TEMPORARY EQUILIBRIUM DYNAMICS WITH BAYESIAN LEARNING*

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ABSTRACT

This paper examines the stability of deterministic steady-states in a class of economies where the state-variable is one dimensional and where agents use Bayesian techniques to form expectations. The dynamics with learning are locally convergent if the prior mean is close to a stable perfect foresight root having modulus less than 1 and if the prior beliefs are held with enough confidence. The dynamics are however divergent if the prior mean or the variance of the prior distribution is sufficiently large.

KEYWORDS: Stability, Bayesian learning.

This paper addresses the stability of deterministic steady-states in a class of economies where the state-variable is one dimensional and where the expectation formation process is based on Bayesian learning.

The study is carried out using a linear approximation to the Temporary Equilibrium Map at the steady-state. The economy evolves as a three dimensional discrete dynamical system which, under the assumptions made on the underlying microeconomic structure, possesses a two dimensional manifold of equilibria corresponding to the steady-state.

The findings of the paper are that the dynamics with Bayesian learning are locally convergent if (i) the prior mean is close to a stable perfect foresight root which has modulus less than one, and (ii) if the beliefs are held with enough confidence, that is, the prior variance is small enough. On the other hand, the learning dynamics are divergent whenever the prior mean or the prior variance is sufficiently large.

Section 1 presents the model. Section 2 describes the dynamics and briefly puts the results in perspective. Section 3 contains the proofs.

1 THE MODEL

The framework is identical to the ones used in [1] and [2] to study the local dynamics with learning around an isolated steady-state of a deterministic economy. The primitive is the Temporary Equilibrium Map F, the solution to which determines the current equilibrium state x_t, assumed to be a real number, as a function of the past equilibrium x_{t-1}, and x_{t+1}^e, the forecast for the next period made at t using information upto $t - 1$. The utility-maximisation problem of the agents is assumed to be such that it suffices to use the expected

value of the state-variable in computing demands. Though agents believe that others may have different forecasts, for simplicity one assumes that all have the same forecast.

The map F is assumed to be well-defined and continously-differentiable in the vicinity of the steady-state under consideration. The steady-state is translated to be 0. The study is conducted using the linear approximation to F at the steady-state. Thus the relevant map becomes

$$b_1 x_{t-1} + b_0 x_t + a x_{t+1}^e = 0 \qquad (1.1)$$

where b_1, b_0 and a are the partial derivatives of F with respect to x_{t-1}, x_t and x_{t+1}^e evaluated at the steady-state. One needs to assume that b_0 and a are different from 0.

The expectation formation process is specified as follows. Agents are Bayesians who use models of the form $x_t = \beta x_{t-1} + \epsilon_t$ to predict deviations from the steady-state, which they are assumed to know is 0. Agents are however uncertain about the adjustment rate β that they believe governs the dynamics outside the steady-state. They have a prior distribution on $\beta \sim N(\mu_{t-1}, \sigma_{t-1}^2)$ at the start of any period t. They believe that the ϵ's are Normally distributed with mean 0 and variance σ_ϵ^2. These are assumed to be i.i.d. and independent of β.

Given any model, the agents form their forecasts by iterating twice on their model. Current values of the state-variable are not allowed to influence the forecast. This procedure is identical to the one used in [2] and removes complications that may arise when the current value of the state-variable and the forecasted value influence each other. Thus given x_{t-1},

$$x_{t+1}^e = E[\beta(\beta x_{t-1} + \epsilon_t) + \epsilon_{t+1}] = x_{t-1}(\mu_{t-1}^2 + \sigma_{t-1}^2)$$

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2   THE DYNAMICS

Given x_0 and x_2^e, x_1 is determined using the market clearing function (1.1).
Thus,

$$x_1 = -\frac{(b_1 + a(\sigma_0^2 + \mu_0^2))x_0}{b_0} \tag{2.1}$$

Agents use the 'realization' x_1 to update the parameters of their prior distribution using Bayes rule. The updated priors are used in conjunction with x_1 to forecast x_3, and the economy evolves as the dynamical system

$$x_t = -\frac{(b_1 + a(\sigma_{t-1}^2 + \mu_{t-1}^2))x_{t-1}}{b_0} \tag{2.2}$$

$$\mu_t = \frac{\sigma_\epsilon^2 \mu_{t-1}}{\sigma_\epsilon^2 + \sigma_{t-1}^2 x_{t-1}^2} + \frac{\sigma_{t-1}^2 x_{t-1}^2}{\sigma_\epsilon^2 + \sigma_{t-1}^2 x_{t-1}^2}\left(-\frac{b_1 + a(\sigma_{t-1}^2 + \mu_{t-1}^2)}{b_0}\right) \tag{2.3}$$

$$\sigma_t^2 = \frac{\sigma_\epsilon^2 \sigma_{t-1}^2}{\sigma_\epsilon^2 + \sigma_{t-1}^2 x_{t-1}^2} \tag{2.4}$$

The dynamical system described by (2.2), (2.3) and (2.4) is defined on the set $\Phi = \{(x, \mu, \sigma) | x \in R, \mu \in R, \sigma \geq 0\}$. A set of equilibria of the system is the set $S^0 = \{(x, \mu, \sigma) | x = 0, \mu \in R, \sigma \geq 0\}$.

A feature of the dynamics is that as the economy evolves, the agents become progressively certain about their beliefs, and if they become completely certain, they cease to update the means of their priors. In the case where the agents are completely certain about their beliefs, i.e. $\sigma^2 = 0$, one identifies perfect-foresight paths as ones where $\frac{x_t}{x_{t-1}}$, the adjustment rate observed in the x-component of the dynamics, coincides with the adjustment rate the agents are certain about. These are thus the solutions to

$$-\left(\frac{b_1 + \mu^2 a}{b_0}\right) = \mu. \tag{2.5}$$

I assume that the above quadratic has two real roots, referred to as the perfect foresight roots, that are different from each other. The root of smaller modulus is denoted λ^s and the other λ^u. Assume that a and b_0 have opposite signs for

the remainder of the paper. This ensures that $\lambda^u > 0$. The excluded case can be treated analogously.

Proposition 1 states that one gets stable learning dynamics if the initial mean is close to $|\lambda^s| < 1$, provided that the beliefs are held with enough confidence.

Proposition 1 : *Assume that $a \neq 0$, $b_0 \neq 0$ have opposite signs and that the two perfect foresight roots are distinct and real with $\lambda^s < \lambda^u$ and $|\lambda^s| < 1$. Consider an initial state (x_0, μ_0, σ_0) and the corresponding sequence (x_t, μ_t, σ_t) generated by the dynamics with Bayesian learning.*

For any fixed $\bar{x} > 0$, the dynamics with Bayesian learning generates convergent trajectories with $(x_t, \mu_t, \sigma_t) \to (0, \bar{\mu}, \bar{\sigma})$ if $|x_0| < \bar{x}$, provided that the initial mean μ_0 is close enough to λ^s and that the beliefs are held with enough confidence, i.e. σ_0 is small enough.

Proposition 2 states that the learning dynamics are unstable whenever the initial mean μ_0 is large enough (and has the same sign as λ^u), or when the initial variance σ_0^2 is sufficiently large.

Proposition 2 : *Assume that $a \neq 0$, $b_0 \neq 0$ have opposite signs. Choose $\lambda^* > 0$ no less than the two perfect foresight roots (if they are real) such that $\Omega(\lambda^*) \geq 1$ where $\Omega(\lambda^*) = -(\frac{b_1 + a\lambda^{*2}}{b_0})$. Consider an initial state (x_0, μ_0, σ_0) with $x_0 \neq 0$, and the corresponding sequence (x_t, μ_t, σ_t) generated by the dynamics with Bayesian learning. Then*

(i) If $\mu_0 > \lambda^*$, the sequence μ_t is nondecreasing, while the sequence $|x_t|$ is increasing and diverges to infinity.

(ii) There exists a unique $\bar{\sigma}(\lambda^*, x_0) > 0$ such that a great initial subjective uncertainty, i.e. $\sigma_0 > \bar{\sigma}(\lambda^*, x_0)$, implies $\mu_1 > \lambda^*$ and $x_1 \neq 0$, independently of

the initial mean μ_0. In that case too the sequence $|x_t|$ diverges to infinity.

These results are now compared to the results of Grandmont and Laroque [2]. A case the authors examine is the stability of the steady-state when expectations are formed using least squares techniques. The dynamics are discontinous at the steady-state. Every neighborhood of the steady-state contains an open set for which the learning dynamics diverge. When $|\lambda^s| < 1$, every neighborhood also contains an open set for which the learning dynamics converge to the steady-state. These sets infact form cones in the space of initial conditions with the steady-state as the vertex. By contrast, the dynamics with Bayesian learning are differentiable[1] at the steady-state, and local stability obtains under the conditions of proposition 1.

On the other hand, the central point emphasized in [2], namely that the learning dynamics tend to be divergent (with differentiable and non-differentiable learning schemes such as the least squares learning schemes, propositions 1 and 2.1, [2]) unless the range of regularities agents extract from past data are restricted, comes out clearly in the instance of Bayesian learning, and takes the form of the restrictions needed on the prior beliefs to prevent instability and guarantee convergence. In this respect the results of [2] and this paper are similar[2]. From the updating rule (2.3) it is clear that the larger the prior variance, the more prepared are the agents to modify and adapt their prior means in accordance with the behaviour of the economic system. One may thus conclude that the more the agents are willing to interact with the system and learn from it, the more likely is the system to diverge.

The divergence to infinity seen under the conditions of proposition 2 should be interpreted as the local divergence of the state-variable from the steady-state, since, as observed in [2], nonlinearities inherent in the Temporary Equilibrium Map, once explicitly incorporated into the dynamics, may keep the locally divergent trajectories globally bounded.

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3 PROOFS

Let $\Omega(\mu) = -(\frac{b_1 + a\mu^2}{b_0})$, $\Omega_\sigma(\mu) = \Omega(\mu) - (\frac{a\sigma^2}{b_0})$ and $m(\sigma, x) = \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma^2 x^2}$. If $x_t = x$, $\mu_t = \mu$, $\sigma_t^2 = \sigma^2$ then the dynamical system to be studied is formulated as follows:

(3.1) $x_{t+1} = \Omega_\sigma(\mu)x$

(3.2) $\mu_{t+1} = m(\sigma, x)\mu + [1 - m(\sigma, x)]\Omega_\sigma(\mu)$

(3.3) $\sigma_{t+1}^2 = m(\sigma, x)\sigma^2$

Under the assumptions, the map Ω has two real fixed points $\lambda^s < \lambda^u$. Since $ab_0 < 0$, the parabola representing Ω has its asymptotic branches going up and $\lambda^u > 0$. Since Ω_σ is obtained from Ω by a vertical upward translation of $-(\frac{a\sigma^2}{b_0})$, it is easily verified by direct inspection that it also has two fixed points $\lambda^s(\sigma) < \lambda^u(\sigma)$ when $0 \le \sigma < \sigma^* = \frac{|\lambda^u - \lambda^s|}{2}$. When σ increases from 0 to σ^*, $\lambda^s(\sigma)$ increases while $\lambda^u(\sigma)$ decreases. The two fixed points coalesce at $\sigma = \sigma^*$ and vanish when $\sigma > \sigma^*$ (Fig. 1).

To prove proposition 1, one assumes $\lambda^s = \Omega(\lambda^s)$ has modulus less than 1. Then by continuity

(3.4) There exist $0 < k < 1, \delta > 0$ and $\bar\lambda_1 < \lambda^s < \bar\lambda_2 < \lambda^u$ such that $|\Omega_\sigma(\mu)| \le k$ and $\lambda^s(\sigma) < \bar\lambda_2 < \lambda^u(\sigma)$ $\forall\mu, \sigma$ satisfying $0 \le \sigma < \delta$ and $\bar\lambda_1 < \mu < \bar\lambda_2$.

The next step is to construct an invariant set W for the dynamics. Let $g(x, \mu, \sigma)$ be the right hand side of (3.2). If $\sigma x = 0$ then $m(\sigma, x) = 1$ and g is identically equal to μ. For fixed $\sigma \ne 0$ and $x \ne 0$, the right hand side of (3.2), considered as a function of μ, is a convex combination, with positive weights, of μ and $\Omega_\sigma(\mu)$. It is therefore represented by an upward sloping parabola that has the same fixed points, i.e. $\lambda^s(\sigma)$ and $\lambda^u(\sigma)$, as Ω_σ. Again by continuity,

(3.5) Given $\bar{x} > 0$ and $\bar{\lambda}_1$, $\bar{\lambda}_2$ as above, one can choose δ in (3.4) small enough so that for all $0 < |x| < \bar{x}$, $0 < \sigma < \delta$ and $\bar{\lambda}_1 < \mu < \bar{\lambda}_2$, g is an increasing function of μ, with $\mu < g(x, \mu, \sigma)$ when $\mu < \lambda^s(\sigma)$, $\mu = g(x, \mu, \sigma)$ for $\mu = \lambda^s(\sigma)$ and $\mu > g(x, \mu, \sigma)$ when $\mu > \lambda^s(\sigma)$.

The picture one then gets when $x \neq 0$, $\sigma \neq 0$ is illustrated in Fig. 2. When $\sigma = 0$ or $x = 0$, the parabola degenerates into the 45° line, since then $g \equiv \mu$.

Let W be the set of states (x, μ, σ) satisfying $0 \leq |x| < \bar{x}$, $\bar{\lambda}_1 < \mu < \bar{\lambda}_2$ and $0 \leq \sigma < \delta$, where $\bar{x}$, $\bar{\lambda}_1$, $\bar{\lambda}_2$ and δ have been fixed as in (3.4) and (3.5). Consider an initial condition (x_0, μ_0, σ_0) in W. If $x_0 = 0$, the sequence (x_t, μ_t, σ_t) generated by (3.1), (3.2), (3.3) stays constant: it thus stays in W and converges trivially. If $\sigma_0 = 0$, the sequence (μ_t, σ_t) stays constant while $|x_1| \leq k|x_0|$ with $k < 1$: thus the sequence (x_t, μ_t, σ_t) stays in W and converges, since $|x_t| \leq k|x_{t-1}|$ goes to 0. If $x_0 \neq 0$ and $\sigma_0 \neq 0$, then the next iterate (x_1, μ_1, σ_1) satisfies $|x_1| \leq k|x_0|$, $0 < \sigma_1 < \sigma_0$ and from Fig. 2, either $\mu_0 < \mu_1 < \lambda^s(\sigma_0)$ or $\lambda^s(\sigma_0) \leq \mu_1 \leq \mu_0$, which implies in particular that $\bar{\lambda}_1 < \mu_1 < \bar{\lambda}_2$. Thus in this case as well the sequence of iterates (x_t, μ_t, σ_t) stays in W. If at some point $x_t = 0$, one gets convergence trivially as above. It remains to consider the case when $x_t \neq 0$ $\forall t$. Then σ_t forms a decreasing sequence of positive numbers which converges to some $\bar{\sigma} \geq 0$. The sequence x_t satisfies $|x_t| \leq k|x_{t-1}|$ and thus goes to 0. Consider now the μ_t sequence. Either it satisfies $\mu_{t-1} < \mu_t < \lambda^s(\sigma_{t-1})$ $\forall t$, in which case it is monotonically increasing and thus converges to some $\bar{\mu}$. Or else $\lambda^s(\sigma_{t-1}) \leq \mu_t \leq \mu_{t-1}$ for some t. Since the sequence $\lambda^s(\sigma_t)$ is decreasing, the sequence μ_τ is then decreasing for $\tau \geq t$, and thus bound to converge in that case too to some $\bar{\mu} \geq \lambda^s$. The case $\lambda^u < 0$ can be treated similarly. ■

For the proof of proposition 2 assume again that a and b_0 have opposite signs. Choose $\lambda^* > 0$ no less than the two fixed points of Ω (if they exist , but

this is not necessary for the proof), such that $\Omega(\lambda^*) \geq 1$. Then by definition, one has $\Omega(\mu) > 1$, $\Omega(\mu) > \mu$, and $\Omega(\mu)$ is increasing, whenever $\mu > \lambda^*$.

Consider now an initial state (x_0, μ_0, σ_0) with $x_0 \neq 0$, and the sequence (x_t, μ_t, σ_t) generated by (3.1), (3.2), (3.3). In all cases the sequence σ_t is non increasing and thus converges to some $\bar{\sigma} \geq 0$.

(i) Assume first that the initial mean is large, so that $\mu_0 > \lambda^*$. Then from (3.1) and the fact that $\Omega_\sigma(\mu) \geq \Omega(\mu)$, when $\mu_0 \geq \lambda^*$, one has $|x_1| \geq \Omega(\mu_0)|x_0|$ with $\Omega(\mu_0) > 1$. From (3.2) one gets $\mu_1 \geq m(\sigma_0, x_0)\mu_0 + [1 - m(\sigma_0, x_0)]\Omega(\mu_0) \geq \mu_0$. By induction, the sequence μ_t is nondecreasing and

$$|x_t| \geq \Omega(\mu_{t-1})|x_{t-1}| \geq \Omega(\mu_0)|x_{t-1}|.$$

Since $\Omega(\mu_0) > 1$, the sequence $|x_t|$ diverges to infinity.

(ii) Assume now that the initial variance σ_0^2 is large. Specifically, it is verified by direct inspection that for fixed $x \neq 0$ and $\sigma \neq 0$, the minimum value of the right hand side $g(x, \mu, \sigma)$ of (3.2), with respect to μ is achieved for

$$\mu = \frac{mb_0}{2a(1-m)} < 0, \text{ with } m = m(\sigma, x).$$

and that it is then equal to

(3.6) $\frac{m^2 b_0}{4a(1-m)} + (1 - m)\Omega(\sigma)$

The minimum (3.6) varies from $-\infty$ to $+\infty$ (it is actually increasing whenever $\Omega(\sigma) \geq 0$) when σ increases from 0 to infinity. There exists thus a unique $\bar{\sigma}(\lambda^*, x) > 0$ defined as the largest σ for which (3.6) is equal to λ^*. The graph of g for that particular value of σ is illustrated in Fig. 3.

For any $\sigma > \bar{\sigma}(\lambda^*, x)$, one has $g(x, \mu, \sigma) > \lambda^* \forall \mu$ by construction. Furthur-

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more, the minimum (3.6) is less than $\Omega(\sigma) = \Omega_\sigma(0) \leq \Omega_\sigma(\mu) \; \forall \mu$. Thus if $\sigma > \bar{\sigma}(\lambda^*, x)$, one also gets $\Omega_\sigma(\mu) > \lambda^* > 0 \; \forall \mu$. In particular, if the initial state satisfies $x_0 \neq 0$, $\sigma_0 > \bar{\sigma}(\lambda^*, x_0)$, one gets $\mu_1 > \lambda^*$ and $|x_1| > \lambda^* |x_0| > 0$, independently of the initial mean μ_0. From $t = 1$ on, one is back to case (i), and the dynamics with Bayesian learning thus diverge. Here too the case where a and b_0 have the same sign is treated similarly. ∎

Footnotes

1. It is easily verified that the Jacobian of the system evaluated at an equilibrium has two eigenvalues equal to 1. The local stability and instability results then follow from the technique of 'center manifolds'(Grandmont [3], Theorem B.5.3) and the results remain valid with nonlinear Temporary Equilibrium Maps under appropriate differentiability assumptions. I am grateful to an anonymous referee for emphasizing this point. This technique however applies only to initial conditions lying sufficiently close to the equilibrium manifold S^0. This paper adopts a more direct technique of proof which illustrates that trajectories generated by initial conditions arbitrarily far from S^0 in the x component may nevertheless converge to S^0 provided the prior mean and variance are appropriately placed, and thus provides some global information about the stable space. Such a method may be applicable to the study of the global dynamics with a nonlinear Temporary Equilibrium Map. It remains to be seen to what extent the introduction of economically relevant nonlinearities preserves or enhances the global structure of the stable space seen here.

2. The results of [2] encompass (p. 264, [2]) the case of least squares learning where the estimate is constrained to lie in a prespecified interval $[\bar{\mu}_1, \bar{\mu}_2]$ which may be interpreted as a 'prior', which however is not revised over time as in this paper. Indeed then (i) if the prior $[\bar{\mu}_1, \bar{\mu}_2]$ is centered around $|\lambda^s| < 1$ and is small, convergence results for all initial conditions and if (ii) the prior is large ($\bar{\mu}_2 > \lambda^u$, $\Omega(\bar{\mu}_2) > 1$), then there is divergence for an open set of initial conditions. The convergence of least squares learning in a stochastic framework is examined in [4].
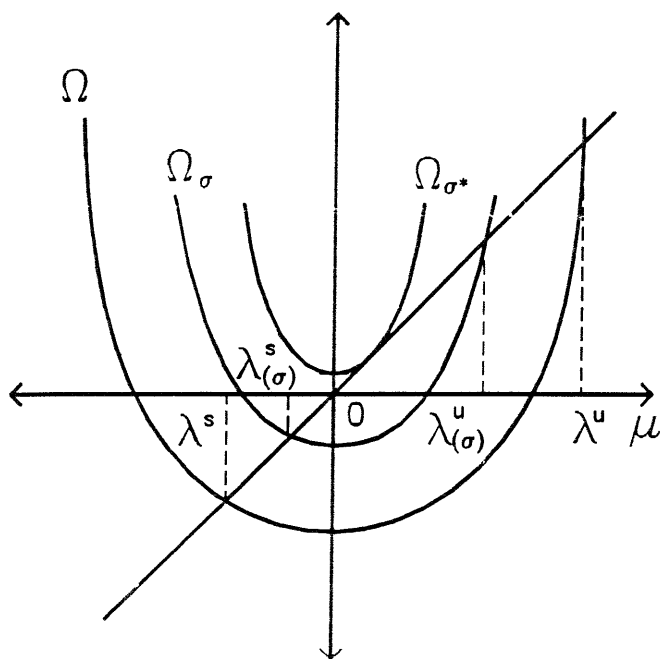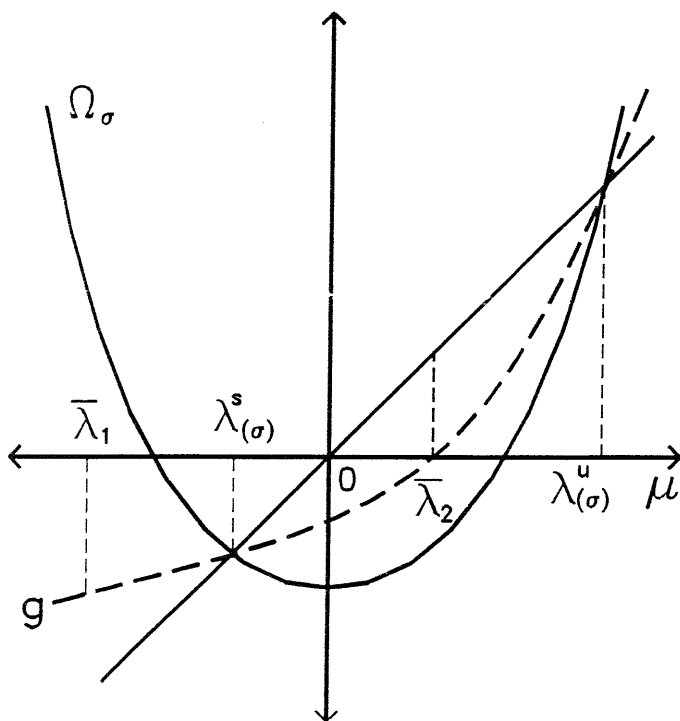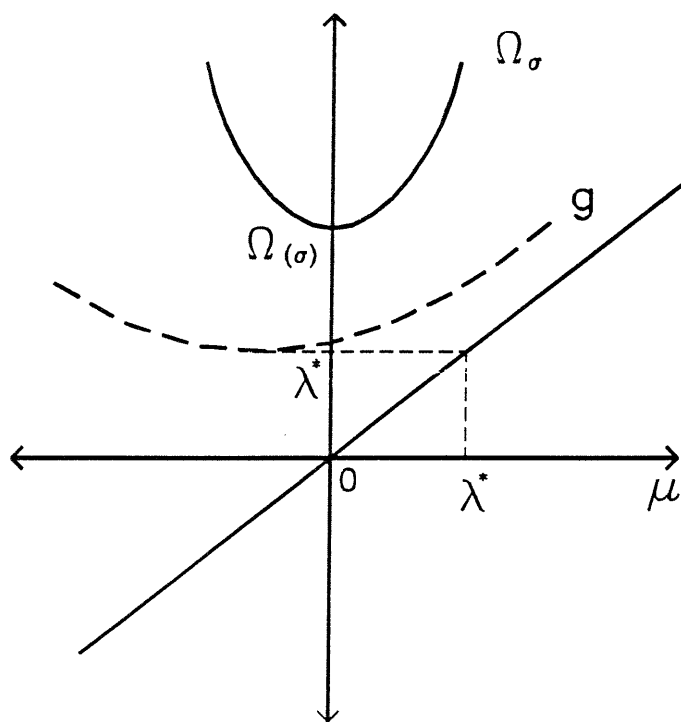
Fig. 1

Fig. 2

Fig. 3

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