $k \in Q_1$ , therefore  $c_k - E_k^*(S, d_P, c_P) > c_k - x_k = \max_{i \in P} c_i - x_i$ , then  $\max_{i \in P} c_i - E_i^*(S, d_P, c_P) > \max_{i \in P} c_i - x_i$  which contradicts proposition 4. ■

The following condition we consider appears in Thomson (forthcoming). It is dual to *consistency*, and it allows us to deduce the solution outcome of a problem involving a large group of agents by resolving the two-person problems obtained from the original one.

*Converse consistency* says that if for some problem, a feasible alternative,  $x$ , is such that its restrictions on all subgroups of cardinality two constitute the solution outcome of the subproblems obtained by the departure of the rest of agents with the utility level corresponding to  $x$ , then  $x$  is the solution outcome for the problem involving all agents.

**(CONV.CONS)** *Converse consistency*: For all  $Q \in \mathcal{P}$ , for all  $(T, d, c) \in \Sigma^Q$ , and for all  $x \in T$ , if for all  $P \subset Q$  with  $|P| = 2$ ,  $(t_P^x(T), d_P, c_P) \in \Sigma^P$  and  $x_P = F(t_P^x(T), d_P, c_P)$ , then  $x = F(T, d, c)$ .

**Proposition 8:**  $E^*$  verifies *converse consistency*.

**Proof:** Let  $Q \in \mathcal{P}$ ,  $(T, d, c) \in \Sigma^Q$  and  $x \in T$  such that for all  $P \subset Q$  with  $|P| = 2$ ,  $(t_P^x(T), d_P, c_P) \in \Sigma^P$  and  $x_P = E^*(t_P^x(T), d_P, c_P)$ . We assume  $c = e_Q$  by T.INV. Then we find two possibilities:

(i) if  $x_i > d_i$  for all  $i \in Q$ , for all  $P \subset Q$  with  $|P| = 2$ ,  $E^*(t_P^x(T), d_P, c_P) = E(t_P^x(T), d_P, c_P)$ , then  $x_i = x_j$  for all  $i, j \in Q$ . Since  $x \in T$ ,  $c - E^*(T, d, c)$  is proportional to  $c - x$ . Now, since  $E^*$  verifies WPO and  $T$  is comprehensive, there is no  $x' \in T$  such that  $x' \gg x$ , otherwise  $x_P \neq E^*(t_P^x(T), d_P, c_P)$ , so by applying again WPO,  $E^*(T, d, c) = x$ .

(ii) if there is  $j \in Q$  such that  $x_j = d_j$ , then  $x_i > d_i$  for all  $i \in Q$ ,  $i \neq j$ , since at  $x$  there is at most one agent with the utility level corresponding to the disagreement point. Otherwise, there is  $P \subset Q$  with  $|P| = 2$  such that  $(t_P^x(T), d_P, c_P) \notin \Sigma^P$ , or, since  $E^*$  verifies WPO,  $x_P \neq E^*(t_P^x(T), d_P, c_P)$ . Therefore for all  $P \subset Q$  with  $|P| = 2$  and  $j \notin P$ ,  $E^*(t_P^x(T), d_P, c_P) = E(t_P^x(T), d_P, c_P)$ , then  $x_i = x_k$  for all  $i, k \neq j$ ,  $i, k \in Q$  and  $x_j \geq x_i$   $i \in Q$ ,  $i \neq j$ . If  $x_j = x_i$ , by using the same argument as in (i), we get the desired conclusion. If  $x_j > x_i$ , there is  $y \in \bar{T}$  with  $y_i = x_i$  for all  $i \in Q$ ,  $i \neq j$  and  $y_j < d_j$  such that  $c - E(\bar{T}, d, c)$  is proportional to  $c - y$ . Since  $E$  satisfies WPO and  $T$  is comprehensive, there is no  $y' \in \bar{T}$  such that  $y' \gg y$ , because if it is the case, for some  $P \subset Q$  with  $|P| = 2$  such that  $j \notin P$ ,  $x_P = y_P \neq E^*(t_P^x(T), d_P, c_P)$ , so by applying again WPO,  $E(\bar{T}, d, c) = y$ . Then by definition 2,  $E^*(S, d, c) = x$ . ■

#### 4.- CHARACTERIZATION OF THE EXTENDED CLAIM-EGALITARIAN SOLUTION

In order to characterize the extended claim-egalitarian solution we consider, as well as some of the conditions presented before, the following independence requirement.

Suppose that, the claims point being fixed, the set of acceptable agreements to all agents enlarges and the solution outcome does not change, so that the new alternatives have been "revealed" to be irrelevant according to an internal argument, that is, to the solution itself. In this case the axiom of *fixed claims independence revealed* requires that adding a subset of such irrelevant elements should not change the players' view of the situation either.

**(FCIR)** *Fixed claims independence revealed:* For all  $P \in \mathfrak{P}$ , for all  $(S^1, d^1, c), (S^2, d^2, c) \in \Sigma^P$  with  $IR(S^2, d^2) \subseteq IR(S^1, d^1)$ , if  $F(S^1, d^1, c) = F(S^2, d^2, c)$ , then for all  $(S^3, d^3, c)$  such that  $IR(S^2, d^2) \subseteq IR(S^3, d^3) \subseteq IR(S^1, d^1)$ ,  $F(S^3, d^3, c) = F(S^1, d^1, c)$ .

This condition is closely related to the straightforward adaptation to bargaining problems with claims of the property *independence of revealed irrelevant alternatives* proposed by Mariotti (1994) for the classic bargaining problem. The main difference is that Mariotti's property only deals with changes in the feasible set whereas *fixed claims independence revealed* consider changes in the set of feasible and individually rational alternatives, on which "rational" people agree. Other conditions which are

close in "spirit" to this one can be found for the classic bargaining problem in Thomson (1981) and for bargaining with claims in Herrero (forthcoming).

**Proposition 9:**  $E^*$  verifies *fixed claims independence revealed*.

**Proof:** Let  $Q \in \mathcal{P}$  and three bargaining problems with claims  $(S^1, d^1, c)$ ,  $(S^2, d^2, c)$   $(S^3, d^3, c)$  in  $\Sigma^Q$  such that:

- [1]  $IR(S^2, d^2) \subseteq IR(S^3, d^3) \subseteq IR(S^1, d^1)$
- [2]  $E^*(S^2, d^2, c) = E^*(S^1, d^1, c)$

By [2] and taking into account that  $E^*$  verifies IR,  $E^*(S^1, d^1, c) \in \bar{S}^2$ . Now by definition 2,  $E(\bar{S}^1, d^1, c) \leq E^*(S^1, d^1, c)$ , therefore  $E(\bar{S}^1, d^1, c) \in \bar{S}^2$ . Since the claims point is fixed  $c - E(\bar{S}^1, d^1, c)$  is proportional to  $c - E(\bar{S}^2, d^2, c)$  but because  $E$  verifies WPO, we get:

$$[3] \quad E(\bar{S}^1, d^1, c) = E(\bar{S}^2, d^2, c).$$

By [1]  $\bar{S}^2 \subseteq \bar{S}^3 \subseteq \bar{S}^1$ , so that  $E(\bar{S}^2, d^2, c) \leq E(\bar{S}^3, d^3, c) \leq E(\bar{S}^1, d^1, c)$ , and from [3] we get:

$$[4] \quad E(\bar{S}^2, d^2, c) = E(\bar{S}^3, d^3, c) = E(\bar{S}^1, d^1, c)$$

Consider now  $E^*(S^3, d^3, c)$ . If  $E_1(\bar{S}^3, d^3, c) \geq d_1^3$ , by definition 2  $E_1^*(S^3, d^3, c) = E_1(\bar{S}^3, d^3, c)$ . By [4]  $E_1(\bar{S}^3, d^3, c) = E_1(\bar{S}^1, d^1, c)$ , and since from [1] we have  $d^3 \geq d^1$ ,  $E_1(\bar{S}^1, d^1, c) = E_1^*(S^1, d^1, c)$ . If  $E_1(\bar{S}^3, d^3, c) < d_1^3$ , by definition 2  $E_1^*(S^3, d^3, c) = d_1^3$ . In this case  $E_1^*(S^1, d^1, c) = d_1^3$ , because on

the one hand, since  $d^1 \leq d^2 \leq d^3$  by [1],  $E_i^*(S^1, d^1, c) < d_i^3$  contradicts  $E^*(S^1, d^1, c) \geq d^2$ , which is deduced from [2] taking into account that  $E^*$  verifies IR; on the other hand,  $E_i^*(S^1, d^1, c) > d_i^3$  implies by definition 2 that  $E_i(\bar{S}^1, d^1, c) > d_i^3$  since  $d^3 \geq d^1$  by [1], and it contradicts [4]. ■

Our characterization of the *extended claim-egalitarian solution* is given in the following theorem.

**Theorem 1:**  $E^*$  is the only solution on  $\Sigma$  satisfying *weak Pareto optimality, symmetry, translation invariance, continuity, independence of non-individually rational alternatives, independence of unclaimed alternatives, fixed claims independence revealed* and *population monotonicity*.

The following lemma is used in proving theorem 1. Its proof is a straightforward adaptation to bargaining with claims of the proof of lemma 1 in Herrero & Marco (1993).

**Lemma 1:** If  $F$  is a solution on  $\Sigma$  satisfying *independence of non-individually rational alternatives, weak Pareto optimality* and *continuity*, then  $F$  satisfies *individual rationality*.

**Proof of theorem 1:** Obviously  $E^*$  satisfies WPO, SY, T.INV, CONT, INIR and IUA. Furthermore, it satisfies POP.MON and FCIR (proposition 1 and proposition 9). In order to prove uniqueness, let  $F$  be a solution satisfying all the axioms. Let  $P \in \mathcal{P}$  and  $(S, d, c) \in \Sigma^P$  be given such that  $WPO(S) = PO(S)$ .

We will distinguish three cases:

(i)  $E(S, d, c) \in r.\text{int}\{WPO(\text{IR}(S, d))\}$ .

By T.INV we can assume  $c = e_p$ , so that  $E^*(S, d, c) = x = \alpha e_p$ , with  $\alpha \in \mathbb{R}$ .

(i)-(a) (Fig.3) Let  $d^1$  such that for all  $i \in P$   $d_i^1 = \min_{j \in P} d_j = t_1$ . Let  $(S^1, d', c^1)$  such that  $d' = 0$ ,  $c^1 = c + (-t_1 e_p) = (1-t_1)e_p$  and  $S^1 = S + \{-t_1 e_p\}$ , then  $E^*(S^1, d', c^1) = \alpha e_p + (-t_1 e_p) = \beta e_p$  with  $\beta \in \mathbb{R}$ . Consider  $S' = S^1 \cap \{x \in \mathbb{R}^P \mid x \leq c^1\}$ . Let  $q \geq |P| + 1$  be the smallest integer such that all  $x \in S'$  fulfills the inequality  $\sum_{i \in P} x_i \leq q\beta$ . Let  $Q \in \mathcal{P}$  with  $P \subset Q$  and  $|Q| = q$  be given. Now we define  $T^1 = \{x \in \mathbb{R}^Q \mid \sum_{i \in Q} x_i \leq |Q|\beta, x \leq (1-t_1)e_Q\}$ , and  $d^*, c^* \in \mathbb{R}^Q$  such that  $c^* = (1-t_1)e_Q$  and  $d_i^* = 0$  for all  $i \in Q$ , so  $(T^1, d^*, c^*) \in \Sigma^Q$ . Then by WPO and SY,  $F(T^1, d^*, c^*) = \beta e_Q$ . Let  $T^2 = \text{Com}\{\beta e_Q\}$ , so that  $(T^2, d^*, c^*) \in \Sigma^Q$  and by WPO and SY  $F(T^2, d^*, c^*) = \beta e_Q$ . Let  $T^3 = \text{ComCo}\{S', \beta e_Q\}$ . Notice that  $(T^3, d^*, c^*) \in \Sigma^Q$ ,  $\text{IR}(T^2, d^*) \subseteq \text{IR}(T^3, d^*)$ ,  $\text{IR}(T^3, d^*) \subseteq \text{IR}(T^1, d^*)$  and  $F(T^1, d^*, c^*) = F(T^2, d^*, c^*)$ . By applying FCIR we get  $F(T^3, d^*, c^*) = \beta e_Q$ . Since  $t_p^d(T^3) = S'$  and  $(d_p^*, c_p^*) = (d', c^1)$ , we obtain by POP.MON that  $F(S', d', c^1) \geq F_p(T^3, d^*, c^*) = \beta e_p$ , and since  $\beta e_p \in \text{PO}(S')$   $F(S', d', c^1) = \beta e_p$ . Now, by IUA  $F(S^1, d', c^1) = F(S', d', c^1) = \beta e_p$  and applying once T.INV we get  $F(S, d^1, c) = \beta e_p$ .

(i)-(b) Let  $d^2$  such that for all  $i \in P$   $d_i^2 = t_2$  and  $t_2 = \min\{t_j \mid t_j \geq d_i \text{ for all } i \in P\}$ . Let  $(S^2, d'', c^2)$  such that  $d'' = 0$ ,  $c^2 = c + (-t_2 e_p) = (1-t_2)e_p$  and  $S^2 = S + \{-t_2 e_p\}$ , then  $E^*(S^2, d'', c^2) = \alpha e_p + (-t_2 e_p) = \gamma e_p$  with  $\gamma \in \mathbb{R}$ . Now, reasoning in the same way as (i)-(a) with the only substitutions being  $S^1$  for  $S^2$ ,  $d^1$  for  $d^2$ ,  $c^1$  for  $c^2$ ,  $\beta$  for  $\gamma$ ,  $t_1$  for  $t_2$  and  $d'$  for  $d''$ , we get  $F(S, d^2, c) = \alpha e_p$ .

Taking into account (i)-(a), (i)-(b) and that  $IR(S,d^2) \subseteq IR(S,d)$ ,  $IR(S,d) \subseteq IR(S,d^1)$ , we can apply FCIR and conclude that  $F(S,d,c) = \alpha e = E^*(S,d,c)$ .

(ii)  $E(S,d,c) \in WPO(IR(S,d) \setminus r.int\{WPO(IR(S,d))\})$ .

We can find a sequence  $\{(S^\nu, d, c)\} \subset \Sigma^P$  for which  $WPO(S^\nu) = PO(S^\nu)$  for all  $\nu$ ,  $E(S^\nu, d, c) \in r.int\{WPO(IR(S^\nu, d))\}$  for all  $\nu$  and  $\lim_{\nu \rightarrow \infty} S^\nu = S$ . Using the same argument as above we get  $F(S^\nu, d, c) = E^*(S^\nu, d, c)$  for all  $\nu$ , and by applying CONT  $F(S, d, c) = E^*(S, d, c)$ .

For an arbitrary element  $(S, d, c) \in \Sigma^P$  such that  $WPO(S) \neq PO(S)$  and  $E(S, d, c) \in IR(S, d)$ , we obtain  $F(S, d, c) = E^*(S, d, c)$  by CONT.

Given  $P \in \mathfrak{P}$  and  $(S, d, c) \in \Sigma^P$ ,  $a_i(S, d)$  will denote the ideal point of the problem  $(S, d, c)$ ,  $a_i(S, d) = \max\{x_i \mid (x_{P \setminus \{i\}}, x_i) \in IR(S, d)\}$  for all  $i \in P$ .

(iii)  $E(S, d, c) \notin IR(S, d)$ .

We apply mathematical induction. Let  $P_1, P_2 \subset Q$  be such that  $P_1 = \{i \in P \mid E_i(S, d, c) \geq d_i\}$  and  $P_2 = P \setminus P_1$ .

(iii)-(a) Let  $P_2 = \{j\}$ . By T.INV we assume  $d = 0$ . Let  $E^*(S, d, c) = x$ , so  $x_j = 0$ . Firstly we will show that  $F_j(S, d, c) = 0$  (Fig.4). Since  $F$  verifies IR (lemma 1),  $F_j(S, d, c) \geq 0$ . Let  $q = |P| + 1$ ,  $Q \in \mathfrak{P}$  with  $P \subset Q$  and  $|Q| = q$ . Let  $Q \setminus P = \{k\}$ . Now we can construct the problem  $(S^*, d^*, c^*) \in \Sigma^Q$  as follows:  $S^* = \text{Com}\{x \in \mathbb{R}^Q \mid x_P \in S, x_k = \gamma\}$ ,  $d^* = 0$  and  $c^* = (c_P, \gamma + \varepsilon)$ , where  $\varepsilon > 0$  is chosen in order to get  $E_{(j,k)}^0(t_{(j,k)}^0(S^*), d_{(j,k)}^*, c_{(j,k)}^*) \geq 0$  with  $E_j(t_{(j,k)}^0(S^*), d_{(j,k)}^*, c_{(j,k)}^*) = 0$ , and  $\gamma$  is selected in such a way that

$E(t_{Q \setminus \{j\}}^0(S^*), d_{Q \setminus \{j\}}^*, c_{Q \setminus \{j\}}^*) \geq 0$  and there is  $h \in Q \setminus \{j\}$  for which  $E_h(t_{Q \setminus \{j\}}^0(S^*), d_{Q \setminus \{j\}}^*, c_{Q \setminus \{j\}}^*) = a_h(S, d)$ .

By (ii)  $F(t_{\{j,k\}}^0(S^*), d_{\{j,k\}}^*, c_{\{j,k\}}^*) = E(t_{\{j,k\}}^0(S^*), d_{\{j,k\}}^*, c_{\{j,k\}}^*)$ , so that and by construction  $F_j(t_{\{j,k\}}^0(S^*), d_{\{j,k\}}^*, c_{\{j,k\}}^*) = 0$ . Now, by applying POP.MON between  $(S^*, d^*, c^*)$  and  $(t_{\{j,k\}}^0(S^*), d_{\{j,k\}}^*, c_{\{j,k\}}^*)$  we get  $F_j(S^*, d^*, c^*) \leq 0$ , and by IR,  $F_j(S^*, d^*, c^*) = 0$ . By POP.MON on  $(S^*, d^*, c^*)$  and  $(t_{Q \setminus \{j\}}^0(S^*), d_{Q \setminus \{j\}}^*, c_{Q \setminus \{j\}}^*)$ ,  $F_h(S^*, d^*, c^*) \leq F_h(t_{Q \setminus \{j\}}^0(S^*), d_{Q \setminus \{j\}}^*, c_{Q \setminus \{j\}}^*)$ , and by using (ii) and taking into account that  $F$  verifies WPO we get  $F_h(S^*, d^*, c^*) = a_h(S, d)$ . Once again by applying POP.MON between  $(S^*, d^*, c^*)$  and  $(S, d, c)$ ,  $F_h(S, d, c) \geq a_h(S, d)$ . By IR,  $F_h(S, d, c) = a_h(S, d, c)$  and because  $WPO(S) = PO(S)$ ,  $F_j(S, d, c) = 0$ .

Now, if  $|P| = 2$  the desired conclusion is obtained by WPO.

If  $|P| > 2$ , by (i)  $F(t_{P_1}^d(S), d_{P_1}, c_{P_1}') = F(t_{P_1}^d(S), d_{P_1}, c_{P_1}) = x_{P_1}$ . Then by POP.MON  $F_{P_1}(S, d, c) \leq x_{P_1}$ . Since  $F_j(S, d, c) = x_j$ , WPO implies  $F_{P_1}(S, d, c) = x_{P_1}$ , so  $F(S, d, c) = x$ .

(iii)-(b) If  $|P_2| = k$  we assume that  $F(S, d, c) = E^*(S, d, c)$ .

(iii)-(c) Let  $|P_2| = k + 1$ . Let  $E^*(S, d, c) = x$  and  $c'$  such that  $c'_i = c_i$  for all  $i \in P_1$ ,  $c'_j - x_j = c'_i - x_i$  for  $j \in P_2$  and  $c'_k = c_k$  for all  $k \neq j$ ,  $k \in P_2$ . Then  $F(S, d, c') = x$  by (iii)-(b). Now, similarly to (iii)-(a) but using several times (iii)-(b) we can get  $F_j(S, d, c) = x_j = d_j$ . Let  $Q = P \setminus \{j\}$ , then we know by (iii)-(b) that



$F(t_Q^d(S), d_Q, c_Q) = F(t_Q^d(S), d_Q, c_Q) = x_Q$ , then by POP.MON  $F_Q(S, d, c) \geq x_Q$ . Since  $F_j(S, d, c) = x_j$ , WPO implies  $F_Q(S, d, c) = x_Q$ , so  $F(S, d, c) = x$ .

For an arbitrary element  $(S, d, c) \in \sum^P$  such that  $WPO(S) \neq PO(S)$  and  $E(S, d, c) \notin IR(S, d)$ , we obtain  $F(S, d, c) = E^*(S, d, c)$  by CONT. ■

It is worth noticing the independence among CONT, INIR, IUA, FCIR and POP.MON in the presence of the rest of axioms characterizing  $E^*$  (theorem 1): it can be checked that the solution that selects the *restricted lexicographic extended claim-egalitarian solution* (see Marco (1995)) for the two person problem and the *extended claim-egalitarian solution* in another case, satisfies all axioms except *continuity*; the *claim-egalitarian solution* fails to be *independent of non-individually rational alternatives* and it verifies the rest of the axioms; consider the solution  $\hat{E}$ , such that  $\hat{E}(S, d, c) = E^*(S, d, \hat{c})$ , where  $\hat{c}_i = \max\{c_i - d_i\}$  for all  $i$ , this solution does not verify *independence of unclaimed alternatives*, and it satisfies the rest of the axioms; the *proportional solution* satisfies all axioms except *fixed claims independence revealed*; finally the reformulations of the  $Yu^P$  solutions (Yu (1973)),  $1 < p < \infty$  for the domain at hand, that is, the claims point playing the role of the ideal point, satisfy all axioms except *population monotonicity*.

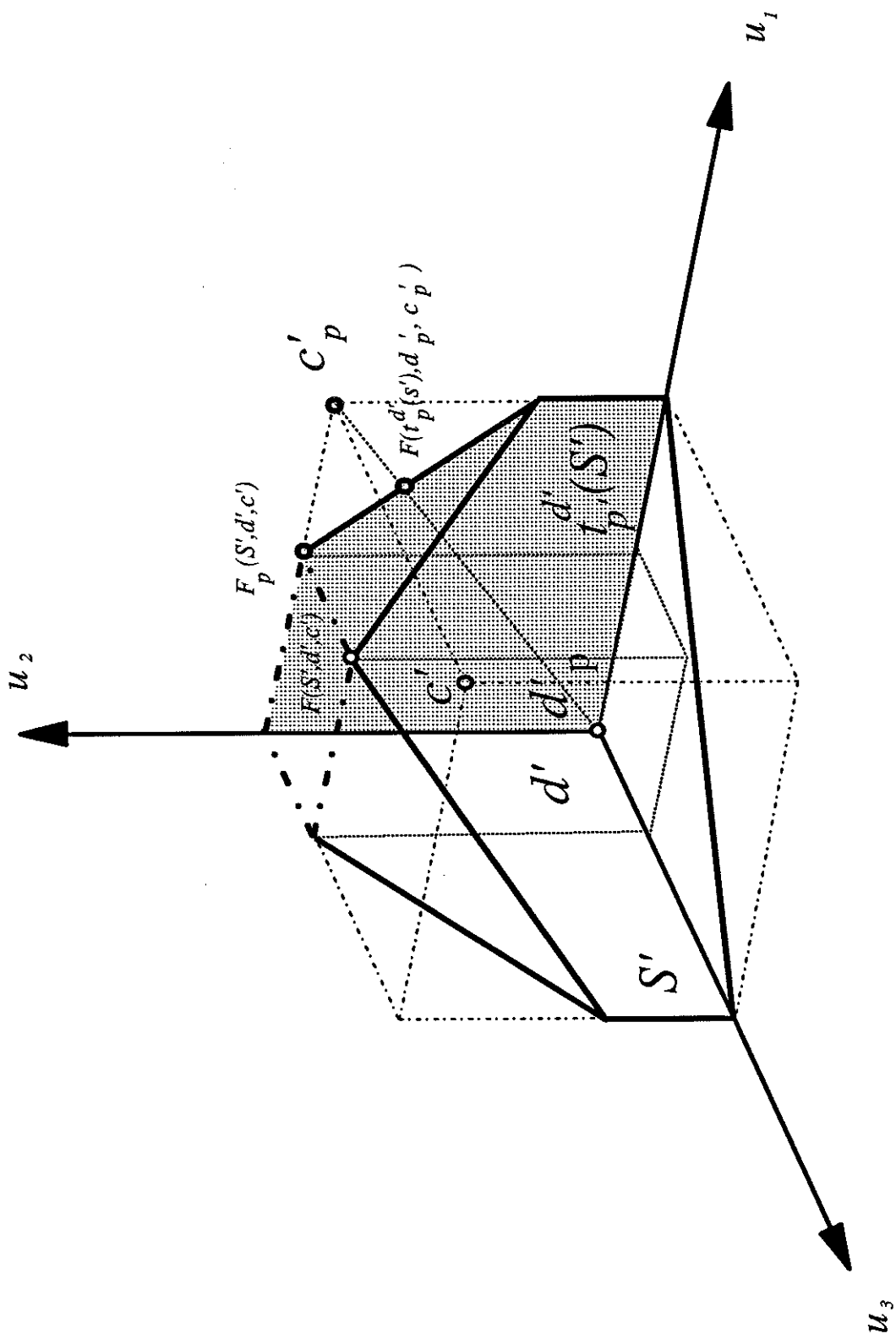


Figure 1

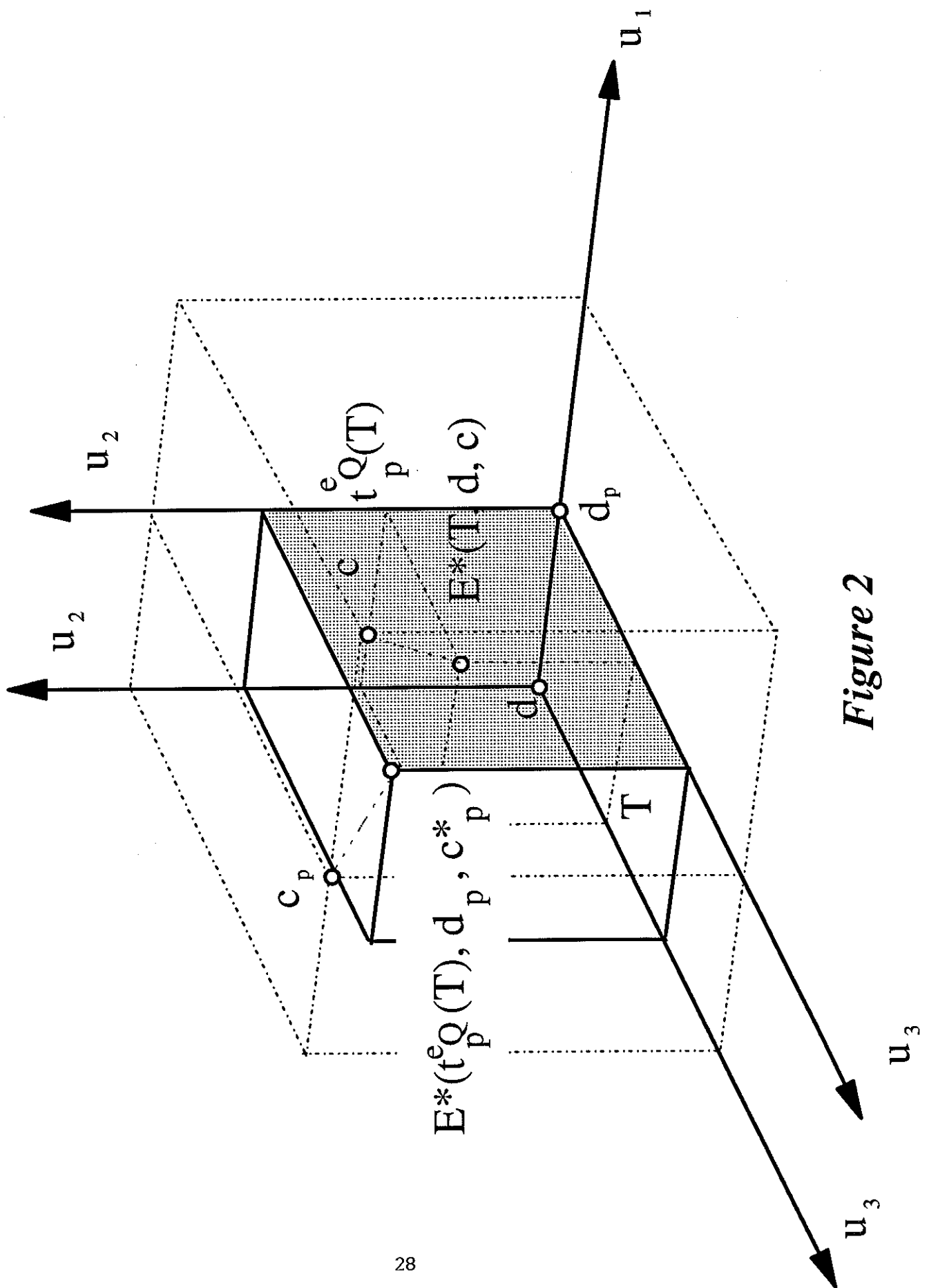
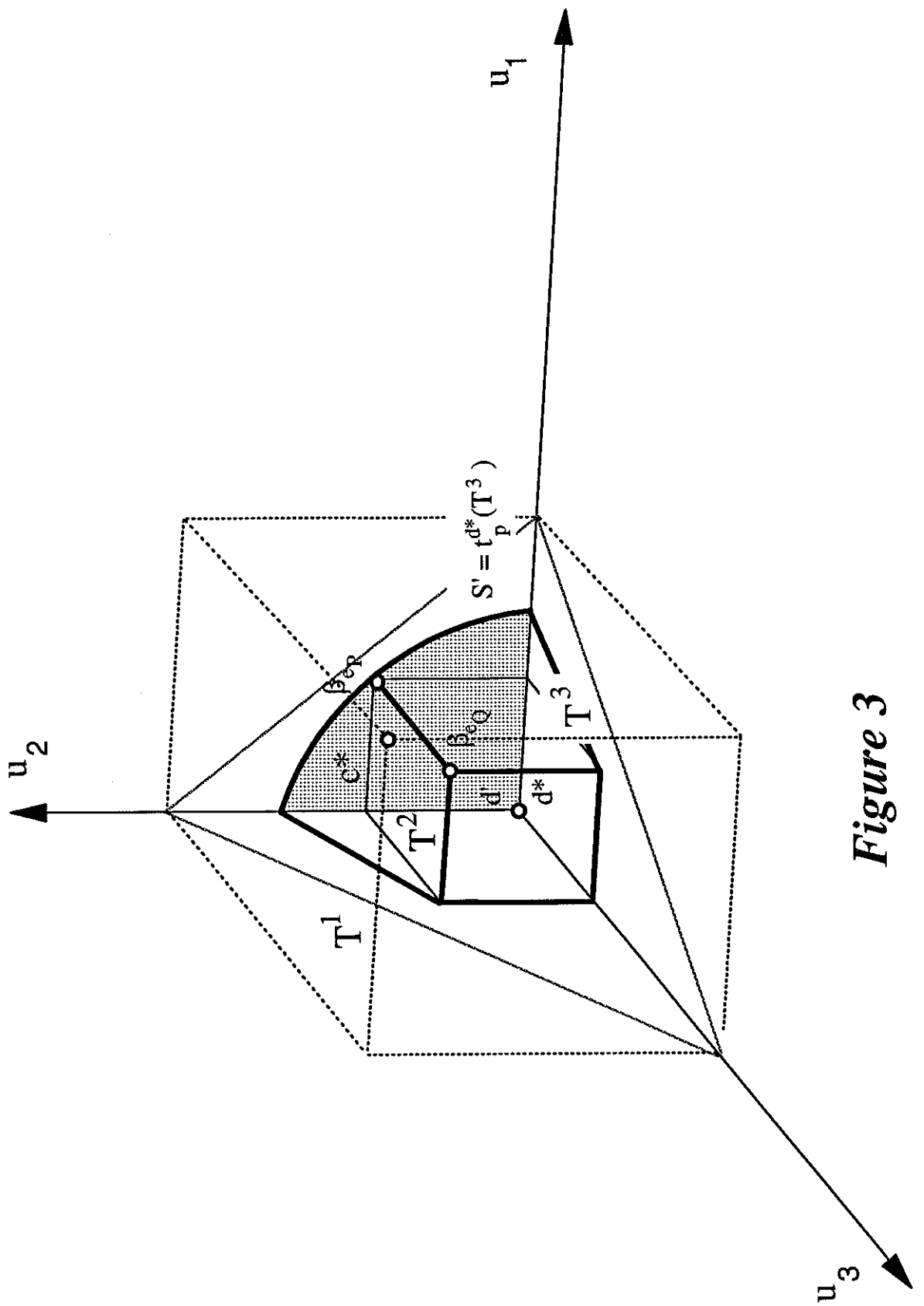


Figure 2



*Figure 3*

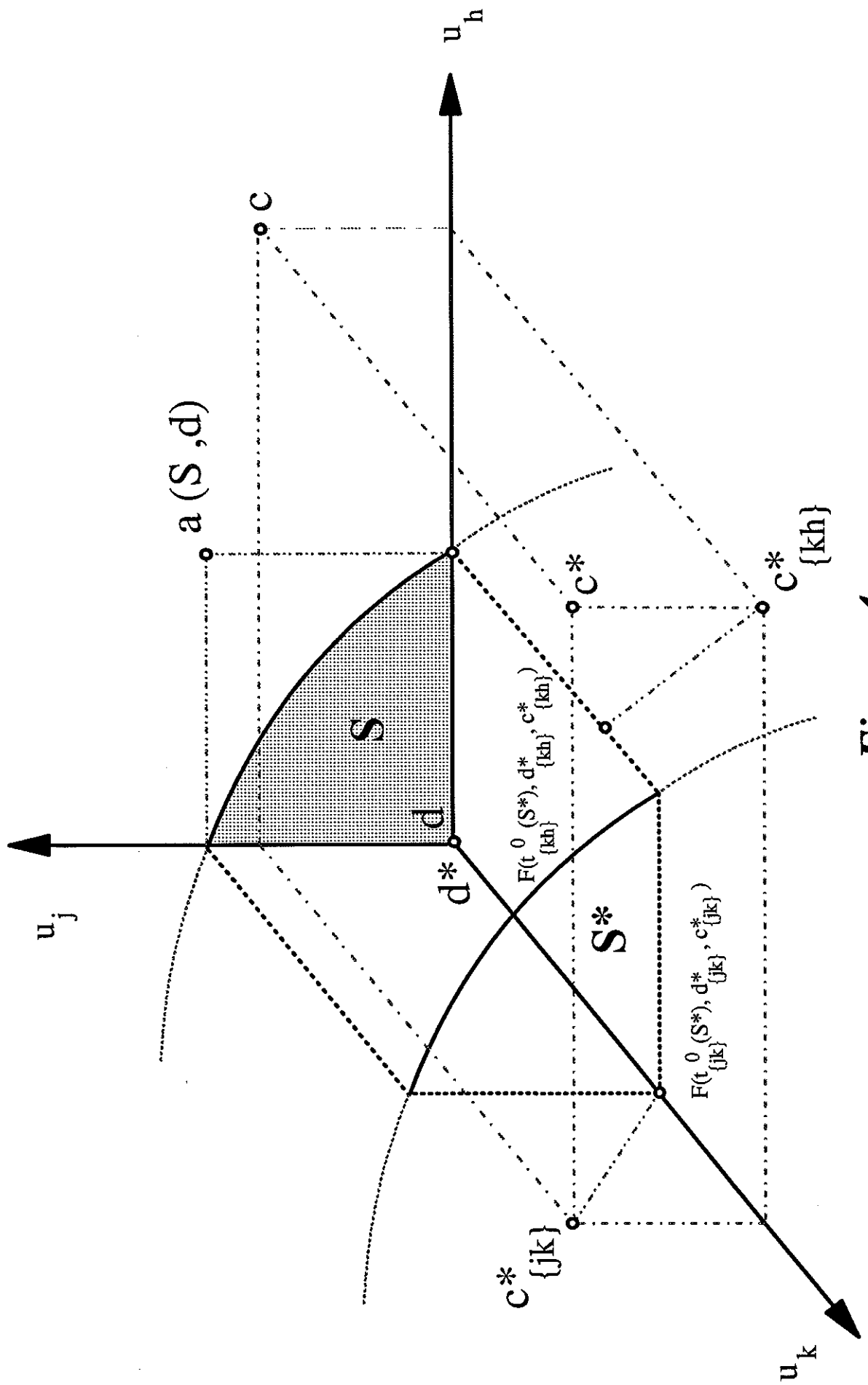


Figure 4

## REFERENCES

R.J.Aumann & M.Maschler (1985) "Game theoretic analysis of a bankruptcy problem from the Talmud". *Journal of Economic Theory* 36, 195-213.

W.Bossert (1993) "An alternative solution to bargaining problems with claims". *Mathematical Social Sciences* 25, 205-220.

Y.Chun & W.Thomson (1992) "Bargaining problems with claims". *Mathematical Social Sciences* 24, 19-33.

J.C.Harsanyi (1959) "A bargaining model for the cooperative N-person game", in *Contributions to the Theory of Games IV* (A.W. Tucker and R.D. Luce eds.), *Annals of Mathematical studies* n° 40, Princeton, Princeton University Press.

C.Herrero (forthcoming) "Endogenous reference points and the adjusted proportional solution for bargaining problems with claims" *Social Choice and Welfare*, forthcoming.

C.Herrero & M.C. Marco (1993) "Rational equal-loss solutions for bargaining problems", *Mathematical Social Sciences* 26, 273-286.

M.C.Marco (1994) "An alternative characterization of the extended claim-egalitarian solution ". *Economics Letters* 45, 41-46.

M.C.Marco (1995) "Efficient solutions for bargaining problems with claims". *Mathematical Social Sciences* 30, 57-69.

M.Mariotti (1994) "The Nash solution and independence of revealed irrelevant alternatives". *Economics Letters* 45, 175-179

J.F.Nash (1950) "The bargaining problem". *Econometrica* 18, 155-162.

W.Thomson (1981) "Independence of irrelevant expansions". *Journal of Game Theory* vol.10, Issue 2, 107-118.

W.Thomson (1983) "Problems of fair division and the egalitarian solution". *Journal of Economic Theory* 31, 211-226.

W.Thomson (1990) "The consistency principle" in: T.Ishiishi, A.Neyman and Y.Tauman,eds., *Game Theory and Applications* (Academic Press, San Diego) 187-215.

W.Thomson (forthcoming) "Bargaining theory: the axiomatic approach". Cambridge University Press, New York.

W.Thomson (1994) "Population monotonic allocation rules". Working paper n° 375. University of Rochester.

W.Thomson & T.Lensberg (1989) "Axiomatic theory of bargaining with a variable number of agents". Cambridge University Press, Cambridge.

P.L.Yu (1973) "A class of solutions for groups decision problems". *Management Science* 19, 936-946.

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