**MARKET LEARNING AND PRICE-DISPERSION***

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## MARKET LEARNING AND PRICE-DISPERSION

**M. Dolores Alepuz & Amparo Urbano**

### A B S T R A C T

This paper analyzes how learning considerations may influence the pricing behavior of a duopoly facing demand uncertainty. We consider a symmetric duopoly game with product differentiation where firms have imperfect information about some parameters of the market demands. Firm learn about these parameters by observing market sales in the two markets.

The central body of this paper consists in showing the conditions under which there exists price dispersion at the equilibrium in pure strategies. In particular, when both -product substitutes- firms experiment in each of the two markets and they have the same ability to make market signals more informative, they will price-disperse as an attempt to increase the informative content of these signals. In this way, a "sampling effect" may arise as the outcome of market learning behavior.

**Keywords:** Learning, Experimentation, Price-dispersion, Asymmetric equilibria.

# 1.- INTRODUCTION

The purpose of this analysis is to characterize equilibrium in markets for consumer goods in which firms set their own prices and they operate under some uncertain environment. Particular attention will be paid to the conditions required for an equilibrium to involve price-dispersion, i.e. the state when some firms charges different prices than others.

Stimulated primarily by Stigler (1961), many economists have devoted considerable effort towards the goal of developing and formalizing the analysis of markets which operate with imperfect information. In this framework, we consider the problem of sellers' behaviour and the joint determination of the price distribution. Once the problem is formulated in this fashion, one important question arises. Under what conditions will equilibrium consist of non-trivial distribution of prices, as opposed to the clasical unique price?.

One difficulty in developing the analysis is that once the problem of information is introduced, one must face the fact that there are many types of markets, varyng according to the structures of information flow, the homogeneity of the product, etc.... The interaction of these factors and others is involved in determining price dispersion, and it is not so obvious which factors and interactions should be singled out as the building blocks of a model. As a start, it is natural to take both one particular market and information flow structures as given, and to derive an equilibrium for that market.

In particular, we focus on uncertain environments where firms devote much effort to the adquisition of information to increase their market profitability. In this paper we study an aspect of this gathering of information, and the induced firms' behavior, known as experimentation. By experimenting firms change their present actions to vary the amount of information available to the future. We are particularly interested in analyzing how symmetric experimental firms' behavior may give rise to different equilibrium prices.

We propose to examine this question in the context of a two stage game, and under price competition. Thus, we consider a non homogeneous product duopoly where firms face demands that may have both the slope parameter and the product substitution parameter unknown to both of them. In particular, they can be one of two possible values. And, in addition there are random noise terms in both markets, that mask the true value of these parameters. Each firm is assumed to have prior beliefs about them that are common and common knowledge.

In period one each firm chooses a price. Then, sales are realized in both markets. We assume that firms observe these market sales as well as both firms' prices. This information is used to update prior beliefs, yielding posterior beliefs that will be used in the second period. Since both firms use the same vector signal -namely the two markets' sales realizations- to revise beliefs, any attempt to make this signal more informative must take in account both the effect of such action on the rival's updating and the effect of the rival's actions on the own updating.

Experimentation has been throughly analyzed for single agent decision problems. Mirman, Samuelson and Urbano (1993a, henceforth MSU), for example, recently characterized the incentives to experiment for the single agent case. They study monopoly models with an uncertain demand function containing a parameter (with two possible values) wich is unknown to the monopolist. In this case all observations lead to partial learning about the true value of unknown parameter. The monopoly makes output decisions in the first period of a two period horizon model. Each output decision implies two possible price distributions. Observations of the price leads to Bayesian updating about the unknown parameter.

However less is known about the outcome of market experimentation in a multi-agent context. This is so because in this case we may find out a variety of learning behavior that depends on the kind of uncertainty that firms face that, in turn, affects the specific market signals that they want to make more informative; this determines each firm rivals' learning actions and hence each firm's experimental behavior. Most of the existing literature on this topic (for instance, Alepuz and Urbano, 1991, 1992, 1993, henceforth AU) concentrates on showing the learning behavior of duopolistic firms that face (symmetric and/or not symmetric) market demands with just an unknwon demand parameter. In these situations each firm conducts its own experimentation in either its own market or in the rival's market and there is no room for experimentation by both firms in each of the two markets. In these models firms never price-disperse as a result of their learning behavior.

But if the demand uncertainty is such that it pushes firms to experiment in both their own and the rivals' markets, then the learning outcome will depend on the different ability of firms to change the informative content of market signals and there is an open room for price dispersion to appear as the consequence of firms experimental behavior.

We model our duopoly game as a game of imperfect information and characterize the first period equilibrium solutions. Our main concern is to show under which conditions information about the demand unknown parameters is acquired more effectively if firms experiment by setting different prices. That is if they price-disperse. Our results show that when firms ability to make market signals more informative is the same for both of them in each market and they are product substitutes, they will price-disperse as an attempt to increase the informative content of these signals. In this way, a "sampling" effect may arise as the global outcome of market learning behavior.

Other explanations of price dispersion have been developed, most of them based on search costs. However, most of the search models of price-dispersion need to assume some asymmetry in the amount of information available to the buyers. For example, many models contain some form of *ex ante* heterogeneity: see Reinganum (1979) (different production costs), Salop and Stiglitz (1976). But, as Burdett and Judd (1983) remarked, what appears to be crucial for equilibrium price dispersion, in this class of models, is an *ex post* heterogeneity in consumer information: see the stochastic advertising mechanism and the stochastic nonsequential search in Wilde

(1977). This *ex post* heterogeneity may still occur when there is no *a priori* reason to expect it. Moreover, some of these models do not exclude the existence of a symmetric equilibrium in pure strategies. In contrast our approach excludes any type of either *ex ante* or *ex post* heterogeneity in the sellers' information and thus it offers a new reason for equilibrium price dispersion. Namely, a *sampling effect* in the adquisition of information by firms, when their ability to influence the informative content of market signals is the same for both of them in each market.

In other models, imperfect information *alone* is insufficient to support price dispersion (Reinganum, 1979). Thus, our model provides a rationale for the existence of the equilibrium price dispersion as long as the imperfect information continues.

The value of information in oligopoly games has been the subject of intensive research. These studies tipically assume either that the firms transmit information by means of "certifiable announcements" or that the signals that yield information to the firms are generated exogenously. Our model differ from these in that the amount of information generated is determined endogenously by the choice of actions in the first period.

The closest model to ours is Aghion, Espinoza y Jullien (1993), who also analyze the price-dispersion phenomenon in a very similar duopoly setting. However, we find out that their results are driven by the specific way of modelling the error terms that affect both market demands and which, in turn, determines which specific market signals the firms want to make more informative. In particular, it determines that firms are interested in making their market sales' difference more informative for each estate of nature. With symmetric demands this is accomplished by price-dispersing. In this sense their sampling effect outcome is implicitely introduced by their concrete specification of demands' random shocks and it is not susprising that when dealing with general (although symmetric) expected demands it represents more a local result than the global outcome of firms learning behavior. In fact, their insight of that information about the elasticity of substitution between competing goods is best acquired if firms set different prices is not generally true -under both general expected demands ans error terms specifications- unless firms ability to make market signals more informative is the same for both of them in each market, like in the Hotelling model. This is why, in this model price dispersión is a global result independently of the demand random shock structure.

In contrast, our model clarifies the learning mechanims which operate -in general learning duopoly models- for the existence of price dispersion. In particular, we offer the sufficient conditions for this phenomenon. In this sense, our results are different and more general that theirs. Thus, we allow for a more general modelization of the market demand randoms terms and for the general class of joint distribution functions of the noises that satisfy the generalized strict monotone likelihood ratio property.

The paper is organizated as follow. Section 2 sets up the duopoly model as a game of incomplete information, and the basic demand and information structure assumptions are laid down in first place. Analysis of the informational characteristics and the experimenting behavior of firms follows. Some examples illustrate the main findings.The characterization of the first period equilibria is the goal of section 3. Section 4 relates the main result with more informative signals. The existence of equilibrium is considered next. Concluding remarks follow.

## 2.- THE GENERAL MODEL

Consider a symmetric duopoly model. The firms, denoted by 1 and 2, produce heterogeneous products over two periods. Symmetric market demands are given by:

$$
\begin{aligned}
Q_1 &= \gamma_1(P_1,P_2,\theta) + \varepsilon_1 \\
Q_2 &= \gamma_2(P_1,P_2,\theta) + \varepsilon_2
\end{aligned}
\tag{1}
$$

where $Q_1$ and $Q_2$ are sales in markets 1 and 2 respectively and $P_1$ and $P_2$ the prices that firms set. $\theta$ represents the unknown parameters of the demand functions, and $\varepsilon_1$ and $\varepsilon_2$ are the realizations of uncorrelated random shocks on demands, $(\tilde{\varepsilon}_1,\tilde{\varepsilon}_2)$, whose joint distribution is characterized by a continously differentiable density $f(\varepsilon_1,\varepsilon_2)$, which has zero means $(\iint \varepsilon_1 f(\varepsilon_1,\varepsilon_2)d\varepsilon_1 d\varepsilon_2 = 0 = \iint \varepsilon_2 f(\varepsilon_1,\varepsilon_2)d\varepsilon_1 d\varepsilon_2)$ and full support on $\mathbb{R}^2$ [1]. We assume that $\theta$ takes on one of two possible values, $\bar{\theta}$ or $\underline{\theta}$, and that firms begin period one with a common and common knowledge prior probability distribution over $\{\bar{\theta},\underline{\theta}\}$. Let $\rho_0$ denote firms' prior belief that $\theta=\bar{\theta}$. We assume that for $\theta=\bar{\theta}$ or $\theta=\underline{\theta}$, $\gamma_i(0,P_j,\theta)>0$, $i=1,2$, $i\neq j$ and that $\gamma_i(P_i,P_j,\theta)$ is decreasing and concave on $P_i$ and increasing and convex in $P_j$ (products are substitute). Let $\bar{\gamma}_i=\gamma_i(P_i,P_j,\bar{\theta})$ and $\underline{\gamma}_i=\gamma_i(P_i,P_j,\underline{\theta})$, and $\frac{\partial\bar{\gamma}_i}{\partial P_i} = \bar{\gamma}'_{i,P_i}$, $\frac{\partial\bar{\gamma}_i}{\partial P_j} = \bar{\gamma}'_{i,P_j}$. We also assume that $|\bar{\gamma}'_{i,P_i}|\geq|\underline{\gamma}'_{i,P_i}|$, and $\bar{\gamma}'_{i,P_j}\geq \underline{\gamma}'_{i,P_j}$, so that we are

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[1] Notice that market sales are not truncated at zero, so that very small realizations of the $\varepsilon$'s can drive sales negative. The analysis is more difficult without this assumption.

concerned about uncertainty that may affect both the slope and the product substitution parameters of the demand functions.

In period one, firms choose prices $P_1$ and $P_2$. For simplicity, we assume that production cost are zero, so that firm i's expected profits, for i=1,2, j≠i, is

$$\pi_i(P_1,P_2;\rho_0)=[\gamma_i(P_1,P_2,\bar{\theta})\rho_0 + \gamma_i(P_1,P_2,\underline{\theta})(1-\rho_0)]P_i \tag{2}$$

After these first period prices are chosen, values of $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$, and therefore values of $Q_1$ and $Q_2$, are realized. We assume that firms observe first period prices and market sales but not the realizations of $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$. Consequently, firms may not be able to determine the value of $\theta$ after the first period. The observation of the market signal $(Q_1,Q_2)$, together with knowledge of $P_1$ and $P_2$, however, leads each firm to revise its beliefs regarding the value of $\theta$. We assume that such revisions proceed according to Bayes'rule, so that firm i's posterior beliefs that $\theta=\bar{\theta}$, $\rho(Q_1,Q_2,P_1,P_2|\rho_0)$, is given by:

$$\frac{\rho_0^2 f(Q_1-\gamma_1(P_1,P_2,\bar{\theta}),\ Q_2-\gamma_2(P_1,P_2,\bar{\theta}))}{\rho_0^2 f(Q_1-\gamma_1(P_1,P_2,\bar{\theta}),Q_2-\gamma_2(P_1,P_2,\bar{\theta}))+(1-\rho_0)^2 f(Q_1-\gamma_1(P_1,P_2,\underline{\theta}),Q_2-\gamma_2(P_1,P_2,\underline{\theta}))} \tag{3}$$

where $Q_1$ and $Q_2$ are market sales realizations in period one and $P_1$ and $P_2$ first period prices.

We shall restrict the density function $f(\varepsilon_1,\varepsilon_2)$ to ensure some monotonic properties, i.e. that larger or lower realizations of the $Q_i$'s,

$i=1,2$, in (3) lead to higher (or lower) common posterior beliefs that $\theta=\bar{\theta}$. While assuming a particular distribution for $f(\tilde{\varepsilon}_1,\tilde{\varepsilon}_2)$ -such as the normal distribution- would achieve this end (as well as simplifying the calculations), it would also limit the applicability of our result. We shall instead merely impose the requeriment that $f(\varepsilon_1,\varepsilon_2)$ satisfy the monotone likelihood ratio property (MLRP) in variables $\varepsilon_1$ and $\varepsilon_2$, namely that

$$\frac{f'_i(\varepsilon_1,\varepsilon_2)}{f(\varepsilon_1,\varepsilon_2)} \tag{4}$$

is strictly decreasing on $\varepsilon_i$, $i=1,2$.

In period two, each firm again chooses a price $P_i$, period two expected profits are therefore $\pi_i(P_1,P_2;\rho)$ (where $P_i$ here denotes second period price for firm i, $i=1,2$, and $\rho$ is calculated by (3)).

We are particularly interested in the subgame perfect equilibrium of this two period game. As is usual, we analyze subgame perfect equilibria by transforming the two-period game into a one-period game by specifiying a value function of posterior beliefs. Observe that, under the conditions imposed on $\gamma_i(P_1,P_2,\bar{\theta})$ and $\gamma_i(P_1,P_2,\underline{\theta})$, $i=1,2$, the second period subgame possesses a unique and symmetric Nash equilibrium for each posterior belief $\rho$; let $(P_1^*(\rho),P_2^*(\rho))=P^*$. Firm i's period two value function is thus given by:

$$V_i(\rho) = [\gamma_i(P^*,P^*,\bar{\theta})\rho + \gamma_i(P^*,P^*,\underline{\theta})(1-\rho)]P^* = V_j(\rho) \tag{5}$$

Now, in period one, the posterior belief $\rho$ is not known, but rather is a random variable whose distribution depends upon first period prices (as well as the joint distribution of first period market sales $(Q_1,Q_2)$ induced by $(\tilde{\varepsilon}_1,\tilde{\varepsilon}_2)$). We may therefore write each firm's two period expected profits as a function of first period prices:

$$\Pi_i(P_1,P_2;\rho_0) = \pi_i(P_1,P_2;\rho_0) + \delta\iint V_i(\rho(Q_1,Q_2,P_1,P_2))h(Q_1,Q_2)dQ_1dQ_2 \tag{6}$$

where $\delta$ is the (common) discount factor and

$$h(Q_1,Q_2) = \rho_0^2 f(Q_1-\bar{\gamma}_1,\ Q_2-\bar{\gamma}_2) + (1-\rho_0)^2 f(Q_1-\underline{\gamma}_1,\ Q_2-\underline{\gamma}_2) \tag{7}$$

(recall that $\bar{\gamma}_1 = \gamma_1(P_1,P_2,\bar{\theta})$, $\underline{\gamma}_1 = \gamma_1(P_1,P_2,\underline{\theta})$, etc.)

Let G denote the game whose payoff functions are given by (6) and let $\sigma_i(P_j) = \{P_i \in \underset{P_i}{\text{Argmax}}\ \Pi_i(P_1,P_2;\rho_0)\}$ denote firm i's best reply correspondence. A Nash equilibrium of G is a pair $(P_1^{**},P_2^{**}) \in \sigma_1(P_2^{**}) \times \sigma_2(P_1^{**})$. Any such equilibrium will be on the equilibrium path of a subgame perfect equilibrium of the two period game.

$$\text{Let} \qquad W_i(P_1,P_2) = \iint V_i(\rho(Q_1,Q_2,P_1,P_2))h(Q_1,Q_2)dQ_1dQ_2 \tag{8}$$

Note that by the properties of the demand functions and those of f, $W_i$ is diffentiable, in $P_1$ and $P_2$.

<u>**Value Functions.**</u>

The first step to determine whether learning considerations creates an incentive for firm i to set up a price P different from P , i.e. to price-disperse, is to analyze the behavior of $W_i(P_1,P_2)$ in $P_i$, for a given $P_j$. This section, accordingly, derives an expression for $\frac{\partial W_i}{\partial P_i}$ and establishes conditions under which the function $W_i(P_1,P_2)$ attains a minimun at $P_1=P_2$. The next section will use these results to examine price-dispersion.

To evaluate $\frac{\partial W_i}{\partial P_i}$, first differentiate (8) to obtain

$$\frac{\partial W_i}{\partial P_i} = \iint V_i'(\rho)\ \frac{\partial \rho}{\partial P_i}\ h(Q_1,Q_2)dQ_1dQ_2 + \iint V_i(\rho)\ \frac{\partial}{\partial P_i}\ h(Q_1,Q_2)dQ_1dQ_2 \qquad (9)$$

This expression requires some manipulation before it is useful. Let us consider first the relation among the market signals $Q_1$ and $Q_2$ and firms' posterior beliefs $\rho$. We have imposed the MLRP in variable $\varepsilon_1$ for the function $f(\varepsilon_1,\varepsilon_2)$. Let $\varepsilon_1 = Q_1-\gamma_1(P_1,P_2,\overline{\theta})$, $(\varepsilon_1 = Q_1-\gamma_1(P_1,P_2,\underline{\theta}))$, and let $\overline{\varepsilon}_2 = Q_2 - \overline{\gamma}_2$ $(\underline{\varepsilon}_2 = Q_2 - \underline{\gamma}_2)$, then $f(\varepsilon_1,\overline{\varepsilon}_2)$ $(f(\varepsilon_1,\underline{\varepsilon}_2))$ gives us the probability of the realization of quantity pair $(Q_1,Q_2)$ given that firms' demand are the high value parameter one $(\theta=\overline{\theta})$ (or the lower value, $\theta=\underline{\theta}$). The MLRP will allow us to give a sign to $\frac{\partial \rho}{\partial Q_1}$, i.e. if high or low quantity sales on market 1 will increase or decrease the probability of $\theta=\overline{\theta}$. We similarly require the MLRP in variable $\varepsilon_2$, which will give us a sign to $\frac{\partial \rho}{\partial Q_2}$. Then it is obtained, *Lemma 1:* If both the slope and the product subtitution demand parameters are unknown, then there exist a k, k>0 with $k \lesseqgtr 1$, such that [2]

$$\frac{\partial \rho}{\partial Q_1} \lesseqgtr 0 \text{ depending on } P_1 \gtrless kP_2$$

$$\frac{\partial \rho}{\partial Q_2} \lesseqgtr 0 \text{ depending on } P_2 \gtrless kP_1$$

*Proof:*

Let $\bar{\varepsilon}_i = Q_i - \bar{\gamma}_i$, $\underline{\varepsilon}_i = Q_i - \underline{\gamma}_i$, then $\bar{\varepsilon}_i - \underline{\varepsilon}_i = \underline{\gamma}_i - \bar{\gamma}_i$. Since $\gamma_i$ decreases in $P_i$, and $|\bar{\gamma}'_{i,P_i}| \geq |\underline{\gamma}'_{i,P_i}|$, but increases in $P_j$, and $\bar{\gamma}'_{i,P_j} \geq \underline{\gamma}'_{i,P_j}$, the sign of $\underline{\gamma}_i - \bar{\gamma}_i$ is not clear. However by continuity there exist a k, $0<k \gtrless 1$, such that if $P_1 \gtrless kP_2$, then $\underline{\gamma}_i - \bar{\gamma}_i \gtrless 0$, and hence $\bar{\varepsilon}_i \gtrless \underline{\varepsilon}_i$. In the Appendix it is shown that the MLRP implies that $\frac{\partial \rho}{\partial Q_i} \lesseqgtr 0$ as $\bar{\varepsilon}_i \gtrless \underline{\varepsilon}_i$. ■

*Remark.* Note that the sign of $\frac{\partial \rho}{\partial Q_i}$ depends on the relationship between $P_1$ and $P_2$. This is due to the fact that both the slope and the product substitution demand parameter are unknowns and that products are substitute. With just one parameter unknown, say the slope demand parameter (the product substitution demand parameter), $\bar{\gamma}_i < \underline{\gamma}_i$, for all pairs $(P_1,P_2)$ ($\bar{\gamma}_i > \underline{\gamma}_i$), and consequently $\frac{\partial \rho}{\partial Q_i} < 0$, for all pairs $(P_1,P_2)$ ($\frac{\partial \rho}{\partial Q_i} > 0$). Also if $\gamma'_{i,P_j} < 0$

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[2] Note that if the intercept (in case it exists) were also unknown, then the statement would be $\frac{\partial \rho}{\partial Q_1} \lesseqgtr 0$ depending on $P_1 \gtrless kP_2 + \beta$, where $\beta > 0$. We abstract from this case, since it is not relevant for our purposes.

(products are complements) (with $|\bar{\gamma}'_{i,P_j}|>|\underline{\gamma}'_{i,P_j}|$), then even with unknown slope, $\underline{\gamma}_i > \bar{\gamma}_i$ for all pairs $(P_1,P_2)$.

Since $\rho = \rho(Q_1,Q_2,P_1,P_2)$ the following result, that is proved in the Appendix, gives the behavior of $\rho$ as a function of $P_i$:

<u>*Lemma 2.*</u>

$$\frac{\partial \rho}{\partial P_i} = -\bar{\gamma}'_{i,P_i}\frac{\partial \rho}{\partial Q_i} - \frac{\rho(1-\rho_0)^2}{D}\left[(\bar{\gamma}'_{i,P_i} - \underline{\gamma}'_{i,P_i})f_i(\underline{\varepsilon}_1,\underline{\varepsilon}_2)\right]$$
$$+(-\bar{\gamma}'_{j,P_i})\frac{\partial \rho}{\partial Q_j} - \frac{\rho(1-\rho_0)^2}{D}\left[(\bar{\gamma}'_{j,P_i} - \underline{\gamma}'_{j,P_i})f_j(\underline{\varepsilon}_1,\underline{\varepsilon}_2)\right]$$

and / or

$$\frac{\partial \rho}{\partial P_i} = -\underline{\gamma}'_{i,P_i}\frac{\partial \rho}{\partial Q_i} - \frac{(1-\rho)\rho_0^2}{D}\left[(\bar{\gamma}'_{i,P_i} - \underline{\gamma}'_{i,P_i})f_i(\bar{\varepsilon}_1,\bar{\varepsilon}_2)\right]$$
$$+(-\underline{\gamma}'_{j,P_i})\frac{\partial \rho}{\partial Q_j} - \frac{(1-\rho)\rho_0^2}{D}\left[(\bar{\gamma}'_{j,P_i} - \underline{\gamma}'_{j,P_i})f_j(\bar{\varepsilon}_1,\bar{\varepsilon}_2)\right]$$

Note that it takes account on the fact that $P_i$ influences not only $Q_i$ but also $Q_j$.

Tedious algebraic manipulation of (9) in the appendix (and using lemma 2), give us the following more simplified expresion:

$$\frac{\partial W_i(P_1,P_2)}{\partial P_i} = \iint V_i''(\rho)\left[(\bar{\gamma}'_{i,P_i} - \underline{\gamma}'_{i,P_i})\frac{\partial \rho}{\partial Q_i} + (\bar{\gamma}'_{j,P_i} - \underline{\gamma}'_{j,P_i})\frac{\partial \rho}{\partial Q_j}\right](1-\rho)\rho_0^2 f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)\,dQ_1\,dQ_2 \tag{10}$$

Observe the meaning of (10). Firm i wants to learn about $\theta$, and it uses $P_i$ to this purpose, in order to make market sales $(Q_i,Q_j)$ more informative about the true value of $\theta$. Then, the first term in the brackets represent the "experimentation" that firm i undertakes trough market i (by making $Q_i$ more informative) and the second term the one which goes though market j (by increasing the informative content of $Q_j$). However, firm j also chooses a $P_j$, that also affects the informative content of both market signals, $(Q_i,Q_j)$, so that firm i has to consider the possible influence of $P_i$ on the informative content of $Q_i$ and $Q_j$, given that $P_j$ is also affecting them. This translates to that the ability of i, to make market signals more informative through $P_i$ may not be independent of $P_j$. Here, we consider the case when this ability increases in the absolute price-difference.

Now, we establish some necessary conditions for $W_i$ to be increasing in the distance $|P_i-P_j|$. Consider first the necessary conditions for $\dfrac{\partial W_i}{\partial P_i} \neq 0$, for any given $P_j$, $j\neq i$, $i=1,2$, and for all $(P_i,P_j)$. First, it should be the case that variations in the first period prices, $P_i$, $i=1,2$ should affect the informative content of market sale signals. To formalize this conditions, let $H(\rho,P_1,P_2)$ denote the cumulative distribution fuction of $\rho$ induced by f and $(P_1,P_2)$. Changes in prices do not affect the informative content of market sales if $\dfrac{\partial H(\rho,P_1,P_2)}{\partial P_i}=0$ for all $(\rho,P_1,P_2)$ and $i=1,2$. Clearly if $\dfrac{\partial H}{\partial P_i} = 0$, $i=1,2$, then $\dfrac{\partial W_i}{\partial P_i} = 0$. Observe that $\dfrac{\partial H}{\partial P_i}$ may be zero for two reasons. First, we could have $\bar{\gamma}'_{i,P_i} = \underline{\gamma}'_{i,P_i}$ and $\bar{\gamma}'_{i,P_j} = \underline{\gamma}'_{i,P_j}$, $i=1,2$, $i\neq j$, then the two demand functions have the same slope and product substitution demand parameters for each pair $(P_i,P_j)$, so that $(\bar{\gamma}'_{i,P_i} - \underline{\gamma}'_{i,P_i})$ and $(\bar{\gamma}'_{j,P_i} - \underline{\gamma}'_{j,P_i})$ are zero. In this case, for any realization of $(\tilde{\varepsilon}_1,\tilde{\varepsilon}_2)$, $\rho$ is independent of $P_i$, i=1,2, so that varying $P_i$, i=1,2, does not affect the informative content of market sales, for any $P_j$, j≠i. So a necessary condition for $\frac{\partial W_i}{\partial P_i} \neq 0$ is that $\bar{\gamma}'_{i,P_i} \neq \underline{\gamma}'_{i,P_i}$ or $\bar{\gamma}'_{i,P_j} \neq \underline{\gamma}'_{i,P_j}$ over some interval of prices $(P_i,P_j)$. However, observe that, by the remark of the end of the lemma 1, if just $\gamma'_{i,P_i}$ (or just $\gamma'_{i,P_j}$) is unknown and $\bar{\gamma}'_{i,P_i} - \underline{\gamma}'_{i,P_i} < 0$ $(\bar{\gamma}'_{i,P_j} - \underline{\gamma}'_{i,P_j} > 0)$ for all $(P_i,P_j)$, then $\frac{\partial \rho}{\partial Q_i} < 0$, i=1,2 $(\frac{\partial \rho}{\partial Q_i} > 0$, i=1,2) and by (10) $\frac{\partial W_i}{\partial P_i} > 0$, i=1,2 for any $P_j$, j≠i, higher prices yield a more informative market sale signal, for all price distances $|P_i-P_j|$, so that $W_i$ is independent of $|P_i-P_j|$. This is so, because in the above situation the "experimentation" (the active learning behavior of the firms) is just conducted in one market, i.e. if the slope is unknown parameter, each firm will experiment in its own market and if it is the product substitution parameter they will do it in the rival's market [3].

Then, in order that $W_i(P_1,P_2)$ depend on the distance $|P_i-P_j|$ we need that the firms experiment in both markets at the same time. So a necessary condition for that is that both $\gamma'_{i,P_i}$ and $\gamma'_{i,P_j}$ are unknown parameters.

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3 Intuitively, if $\bar{\gamma}'_{i,P_i} < \underline{\gamma}'_{i,P_i}$ (or $\bar{\gamma}'_{i,P_j} > \underline{\gamma}'_{i,P_j}$), then an increase in $P_i$ spreads the two expected demand curves of market i apart (market j), so that, loosely speaking, it is easier to distinguish between them. The opposite result follows if $\bar{\gamma}'_{i,P_i} > \underline{\gamma}'_{i,P_i}$.

The second condition under which $\frac{\partial H(\rho,P_1,P_2)}{\partial P_i}$ =0 is $\frac{\partial\rho}{\partial Q_i}$ <0, i=1,2, for all $(Q_i,Q_j)$ that belong to the supp $(h(Q_i,Q_j))$.This later condition, in turn, holds under the MLRP if the support of f is "small enough" so that, for any pair $(P_i,P_j)$, the firms learn the value of $\theta$ with probability one. Since we have assumed that if has support on $\mathbb{R}^2$ then we have ruled out this possibility.

Finally, we need that information is valuable to each firm. This translates to the strict convexity of the value function $V(\rho)$. To see this it suffices to compare between firms expectations of second period profits when the true value of the unknown demand parameters is to be learnt between period 1 and 2, and when it is not. Since in the first "informed" case expected second period profits for firm i, i=1,2 are equal to:

$$V_i(1)\rho + V_i(0)(1-\rho)$$

and in the second "uninformed" case, expected second period profits for firm i, i=1,2 are simply, $V_i[E(\theta)]=V_i(\rho)$, then information is <u>valuable</u> <u>ex-ante</u> for each firm if:

$$E[V_i(\theta)]=V_i(1)\rho + V_i(0)(1-\rho) > V_i[E(\theta)]=V_i(\rho),\ i=1,2 \tag{11}$$

so that if $V_i(\rho)$ is strictly convex, information is valuable ex-ante for each firm. Deriving conditions under which the value of information is positive turns out to be difficult and lies beyond the scope of this paper [4].

We now turn to examine the conditions under which $W_i(P_i,P_j)$ increases in the distance $|P_i-P_j|$. One set of sufficient conditions is given by the following proposition.

*Proposition 1:* Suppose that the MLRP as given in (4) holds. Then, $W_i(P_1,P_2)$ is strictly convex in $P_i$, i=1,2, for a given $P_j$, j≠i, with a minimum at $P_i=P_j$ for all $\rho_0 \in (0,1)$ if

(i) Products are substitutes and both the slope and the substitution demands parameters are unknown, i.e.

$$|\bar{\gamma}'_{1,P_i}|>|\underline{\gamma}'_{1,P_i}|,\ \bar{\gamma}'_{1,P_j}>\underline{\gamma}'_{1,P_j}>0$$

(ii) $\mathrm{supp}(f(Q_1-\bar{\gamma}_1(P_1,P_2),Q_2-\bar{\gamma}_2(P_1,P_2))\cap\mathrm{supp}(f(Q_1-\underline{\gamma}_1(P_1,P_2),Q_2-\underline{\gamma}_2(P_1,P_2))\neq\emptyset$

(iii) Information is valuable: $V(\rho)$ strictly convex.

(iv) If $P_1=P_2$, then $\bar{\gamma}_i=\underline{\gamma}_i$, i=1,2. In other words, k, as stated in lemma 1 is equal to 1.

*Proof*. Recall that the MLRP in (4), and condition (i) and (iv) imply that by lemma 1, $\frac{\partial\rho}{\partial Q_i} \gtrless 0$ depending on $P_i \lessgtr P_j$. By (i), (iii) and equation (10) we have that $\frac{\partial W_i}{\partial P} \lessgtr 0$, for $P_i \lessgtr P_j$, i=1,2, j≠i. ∎

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4 Note that in the duopoly case in contrast with the monopoly case, each firm's second period value fuction, although derived from payoff functions that are linear in $\rho$, is no longer the <u>supremum</u> of a colection of such functions, since the other firm's price enters as an argument in its payoff function.

Since conditions (i)-(iii) have already discussed let us now consider condition (iv). This condition seems very strong, but it has a very intuitive explanation. The key to understand the meaning of (iv) is the following. Each firm experiments through the two markets i and j to make the signals $Q_i$ and $Q_j$ more informative, i.e to separate the means of the two possible distribution firm which they can come from, $(\bar{\gamma}_i, \underline{\gamma}_i)$ and $(\bar{\gamma}_j, \underline{\gamma}_j)$. Also the ability of say firm i to experiment, through $P_i$, in each market may be different from that of firm j, through $P_j$, in that market. By lemma 1, given that products are substitutes, $\bar{\gamma}_i \neq \underline{\gamma}_i$, $\bar{\gamma}_j \neq \underline{\gamma}_j$ if firms set prices such that $P_i \neq kP_j$ and $P_j \neq kP_i$ respectively. Thus k reflects the firms different abilities for experimentation in each market. However, if this ability to increase the informative content of market signals $Q_i$ and $Q_j$ through $P_i$ and $P_j$ is the same for both firms in each market i, i=1,2, then, given that the products are substitutes and market demands are symmetric, k=1; in other words, $\bar{\gamma}_i \neq \underline{\gamma}_i$, i=1,2, if $P_i \neq P_j$, j≠i. This behavior gives place to a "sampling" effect. Since market demand are symmetric, when $P_i = P_j$, also $\gamma_i = \gamma_j$, and $\bar{\gamma}_i - \bar{\gamma}_j = \underline{\gamma}_i - \underline{\gamma}_j = 0$. Hence if learning considerations make firms to set $P_i \neq P_j$, in order to separate $\bar{\gamma}_i$ from $\underline{\gamma}_i$, i=1,2, it also separates $\gamma_i$ from $\gamma_j$, or in other words, the means of $Q_i$ and $Q_j$. Thus, price dispersion is a sampling phenomenon that may appear when firms have the same ability to make market signals more informative in the markets they experiment at the same time[(5)].

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5 Note that this phenomenon will not appear in our model in the case where just the demand slope (or the product substitution parameter) is the unknown parameter. This is so because now each firm only has the ability to experiment in its own market (in the rival's market) and not in the rival market (in its own one). Hence both firms experimentation does not coincide in the same market. As it was said before, the behavior of $\rho$ in $Q_i$, i=1,2, does not depend on the relationship between $P_i$ and $P_j$.

Then, (iv) is satisfied in some models and under some error specifications. In particular, in models where the ability of firms to experiment in both markets is the same, like Hotelling's type models,and linear demand structure models with slope and product substitution demands parameters unknown, where both parameters variation is the same, and in those models where the demand random shock are just a random shift from one market to the other (see Aghion, Espinosa and Jullien, 1993). The simplest specifications for which (iv) hold are the following.

*Example 1.* Linear Demands. Let $\gamma_i = a - bP_i + cP_j$, i=1,2, j≠i, with $b=\{\bar{b},\underline{b}\}$, $c=\{\bar{c},\underline{c}\}$, $(\bar{b}-\underline{b})=(\bar{c}-\underline{c})$, and $\rho_0=$ Prob$\{(b,c)=(\bar{b},\bar{c})\}$, that is common and common knowledge for the firms. The stochastic market demand are:

$$\tilde{Q}_1 = \gamma_1 + \tilde{\varepsilon}_1 = a - bP_1 + cP_2 + \tilde{\varepsilon}_1$$
$$\tilde{Q}_2 = \gamma_2 + \tilde{\varepsilon}_2 = a - bP_2 + cP_1 + \tilde{\varepsilon}_2$$

where $f(\tilde{\varepsilon}_1,\tilde{\varepsilon}_2)$ is the joint distribution of the error terms has full suport on $\mathbb{R}^2$ and satisfy the MLRP in variables $\varepsilon_1$ and $\varepsilon_2$. Also $E[\tilde{\varepsilon}_1]=E[\tilde{\varepsilon}_2]=0$. Let $\rho$ be the posterior of $\rho_0$, after the firms set first period prices, $P_1$ and $P_2$ and observe the realizations of market sales $Q_1$ and $Q_2$, i.e. $\rho(Q_1,Q_2,P_1,P_2)$

is
$$\frac{\rho_0^2 f(Q_1-a+\bar{b}P_1-\bar{c}P_2, Q_2-a+\bar{b}P_2-\bar{c}P_1)}{\rho_0^2 f(Q_1-a+\bar{b}P_1-\bar{c}P_2,Q_2-a+\bar{b}P_2-\bar{c}P_1)+(1-\rho_0)^2 f(Q_1-a+\underline{b}P_1-\underline{c}P_2,Q_2-a+\underline{b}P_2-\underline{c}P_1)} \tag{12}$$

Note that since $(\bar{b}-\underline{b})=(\bar{c}-\underline{c})$

$$\bar{\gamma}_i=a-\bar{b}P_i+\bar{c}P_j \gtrless a-\underline{b}P_i+\underline{c}P_j=\underline{\gamma}_i \text{ depending } P_i \lessgtr P_j \tag{13}$$

so that by lemma 1, and by (12) and (13) $\frac{\partial\rho}{\partial P} \gtrless 0$ depending on $P_i \lessgtr P_j$, i=1,2, j≠i.

Also the second period value function $V(\rho)=V_i(\rho) = \frac{a^2\hat{b}}{(2\hat{b}-\hat{c})^2} = V_j(\rho)$ is strictly convex in $\rho$, where $\hat{b}=\rho\bar{b}+(1-\rho)\underline{b}$, and $\hat{c}=\rho\bar{c}+(1-\rho)\underline{c}$. Then equation (10) specializes here to:

$$\frac{\partial W_i(P_1,P_2)}{\partial P_i} = \frac{\partial E[V(\rho(Q_1,Q_2,P_1,P_2)]}{\partial P_1} =$$

$$\iint V''(\rho)[(\bar{b}-\underline{b})\left(-\frac{\partial\rho}{\partial Q_i}\right)+(\bar{c}-\underline{c})\frac{\partial\rho}{\partial Q_j}](1-\rho)\rho_0 f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)dQ_1dQ_2 \lessgtr 0 \text{ depending } P_i \lessgtr P_j. \tag{14}$$

Since if $P_i<P_j$, $\frac{\partial\rho}{\partial Q_i}>0$, $\frac{\partial\rho}{\partial Q_j}<0$, and $V''>0$, imply that (14) is negative; if $P_i>P_j$, $\frac{\partial\rho}{\partial Q_i}<0$, $\frac{\partial\rho}{\partial Q_j}>0$, and (14) is positive. Finally if $P_i=P_j$, $\frac{\partial\rho}{\partial Q_i}=\frac{\partial\rho}{\partial Q_j}=0$ and then (14) attains a minimum.

*Example 2.* The Hotelling model. Let

$$\tilde{Q}_1= \gamma_1(P_1,P_2,\theta) + \tilde{\varepsilon}_1= \frac{1}{2} + \frac{(P_2-P_1)\theta}{2} + \tilde{\varepsilon}_1$$

$$\tilde{Q}_2= \gamma_2(P_1,P_2,\theta) + \tilde{\varepsilon}_2= \frac{1}{2} + \frac{(P_1-P_2)\theta}{2} + \tilde{\varepsilon}_2$$

where $\theta=\{\underline{\theta},\bar{\theta}\}$, $\underline{\theta}<\bar{\theta}<2\underline{\theta}$, $f(\tilde{\varepsilon}_1,\tilde{\varepsilon}_2)$ is the joint density of the shock disturbances, with full support on $\mathbb{R}^2$ and satisfy the MLRP in variables $\varepsilon_1$ and $\varepsilon_2$. Also, $E[\tilde{\varepsilon}_1]=E[\tilde{\varepsilon}_2]=0$.

Note that the <u>mean demands</u> here correspond to a Hotelling model in which two firms, 1 and 2, are located at the extreme points of the interval [0,1]. The firms costlessly produce two goods 1 and 2 that are identical except for their location and they compete in mill price and cannot change location. Consumers are uniformly distributed on the same interval and each of them purchases only one unit of good per period, provided the total payment (mill price plus transportation cost) is less than his reservation value. The cost of transporting one unit of good is $\frac{1}{\theta}$ per unit of distance. At the beginning of period one, the firms do not know the exact value of $\theta$, but they have the same prior beliefs about $\theta$. Let $\rho_0=\text{Prob}\{\theta=\bar{\theta}\}$, and this is common knowledge (the consumers know the true value of $\theta$, since they directly bear it). It is also assumed that the consumers reservation value is sufficiently large for the whole market to be served by firms 1 and 2[6].

Firms set prices $P_1$ and $P_2$ in the first period and observe the realizations of $Q_1$ and $Q_2$. Consequently, the posterior $\rho(Q_1,Q_2,P_1,P_2)$ is

$$\frac{\rho_0^2 f(Q_1-\frac{1}{2}-\frac{1}{2}(P_2-P_1)\bar{\theta},\ Q_2-\frac{1}{2}-\frac{1}{2}(P_1-P_2)\bar{\theta})}{\rho_0 f(Q_1-\bar{\gamma}_1(P_1,P_2,\bar{\theta}),Q_2-\bar{\gamma}_2(P_1,P_2,\bar{\theta}))+(1-\rho_0)f(Q_1-\underline{\gamma}_1(P_1,P_2,\underline{\theta}),Q_2-\underline{\gamma}_2(P_1,P_2,\underline{\theta}))} \tag{15}$$

Also notice that

$$\bar{\gamma}_1(P_1,P_2,\bar{\theta})=\frac{1}{2}+\frac{(P_2-P_1)\bar{\theta}}{2} \gtrless \frac{1}{2}+\frac{(P_2-P_1)\underline{\theta}}{2}=\underline{\gamma}_1(P_1,P_2,\underline{\theta}) \tag{16}$$

6 Note, again, that market sales are not truncated at zero, so that very small realizations of the ε's can drive negative sales .

depending on $P_1 \lesseqgtr P_2$, so that for $\rho$ as defined in (15), $\frac{\partial \rho}{\partial Q_1} \gtrless 0$ depending on $P_1 \lesseqgtr P_2$, $\frac{\partial \rho}{\partial Q_2} \lesseqgtr 0$ depending on $P_1 \lesseqgtr P_2$, and since $\bar{\gamma}'_{i,P_i} = -\bar{\gamma}'_{i,P_j}$, $\underline{\gamma}'_{i,P_i} = -\underline{\gamma}'_{i,P_j}$ and the firms second period value function $V_i(\rho)=\frac{1}{2\hat{\theta}}=V_j(\rho)=V(\rho)$ is strictly convex, then $\frac{\partial W_i(P_1,P_2)}{\partial P_i} = \frac{\partial E[V(\rho)]}{\partial P_1}$ is equal to

$$(\bar{\theta}-\underline{\theta})\iint V''(\rho)[\left(-\frac{\partial \rho}{\partial Q_i}\right)+\frac{\partial \rho}{\partial Q_j}](1-\rho)\rho_0 f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)dQ_1dQ_2 \lesseqgtr 0 \text{ depending on } P_i \lesseqgtr P_j. \tag{17}$$

so that $W_i(P_1,P_2)$ is strictly convex in $P_i$, i=1,2, with a minimum at $P_i=P_j$, $j\neq i$.

The next example shows that (iv) is fulfilled under some shock terms specifications.

*Example 3.* The Aghion et alia (1993), error term specification.

Let us consider the Aghion et alia error term specification where two duopolists A and B face the following symmetric demands for their products at each period t=1,2:

$$\begin{aligned} \tilde{X}_A &= D_A(P_A,P_B,\theta) + \tilde{\varepsilon}_a + \tilde{\varepsilon}_s \\ \tilde{X}_B &= D_B(P_A,P_B,\theta) + \tilde{\varepsilon}_a - \tilde{\varepsilon}_s \end{aligned} \tag{18}$$

where $\theta=\{\underline{\theta},\bar{\theta}\}$, represent the unknown parameters of the model and $(\tilde{\varepsilon}_a,\tilde{\varepsilon}_s)$ is a random shock on demand. The agregate component $\tilde{\varepsilon}_a$ is interpreted as a shock on total demand (reflects all the random influences that affect both firms equally). The other component $\tilde{\varepsilon}_s$ is interpreted as a shift of market share from one firm to the other which leaves total demand unchanged. Let $f(\tilde{\varepsilon}_a,\tilde{\varepsilon}_s)$ represent the joint distribution function of the error with support on $\mathbb{R}^2$, and with $E[\tilde{\varepsilon}_a]=E[\tilde{\varepsilon}_s]=0$, and $\rho_0$, as before, the initial common and common knowledge a prior' probability of $\theta=\bar{\theta}$.

We assume here that the density $f(\tilde{\varepsilon}_1,\tilde{\varepsilon}_2)$ has the MLRP in variables $\tilde{\varepsilon}_a$ and $\tilde{\varepsilon}_s$. As it is usual, in the first period firms set prices $P_A$ and $P_B$ and observe the market sales realizations $X_A$ and $X_B$. Then, the posterior probability of $\theta=\bar{\theta}$, $\rho$, is

$$\rho(X_A+X_B,X_A-X_B,P_A,P_B)=\frac{\rho_0^2 f(\bar{\varepsilon}_a,\bar{\varepsilon}_s)}{\rho_0^2 f(\bar{\varepsilon}_a,\bar{\varepsilon}_s)+(1-\rho_0)^2 f(\underline{\varepsilon}_a,\underline{\varepsilon}_s)} \tag{19}$$

where from (18)

$$\varepsilon_a=\frac{X_A+X_B}{2}-\frac{(D_A+D_B)}{2},\qquad \varepsilon_s=\frac{X_A-X_B}{2}-\frac{(D_A-D_B)}{2} \tag{20}$$

and letting $\bar{D}_A= D_A(P_A,P_B,\bar{\theta})$, $\underline{D}_A=D_A(P_A,P_B,\underline{\theta})$, etc, then

$$\bar{\varepsilon}_a=\frac{X_A+X_B}{2}-\frac{(\bar{D}_A+\bar{D}_B)}{2},\qquad \bar{\varepsilon}_s=\frac{X_A-X_B}{2}-\frac{(\bar{D}_A-\bar{D}_B)}{2},\ \text{etc...}$$

Let $\mathcal{X}=\frac{X_A+X_B}{2}$ and $\mathcal{Y}=\frac{X_A-X_B}{2}$, then here firms are interested in making more informative market signals $\mathcal{X}$ and $\mathcal{Y}$. Note that,

$$\frac{\partial \rho}{\partial \mathcal{X}} = \frac{(\bar{f}'_1\underline{f} - \underline{f}'_1\bar{f})}{D^2}\,\rho_0^2(1-\rho_0)^2, \text{ and} \tag{21}$$

$$\frac{\partial \rho}{\partial \mathcal{Y}} = \frac{(\bar{f}'_2\underline{f} - \underline{f}'_2\bar{f})}{D^2}\,\rho_0^2(1-\rho_0)^2, \tag{22}$$

where $f'_1$ and $f'_2$ denote the derivatives of f with respect to its first and second arguments respectively, and $\bar{f}=f(\bar{\varepsilon}_a,\bar{\varepsilon}_s)$, $\underline{f}=f(\underline{\varepsilon}_a,\underline{\varepsilon}_s)$ and $D=\bar{f}\rho_0^2+\underline{f}(1-\rho_0)^2$.

If $f(\varepsilon_a,\varepsilon_s)$ has the MLRP in variable $\varepsilon_a$ it means that by (20), $\frac{\partial \rho}{\partial \mathcal{X}} \gtrless 0$ depending on $\bar{D}_A+\bar{D}_B \gtrless \underline{D}_A+\underline{D}_B$ (i.e. $\bar{\varepsilon}_a \lessgtr \underline{\varepsilon}_a$). Note that this does not depend on the difference of prices. In fact, it depends on the sum of them. For instance, if $D_A$ and $D_B$ are linear, as in example 1: $\bar{D}_A+\bar{D}_B \gtrless \underline{D}_A+\underline{D}_B$ depending on $(\bar{b}-\underline{b})(P_A+P_B) \lessgtr (\bar{c}-\underline{c})(P_A+P_B)$, so that if $(\bar{b}-\underline{b}) = (\bar{c}-\underline{c})$, $\frac{\partial \rho}{\partial \mathcal{X}}=0$. Also, in the Hotelling model of example 2, $\bar{D}_A+\bar{D}_B = \underline{D}_A+\underline{D}_B=0$, so that $\frac{\partial \rho}{\partial \mathcal{X}}=0$ again. The key feature of $\frac{\partial \rho}{\partial \mathcal{X}}$ is that it will be alway positive or negative (or zero) for any price pair combination, and does not depend on $|P_1-P_2|$.

However $\frac{\partial \rho}{\partial \mathcal{Y}}$ is different. By (20) and the MLRP $\frac{\partial \rho}{\partial \mathcal{Y}} \gtrless 0$ depending on $\bar{D}_A-\bar{D}_B \gtrless \underline{D}_A-\underline{D}_B$(or $\bar{D}_A-\underline{D}_A \gtrless \bar{D}_B-\underline{D}_B$), that in turn is the case whenever $P_A \lessgtr P_B$. Note that if $P_A=P_B$, then $D_A=D_B$ and hence $\bar{\varepsilon}_s=\underline{\varepsilon}_s$ that by the MLRP implies $\frac{\partial \rho}{\partial \mathcal{Y}}=0$; if $P_A \lessgtr P_B$ then by the symmetry of demands $D_A \gtrless D_B$ and $\bar{D}_A-\bar{D}_B \gtrless \underline{D}_A-\underline{D}_B$ since $\bar{\theta}>\underline{\theta}$. For instance, in the linear demand structure of example 1, $\frac{\partial \rho}{\partial \mathcal{Y}} \gtrless 0$ depending on $(\bar{b}+\bar{c})(P_A-P_B) \lessgtr (\underline{b}+\underline{c})(P_A-P_B)$, that, in turn, is the case whenever $P_A \lessgtr P_B$, and in the Hotelling model: $\frac{\partial \rho}{\partial \mathcal{Y}} \gtrless 0$ depending on $\bar{\theta}(P_A-P_B) \lessgtr \underline{\theta}(P_A-P_B)$, or, on what is the same, $P_A \lessgtr P_B$.

Let $V(\rho)=V_A(\rho)=V_B(\rho)$ the firms second period value function and assume that the second period equilibrium is unique and let $W(P_A,P_B)=E[V(\rho(\frac{X_A+X_B}{2},\frac{X_A-X_B}{2},P_A,P_B))]$. Also assume that information is valuable, i.e. $V(\rho)$ is strictly convex. Some calculations (see the Appendix) show that:

$$\frac{\partial W(P_A,P_B)}{\partial P_A}=\iint V''(\rho)[(A+B)\frac{\partial\rho}{\partial\mathcal{X}}+(A-B)\frac{\partial\rho}{\partial\mathcal{Y}}](1-\rho)\rho_0^2 f(\bar{\varepsilon}_a,\bar{\varepsilon}_s)dX_A dX_B \tag{23}$$

where $A=\bar{D}'_{A,P_A}-\underline{D}'_{A,P_A}<0$, $\bar{D}'_{A,P_A}=\frac{\partial\bar{D}_A}{\partial P_A}$, $\underline{D}'_{A,P_A}=\frac{\partial\underline{D}_A}{\partial P_A}$, and

$B=\bar{D}'_{B,P_A}-\underline{D}'_{B,P_A}>0$, $\bar{D}'_{B,P_A}=\frac{\partial\bar{D}_B}{\partial P_A}$, $\underline{D}'_{B,P_A}=\frac{\partial\underline{D}_B}{\partial P_A}$

Note that (23) reflects the experimentation that firm A undertakes in order to make the market signals $\mathcal{X}$ and $\mathcal{Y}$ more informative and hence $\rho$ more precise. Also, notice that here, in contrast with our model, firm A is not interested in making $X_A$ and $X_B$ more informative by themselves, i.e. to separate $\bar{D}_A$ from $\underline{D}_A$ and $\bar{D}_B$ from $\underline{D}_B$, but in separating $\bar{D}_A+\bar{D}_B$ form $\underline{D}_A+\underline{D}_B$ and $\bar{D}_A-\bar{D}_B$ from $\underline{D}_A-\underline{D}_B$. This makes a significant difference from the general model, since now the firms' ability to make each market signal ($X_A$ and $X_B$) more informative is not important at all. However, the "sampling" effect that appeared there, when this ability was the same for the two firms, is explicitely introduced here through the special structure of the random shocks that already incorporates the "sampling" effect: by means of $\varepsilon_s$. By (20), $\bar{\varepsilon}_s=\underline{\varepsilon}_s$ whenever $\bar{D}_A-\bar{D}_B\neq\underline{D}_A-\underline{D}_B$, and since at $P_A=P_B$, $D_A=D_B$, the price- dispersion results would follow trivially from the modelling of $\varepsilon_s$, if $\varepsilon_a$ were absent (or for a given level of $\varepsilon_a$).

More formally, note that (A-B)<0, so that if $P_1 \lessgtr P_2$, then $\frac{\partial \rho}{\partial y} \gtrless 0$ and $(A-B)\frac{\partial \rho}{\partial y} \lessgtr 0$. Suppose that $\frac{\partial \rho}{\partial x} = 0$ (as in the Hotelling model of example 2, or in the linear demand estructure of example 1), then $\frac{\partial W}{\partial P_A} \lessgtr 0$ whenever $P_A \lessgtr P_B$, i.e. $W(P_A,P_B)$ is strictly convex in $P_A$ with a minimum at $P_A=P_B$. (Notice that:

$$\frac{\partial W(P_A,P_B)}{\partial P_B} = \iint V''(\rho)\left[(A+B)\frac{\partial \rho}{\partial x} - (A-B)\frac{\partial \rho}{\partial y}\right](1-\rho)\rho_0 f(\bar{\varepsilon}_a,\bar{\varepsilon}_s)dX_A dX_B \quad \text{so that if}$$

$$\frac{\partial \rho}{\partial x}=0, \text{ then } \frac{\partial W}{\partial P_B} \lessgtr 0 \text{ whenever } P_B \lessgtr P_A).$$

Thus, *if the demand random disturbances are just random shifts from one market to the other, then our condition (iv) of Proposition 1 is trivially satisfied*[7], and the function $W(P_A,P_B)$ is strictly convex in $P_A$ and in $P_B$ with a minimum at $P_A=P_B$. We will see next that this implies price dispersion at the equilibrium in pure strategies. So we have

*Proposition 2.* The price-dispersion phenomenon will appear if the demands specification is of the form:

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7 Note that for this result we do not need, as in our model, that the slope and the product substitution demand parameters are both unknown. With just one of them unknown the result would also follow.

(i) $$Q_1=\gamma_1(P_1,P_2,\theta) + \varepsilon$$
$$Q_2=\gamma_2(P_1,P_2,\theta) - \varepsilon$$

where $\theta$ is the unknown demand parameter and $f(\varepsilon)$ is the density of $\varepsilon$, that satisfies the MLRP, or

(ii) $$Q_1=\gamma_1(P_1,P_2,\theta) + \varepsilon_a + \varepsilon_s$$
$$Q_2=\gamma_2(P_1,P_2,\theta) + \varepsilon_a - \varepsilon_s$$

and $\gamma_i(P_1,P_2,\theta)$ is as either in example 1 or in 2 (Hotelling model), and $f(\varepsilon_a,\varepsilon_s)$ satisfies the MLRP with respect to variables $\varepsilon_a$ and $\varepsilon_s$.

Next, what is the sign of (23) when $\frac{\partial\rho}{\partial\mathcal{X}} \neq 0$?. Note that the sign of (A+B) is the same than that of $\frac{\partial\rho}{\partial\mathcal{X}}$ (more properly the sign of (A+B) determines that of $\frac{\partial\rho}{\partial\mathcal{X}}$), so that the term $(A+B)\frac{\partial\rho}{\partial\mathcal{X}}$ (in (23)) is always nonnegative. In the case we are considering it is positive, implying that if $\frac{\partial\rho}{\partial\mathcal{Y}}=0$, then $\frac{\partial W}{\partial P_i} >0$, i=A,B, i.e. $W(P_A,P_B)$ is increasing on $P_i$, i=A,B. In other words, assuming that information is valuable the duopolist will increase first period prices with respect to the one-shot optimal ones, for learning considerations ("experimentation" purposes), as in the case of demands with just unknown slope. Then, if the random disturbances are such that they just affect both firms equally, the firms have an incentive to increase prices respect to the myopic ones.

Then the term $(A+B)\frac{\partial\rho}{\partial\mathcal{X}}$ adds something positive to the term $(A-B)\frac{\partial\rho}{\partial\mathcal{Y}}$, implying the (provided that information is valuable) following "local" results:

*Proposition 3.* If the densities $f(\tilde{\varepsilon}_a,\tilde{\varepsilon}_s)$ satisfy the MLRP in variables $\varepsilon_a$ and $\varepsilon_s$, and under the Aghion et alia, error terms especification, then

(i) For a fixed level of $\varepsilon_a$, $W(P_A,P_B)$ is increasing on $|P_A-P_B|$,

(ii) For a fixed level of $\varepsilon_s$, $W(P_A,P_B)$ is increasing on $P_i$, i=A,B.

Note that (i) is obtained in Aghion et alia (1993) under the assumption that $f(\varepsilon_a,\varepsilon_s)$ is symmetric and quasi-concave in its second argument $\varepsilon_s$. Here, in contrast we merely need that $f(\tilde{\varepsilon}_a,\tilde{\varepsilon}_s)$ satisfy teh MLRP in variables $\varepsilon_a$ and $\varepsilon_s$. Notice, also, that this is a "local" result, since $\varepsilon_a$ is kept fixed, and the global learning behavior of firms is driven by the combination of (i) and (ii).

Thus, firm have an incentive to both price-disperse and to increase prices in order to learn about θ. Now, the question is what is the sign of (23) when these two incentives are operating at the same time?. It is clear that for any $P_i$, i=A,B, $P_i>P_j$, j≠i, $W(P_A,P_B)$ is strictly increasing on $P_i$, since W increases in $P_i$, i=A,B and in the distance $|P_i-P_j|$, j≠i. However, for $P_i<P_j$, the result is not so clear. Hence, we have two posible behaviors of W in $P_i$, i=A,B and none of them guarantees the "price-dispersion" outcome globally

(iii). Either the function $W(P_A,P_B)$ is increasing in $P_i$, i=A,B, for all price pairs $(P_A,P_B)$ meaning that W will be concave for $P_i<P_j$, and convex for $P_i>P_j$, i.e. the second period expected value function of each firm i is higher the higher is its own price and the higher the distance $P_i-P_j>0$.

(iv). Or W can be still strictly convex, but the minimum is not at $P_A=P_B$, but to the left of it, since the firm always benefit in the future from the higher information adquire though higher prices.

Neither (iii) nor (iv) can guarantee that firms choose to disperse prices as an equilibrium outcome of the rolled-back game.

## 3.- CHARACTERIZATION OF EQUILIBRIUM PRICES

In this section we characterize the equilibria prices for the first period of the rolled back game, when the conditions of the Proposition 1 are satisfied, i.e. when $E[V(\rho)]$ is strictly convex in $P_i$, i=1,2. We show that any equilibrium in pure strategies of this game involves prices dispersion in period one. The existence of an equilibrium in pure estrategies will be analyzed in a next section, so that suppose, for the moment, that an equilibrium in pure strategies exists.

*Proposition 4.* If the densities $\{f(Q_1-\gamma_1(P_1,P_2,\theta),Q_2-\gamma_2(P_1,P_2,\theta)\}$, $\theta=\{\underline{\theta},\overline{\theta}\}$, satisfy the MLRP in variables $\varepsilon_1$ and $\varepsilon_2$ and if the conditions of Proposition 1 are satisfied (i.e. $E[V(\rho)]$ is strictly convex in $P_i$, i=1,2 with a minimum at $P_i = P_j$), then an equilibrium solutions for the first period of the rolled back game is characterized by a pair $P_1^{**}$, $P_2^{**}$ such that

$$P_1^{**} \neq P_1^m$$
$$P_2^{**} \neq P_1^m$$

and either $P_1^{**}>P_1^m>P_2^{**}$, or $P_1^{**}<P_1^m<P_2^{**}$. In other words there exist no symmetric equilibrium in pure strategies.

*Proof.* For any given $P_j$ chosen by firm j in period one, firm i will determine its best response to P , by (see (6)),

$$P_i^{**}(P_j) = \underset{P_i}{\text{Argmax}} \; [\; \pi_i(P_1,P_2;\rho_0) + \delta E[V_i(\rho(Q_1,Q_2))]] \tag{24}$$

Let us define the one-shot (myopic) best response to $P_j$, by

$$\hat{P}_i(P_j) = \underset{P_i}{\text{Argmax}}\ [\pi_i(P_1,P_2;\rho_0)] \tag{25}$$

Observe that since $\hat{P}_i(P_j)$ is the myopic reacction function $\hat{P}_i \gtrless P_j$ depending on $P_j \lessgtr P^m_1$.

Next, we derive the best response correspondence for firm i.

Suppose first that $P_j > P^m_1$. Then $\hat{P}_i(P_j) < P_j$. Since $E[V(\rho)]$ is strictly convex in $P_1$, with at minimum at $P_i = P_j$, this minimun is to the right of the maximum of the function $\pi_i(P_1,P_2)$, that is to the right of $\hat{P}_i(P_j) < P_j$. Then, since

$$\hat{P}_i(P_j) - P_j < 0, \tag{26}$$

the one-shot best response is not the best that firm i can do. In fact, at $\hat{P}_i(P_j)$, by (26) and Proposition 1, $E[V_i(\rho)]$ is decreasing and since $\pi_i(P_1,P_2;\rho_0)$ is strictly concave in $P_i$, then the first order conditions imply that $P^*_i < \hat{P}_i(P_j)$, since the loss of short run profits is small compared to the gains of information. See figure 1

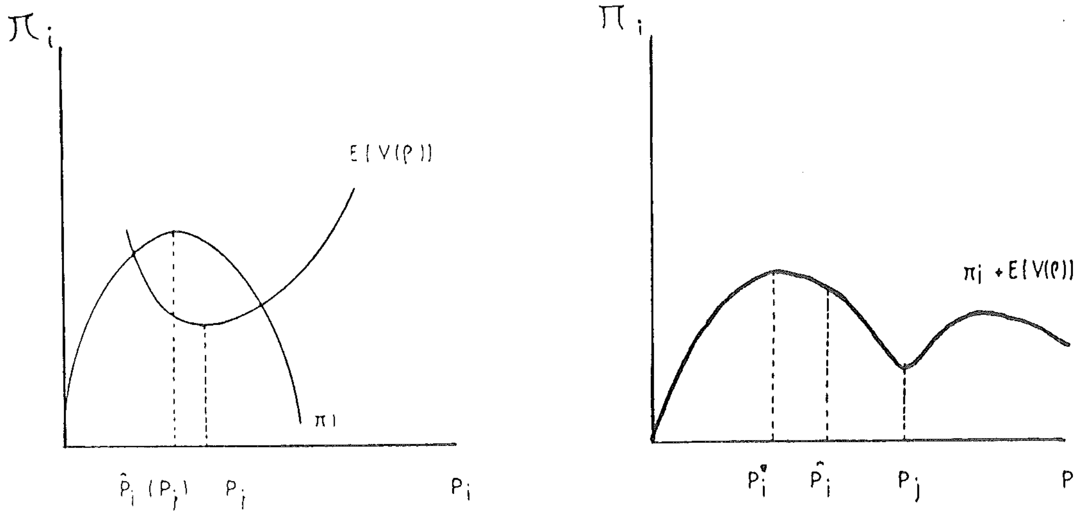


FIGURE 1

Next, suppose that $P_j<P^m_1$, then $\hat{P}_i(P_j) > P_j$, and the function $E[V_i(\rho)]$ attains a minimum at $P_i=P_j$, to the left of $\hat{P}_i(P_j)>P_j$, the maximum of $\pi_i(P_1,P_2;\rho_0)$. Then, since $\hat{P}_i(P_j)-P_j>0$, the convexity of $E[V_i(\rho)]$ with respect to $P_i$ implies that it is increasing at $\hat{P}_i$, so that by the first order conditions and the concavity of $\pi_i(P_1,P_2;\rho_0)$ in $P_i$, $P_i^{**}>\hat{P}_i(P_j)$. See Figure 2.

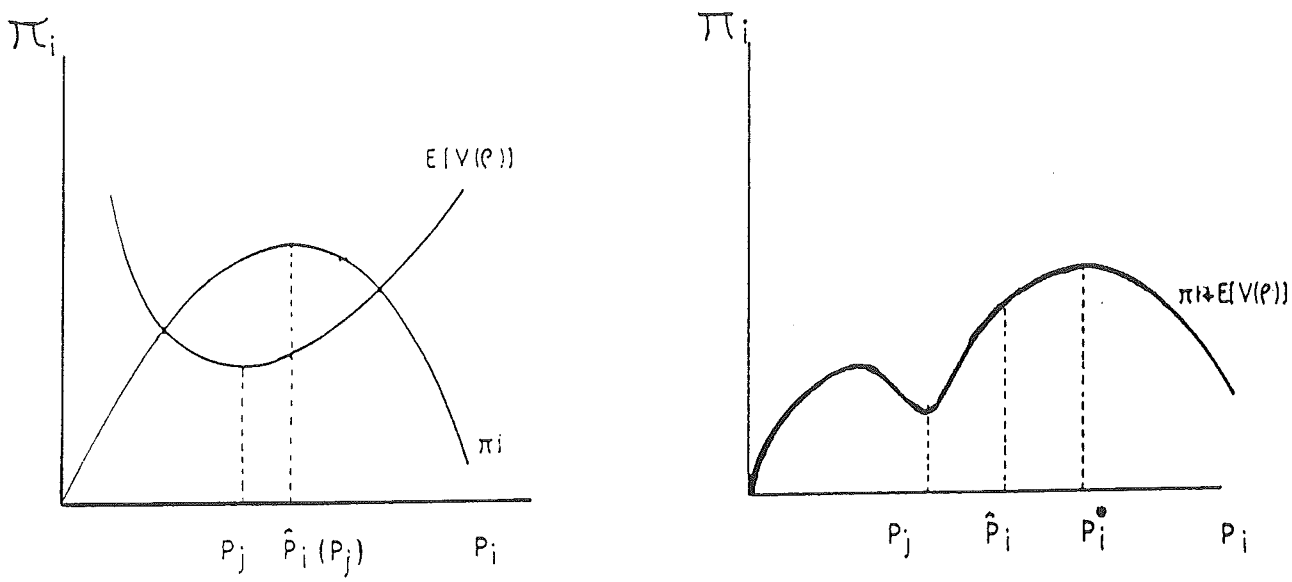


FIGURE 2

Finally, if $P_j= P^m_1$, $\hat{P}_i(P_j) = P_j= P^m_1$. Then, the function $E[V(\rho)]$ has a minimum at $\hat{P}_i(P_j)= P_j= P^m_1$, so that by the above argument, $P_i^{**} \gtrless \hat{P}_i$, depending on

$$[\pi_i(P_1,P_2;\rho_0) + \delta E[V_i(\rho(Q_1,Q_2))]]_{P_i^{**}>\hat{P}_{i1}} \gtrless [\pi_i(P_1,P_2;\rho_0) + \delta E[V_i(\rho(Q_1,Q_2))]]_{P_i^{**}<\hat{P}_{i1}}$$

Then, there is a jump in the reaction correspondence of firm i at $\hat{P}_i=P_j=P^m_1$. Hence both firms' response correspondences never cross the diagonal: firm i's (respt. firm j) correspondence has a discontinuity at $P_j=P^m_1$ (respt. $P_i= P^m_1$). Note that this type of best responses precludes the existence of symmetric pure strategy equilibrium, i.e. there is always price dispersion in equilibrium. See figure 3. ∎

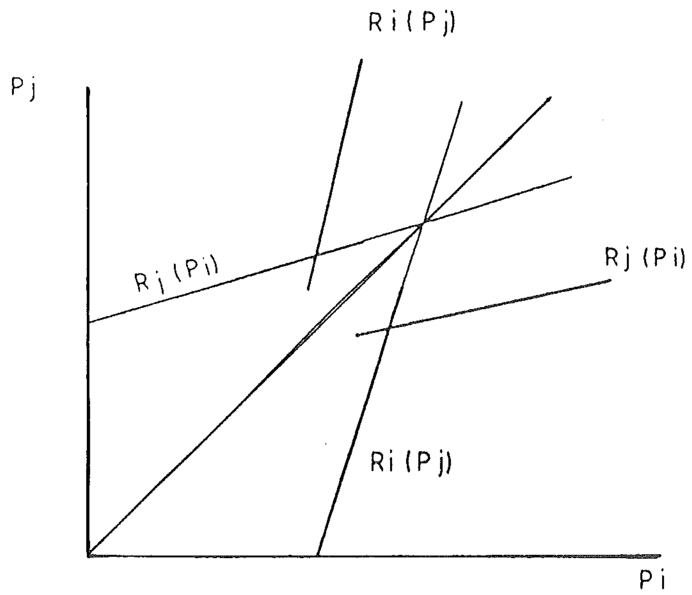


FIGURE 3

*Remark.* Note that the above equilibria in pure strategies implies a coordination problem. In fact the choice of the equilibrium could be viewed as a pure coordination symmetric one-shot game. Moreover, since any symmetric game has a symmetric Nash equilibrium, our duopoly game has in addition to the asymmetric equilibria a symmetric solution in mixed strategies. We state without proof the following result.

*Lemma 3.* The duopoly game has a unique symmetric Nash Equilibrium in mixed strategies, with both first period prices $P_1^{**}$ and $P_2^{**}$ distributed on the same compact interval. This interval is centered at the myopic equilibrium $P_1^m$.

Since it is seems that in most of the cases the natural solution for symmetric games is a symmetric equilibrium, the justification for the asymmetric solution, that yields the highest expected profits to the firms, has to rely mainly in two types of arguments. One belongs to the preplay communication or "cheap talk" stories. For example, firms would be interested in talking before the starting of the play to coordinate in which equilibrium to play. They could even send signals to this end. The second argument is based on focal points and/or preplay reputation about players' way to play, i.e. firms way have same preplay reputation of aggressive pricing, or one firm can be located in a "luxury" area so that it may price higher than the other, etc.

Assuming, then, that firms can coordinate themselves in the asymmetric equilibrium, we analyze next the informational properties of the solution.

## 4.- INTERPRETATION OF EQUILIBRIUM PRICE-DISPERSION

Proposition 4 tell us that both firms choose first period equilibrium prices, $P_1^{**}$ and $P_2^{**}$, that are different from the firs period myopic price $P_1^m$, and that are also different from each other, i.e. firms price-disperse at the equilibrium. The reason of this behaviour is that firms follow it to collect information about Nature's choice of θ. Let us be more specific. Observe that in the second period the posterior belief about θ is represented by $\rho(Q_1,Q_2)$, where ρ is defined as (3). However, in the first period, before $P_1^{**}$ and $P_2^{**}$ have been chosen, ρ itself is a random variable that is generated by the signals of the parameters θ, $Q_1$ and $Q_2$.

For any pair of first period prices, $P_1$ and $P_2$, let $\bar{S}^1_{P_1,P_2} = S^1_{P_1,P_2}(\bar{\theta})=\gamma_1(P_1,P_2,\theta) + \tilde{\varepsilon}_1$, be the signal generated by $P_1$ and $P_2$ in market 1, when $\theta=\bar{\theta}$, and let $\bar{S}^2_{P_1,P_2} = S^2_{P_1,P_2}(\bar{\theta})= \gamma_2(P_1,P_2,\bar{\theta}) + \tilde{\varepsilon}_2$, be the signal generated by $P_1$ and $P_2$ in market 2, when again $\theta=\bar{\theta}$. Defined similarly, $\underline{S}^1_{P_1,P_2}$, $\underline{S}^2_{P_1,P_2}$ and $S^1_{P_1,P_2}(\theta)$, $S^2_{P_1,P_2}(\theta)$, when θ is not specified.

Let $\bar{S}_{P_1,P_2}=S_{P_1,P_2}(\bar{\theta})=(\bar{S}^1_{P_1,P_2},\bar{S}^2_{P_1,P_2})$, $\underline{S}_{P_1,P_2}=S_{P_1,P_2}(\underline{\theta})=(\underline{S}^1_{P_1,P_2},\underline{S}^2_{P_1,P_2})$, and $S_{P_1,P_2}(\theta)$, be the vector of market signals generated by $P_1$ and $P_2$. The market signals vector $S_{P_1,P_2}(\theta)=\{\gamma_1(P_1,P_2,\theta)+\tilde{\varepsilon}_1,\ \gamma_2(P_1,P_2,\theta)+\tilde{\varepsilon}_2\}$, $\theta=\{\bar{\theta},\underline{\theta}\}$, is distributed as the joint distribution of the random variables $\gamma_1(P_1,P_2,\theta)\ +\tilde{\varepsilon}_1$

and $\gamma_2(P_1,P_2,\theta)$ $+\tilde{\varepsilon}_2$, and lead to some posterior about $\theta$, $\rho(S_{P_1,P_2}|P_1,P_2)$ = Prob $\{\theta=\bar{\theta}|S_{P_1,P_2}\}$. Define,

$$\rho(S_{P_1,P_2}|P_1,P_2) = \frac{1}{D}\,[\rho_0^2 f(\bar{S}^1_{P_1,P_2} - \gamma_1(P_1,P_2,\theta),\bar{S}^2_{P_1,P_2} - \gamma_2(P_1,P_2,\theta))] \tag{26}$$

where

$$D = \rho_0^2 f(\bar{S}^1_{P_1,P_2}|P_1,P_2,\ \bar{S}^2_{P_1,P_2}|P_1,P_2) + (1-\rho_0)^2 f(\underline{S}^1_{P_1,P_2}|P_1,P_2,\ \underline{S}^2_{P_1,P_2}|P_1,P_2)$$

Consider next, two pairs of first period price values of both firms: the pair $P_1=P_2=y$, and the pair $P_1=x$, $P_2=y$, $x\neq y$. The associated market signal vector is in the first case $S_y(\theta)=(S^1_y(\theta),S^2_y(\theta))$ and $S_{x,y}(\theta)=(S^1_{x,y}(\theta),S^2_{x,y}(\theta))$ in the second case, $\theta=\{\bar{\theta},\underline{\theta}\}$. These market signal vectors generate the posteriors about $\theta$, $\rho(S_y|y)$ and $\rho(S_{x,y}|x,y)$, that are as in (26).

Then, the condition that ensures that any decision market will prefer the signal vector $S_{x,y}(\theta)$ to the signal vector $S_y(\theta)$ is the following.

*Definition 2.*

$S_{x,y}(\theta)$ is more informative than $S_y(\theta)$, if:

$$E_{S_{x,y}}[G(\rho(S_{x,y}|x,y))] > E_{S_y}[G(\rho(S_y|y))] \tag{27}$$

for any strictly convex function $G(\rho)$, i.e. for any strictly convex function $G(\rho)$,

$$\int G(\rho(s|x,y))[\rho_0 g(\bar{s}_{x,y},x,y) + (1-\rho_0)g(\underline{s}_{x,y},x,y)]ds >$$
$$\int G(\rho(s|y))[\rho_0 g(\bar{s}_y,y) + (1-\rho_0)g(\underline{s}_y,y)]ds$$

where $g(s_{x,y}(\theta),x,y)$ is the joint density function of each $S_{x,y}(\theta)$ evaluated at s, i.e. $\bar{S}_{x,y} = (\bar{S}^1_{x,y},\bar{S}^2_{x,y})$ has joint density function.

$$g(s_{x,y}(\bar{\theta}),x,y) = [\rho_0 f(\bar{s}^1_{x,y}-\gamma_1(x,y,\bar{\theta}),\ \bar{s}^2_{x,y}-\gamma_2(x,y,\bar{\theta})]$$

and $g(s_y(\theta),y)$ is similarly the joint density function $S_y(\theta)$ evaluated at s.

In words, the signal $S_{x,y}(\theta)$ is more informative than the signal $S_y(\theta)$ if the future expected profits conditional on observing $S_{x,y}(\theta)$ are higher than those conditional on observing $S_y(\theta)$. i.e. $S_{x,y}(\theta)$ is more valuable than $S_y(\theta)$.

Then, it can be proved, (see Appendix).

*Proposition 5.* Let $P_1^{**}$ and $P_2^{**}$ be the first period equilibrium prices with $P_1^{**} \neq P_2^{**}$ and let $P_1^m = P_2^m = P_1^m$ be the symmetric myopic first period price. Then the signal $S_{P_1^{**},P_2^{**}}$ is more informative than the signal $S_{P_1^m}$.

## 5.- EXISTENCE OF EQUILIBRIA

Recall that firm i's period-one problem is to solve

$$\underset{P_i}{\text{Max}}\ \Pi_i(P_i,P_j^{**}) = \text{Max}\ \{\pi_i(P_i,P_j^{**},\rho_0) + \delta E[V(\rho(Q_1,Q_2;\ P_1,P_2^{**})] \tag{28}$$

The maximization problem in (28) can be taken to be on some compact set. To see that let $\mathcal{P}_i = \{P \in \mathbb{R}^2_+ : Q_i(P)>0\}$ and $\overline{P}_i = \sup\ \{P_i \in \mathbb{R}_+ : P \in \mathcal{P}_i\}$. Note that $0<\overline{P}_i<\infty$ since $E[V(\rho)]$ does not increase without bounds and $\lim_{P_i \Rightarrow \infty} \pi_i(P_i,P_j,\theta) = -\infty$. In fact $E[V(\rho)]$ is bounded above by the monopolist profits for the favorable demand curve.

Let $\mathcal{P} = \mathcal{P}_1 \cap \mathcal{P}_2$. Also note that each demand $Q_i(P)$ is twice-continuously differentiable on $\mathbb{R}^2_+ \setminus \bigcup_{i=1,2} bd(\mathcal{P}_i)$, strictly decreasing in $P_i$, whenever $Q_i(P)>0$, i=1,2, and $\frac{\partial}{\partial P_j} Q_i(P)>0$, $j\neq i$ for $P \in \text{int}\ \mathcal{P}$. Moreover, the properties of our demand system imply the existence of a unique second period equilibrium for each $\rho$, say $P_i^*(\rho)$, twice-continuously differentiable in $\rho$, for $\rho \in (0,1)$. By the properties of the function $f(\varepsilon_1,\varepsilon_2)$, $\rho$ is twice-continuously differentiable in $\mathcal{P}$ and so is $V(\rho)$. Hence $\Pi_i(P_i,P_j)$ is twice-continuously differentiable for $P \in \text{int}\ \mathcal{P}$.

It follows that $\overline{P} = [\overline{P}_1,\overline{P}_2]$ is the unique maximal element of the closure of $\mathcal{P}$. for prices out of $\mathcal{P}$ at least one firm is priced out of the market. The strategy space for firm i is $[0,\overline{P}_i]$. Although revenues may not be necessarily smooth on $[0,\overline{P}_1]x[0,\overline{P}_2]$, but provided that when a rival prices at zero a firm can still make a profit, we can show that all the equilibria will lie in regions of the type,$[0,P^m)x(0,\tilde{P}_1]\cup(P^m,\tilde{P}_j]x[0;P^m)$ with $\tilde{P}_i \leq \overline{P}_2$, i=1,2, i≠j, where best responses are strictly increasing.

Let $\Psi_i(P_j)$ be firm i's best responses, i=1,2. We prove first that the composite best response $\Psi = \Psi_1 \circ \Psi_2$ is always increasing for $\mathcal{P} \in [0,\tilde{P}_1]x[0,\tilde{P}_2]$.

*Lemma 4.* Suppose that firm i, i=1,2, can make positive profits even if its rival charges a price equal to zero, then $\Psi = \Psi_1 \circ \Psi_2$ is increasing for $\mathcal{P} \in [0,\tilde{P}_1]x[0,\tilde{P}_2]$.

*Proof.* Note first that firms' two period payoff functions are the same and hence, by lemma 1 and proposition 1 their best responses are symmetric.

Next, consider firm i, since it can make positive profits even if $P_j=0$, the best reply of firm i will lie in $\mathcal{P}_i$. Let $\tilde{P}_j = \inf\{P_j \in \mathbb{R}_+ : P \in bd(\mathcal{P}_j),\ P_i \in \Psi_i(P_j)\}$. Clearly, $\tilde{P}_j \leq \overline{P}_j$. We claim now that $\Psi_i$ is strictly increasing in $[0,P^m)\cup(P^m,\tilde{P}_j]$. For any $P_j \in [0,P^m)$, $\hat{P}_i(P_j) > P_j$, where $\hat{P}_i(P_j)$ is the myopic best response as defined in (25), and by the proof of the Proposition 4, $\Psi_i(P_j) > \hat{P}_i(P_j)$, for any $P_j \in [0,P^m)$. Since $\hat{P}_i(P_j)$ is strictly increasing, then $\Psi_i(P_j)$ is also strictly increasing for any $P_j \in [0,P^m)$, or since $\Pi_i$ is twice-continuously differentiable,

$$\left.\frac{\partial \Pi_i(P_i,P_j)}{\partial P_j\,\partial P_i}\right]_{P_i>P_j} = \frac{\partial \pi_i}{\partial P_j \partial P_i} + \delta\left[\frac{\partial E[V(\rho)]}{\partial P_j \partial P_i}\right]>0 \tag{29}$$

Now, by Proposition 1, $E[V(\rho)]$ decreases in $P_i$, whenever $P_i<P_j$, for a fixed $P_j$, so that by (10) and by lemma 1, $E[V(\rho)]$ increases in $P_j$, for a fixed $P_i$ such that $P_j>P_i$. In particular, for $P_i=P_j$, we must have that $E[V(\rho(P_i,P_j))]\Big]_{P_i=P_j} < E[V(\rho(P_i,P'_j))]\Big]_{P'_j>P_j=P_i}$ so that the new minimun of $E[V(\rho)]$ as a function of $P_i$, at the new $P'_j$, is at $P'_i=P'_j>P_i$ (i.e. to the right of the old one). Therefore it must be the case that $\frac{\partial E[V\rho)]}{\partial P_j}$ is monotone increasing in $P_j$, for $P_j<P_i$, i.e. $\frac{\partial E[V(\rho)]}{\partial P_j \partial P_i}>0$, then

$$\left.\frac{\partial^2 \Pi_i(P_i,P_j)}{\partial P_j \partial P_i}\right]_{P_i<P_j} = \frac{\partial \pi_i}{\partial P_j \partial P_i} +\delta\left[\frac{\partial^2 E[V(\rho)]}{\partial P_j \partial P_i}\right]>0 \tag{30}$$

This implies that $\Psi_i(P_j)$ is strictly increasing for $P_j\in(P^m,\tilde{P}_j]$. Furthermore $\Psi_i$ cannot jump down out of $\mathcal{P}$ into $\mathcal{P}_i\backslash\mathcal{P}_j$ where firm j is priced out the market since for any $P_j$ firm 1's maximun monopoly profits for the favourable demand is strictly larger than any profit level that firm i may attain if firm j produces a positive amount. Therefore $\Psi_i(P_j)$ is strictly increasing for $P_j>P^m$, till it hits the boundary of $\mathcal{P}_j$ where firm j goes out of bussiness, this is at $P_j=\tilde{P}_j$. Hence $\Psi_i$ is strictly increasing for $P_j\in[0,P^m)\cup(P^m,\tilde{P}_j]$. Similarly, for firm j we can find an analogous $\tilde{P}_i$. $\Psi_1$ and $\Psi_2$ are strictly increasing on $[0,P^m)\cup(P^m,\tilde{P}_2]$ and on $[0,P^m)\cup(P^m,\tilde{P}_1]$ respectively. Note, however, by the proof of Proposition 4, that each $\Psi_i$, has a jump down at $P^m_1$. But since, firms' best responses are symmetric and both jump down at $P^m$, the two jumps cancells out so that the composite best response $\Psi = \Psi_1 \circ \Psi_2$ is continuous and increasing in $[0,\tilde{P}_1]x[0,\tilde{P}_2]$. ■

We can now prove

*Proposition 6.* Suppose that firm i, i=1,2 can make positive profits even if its rival charges a price equal to zero. Then, the equilibrium set is non-empty.

*Proof.* The proof of this Proposition is based on one application of Tarski existence theorem (see either the existence Theorem 4.2 in Vives (1990) or Theorem 4 in Milgron and Roberts (1990)), who showed that if $\Psi$ is continous and increasing in the compact sets where it is defined, then the equilibrium set is non-empty. By lemma 4, the results follows. ■

Note that all equilibria in pure strategies will lie in any of the following regions: either in $[0,P^m)x(P^m,\tilde{P}_2]\subset[0,\tilde{P}_1]x[0,\tilde{P}_2]$, that implies $P_2^{**}>P_1^{**}$, or in $(P^m,\tilde{P}_1]x[0,P^m)\subset[0,\tilde{P}_1]x[0,\tilde{P}_2]$, that implies $P_1^{**}>P_2^{**}$. In any of these regions $\Psi_1$ and $\Psi_2$ are strictly increasing and no equilibrium may exists outside of them. Let $E_1$ and $E_2$ be the set of equilibria. Given our assumptions $E \subset \mathrm{Int}\mathcal{P}$, that is all equilibria involve positive prices and production, and $E\subset E_1\cup E_2$. Also, notice that the set $E_1$ (resp. $E_2$) is strictly ordered. Let $P^*$ and $q^*$ be two equilibria that belongs to $E_1$, i.e. $P_2^*>P_1^*$ and $q_2^*>q_1^*$ (resp. $E_2$, i.e. $P_2^*<P_1^*$, $q_2^*<q_1^*$), which are not strictly ordered: $P_1^*>q_1^*$ and $P_2^*\leq q_2^*$. Then, since $\Psi_2$ is strictly increasing in $P_1\in [0,P^m)$ (resp.

$P_1 \in (P^m, \tilde{P}_1])$, then min $\Psi_2(P_1^*)$>max $\Psi_2(q_1^*)$ wich is a contradiction since $P_2^* \geq$ min $\Psi_2(P_1^*)$> max $\Psi_2(q_1^*) \geq q_2^*$.

Finally, notice that given the continuity of firms'payoff functions and compactness of their strategy spaces established above, by the existence Theorem in Glicksberg (1952), a Nash equilibrium always exists in mixed strategies.

## 6.- CONCLUSIONS

In this paper we have tried to clarify the learning mechanisms wich operate behind the price-dispersion phenomenon.

We have considered a symmetric duopoly game with product differentiation where firms have imperfect information about demands. In particular, both the slope and the product substitution parameters of the two demand are unknown to firms and in addition there are random shocks in both markets. Firms learn about these parameter by observing market sales in the two markets.

We have modelled our duopoly market as a game of imperfect information and have characterized the first period equilibrium solutions. Our main concern has been to show under which conditions the information about the demand unknown parameters is acquired more efficiently if firms experiment by setting different prices. Our results show that when firms experiment in all markets at the same time and they have the same ability to make market signals more informative then, provided that products are substitutes, they will price-disperse as an attempt to increase the informative content of these signals. Hence a sampling effect may arise as the global outcome of market learning behaviour.

Some few remarks should be made. First, we have restrict our analysis to the case where the experimenting variables, prices, were publicly observable, and where market signals were also public. In this way we have isolated the learning behavior from signal-jamming considerations (see MSU 1993,b).

Second, we have left aside the case of asymmetric expected demands, i.e. independent (and not perfectly correlated) parameters, because the analysis becomes very cumbersome is this situation. This is so because there are now additional factors which also play a key role in firms' learning actions. In particular, both the strategic relation of firms with respect to information and the possible correlation of demand random shocks, that translates to correlation between market signals, will influence the outcome of the market experimental behavior. The first one is due to the fact that now there are two posterior beliefs (which depend on firms first period prices) and each firm second period value function will depend on both, its own an its rival's posterior. Firms experiment to gather information but at the same time -since market signals are publicly observed- they may also give information to the rival. Hence each firm has to consider whether it wants the rival to be more informed and if a more informed rival is a nicer competitor. In other words, if firms are strategic substitutes or complements in information (see Bulow, Geanakoplos and Klemperer, 1985, and Vives, 1988), that will determine the relationship between each firm second period value function and its rival's posterior. Under price competition firms are strategic complements in information (See AU, 1993), so that they will be willing to give information to the rival. But, in adittion to the above effect, firms also have to consider the effect of a higher informative content of one market signal on the informativeness of the other one, i.e.

the correlation between market signals. This depends on the correlation between the demand random shocks. For instance, a positive covariance of $\varepsilon_1$ and $\varepsilon_2$ means that to make one market signal more informative also makes the other one more informative, so that if firms are strategic complements (strategic substitutes) in information it will encourage (discourage) the production of information, meanwhile if $cov(\varepsilon_1,\varepsilon_2)$ is negative it will discourage (encourage) it. Since both firms experiment in each market, the above mentioned effects are interrelated so that to look for the conditions that guarantee the price-dispersion outcome as the result of market learning behavior, becomes a very complicated issue and although same local conditions could be found it remains an open question.

Third, our analysis has relied on the value of information being positive. Deriving conditions under which this is the case turns out to be difficult and lies beyond the scope of this paper (see AEJ, 1993 for a discussion on this issue).

Finally, the moral of the paper is that the outcome of market experimentation in a multifirm setting is highly sensitive to the kind of uncertainty that firms face which, in turn, affects the specific markets signals that they want to make more informative and their relationship. This will determine whether firms will experiment in the same markets and if so whether they have the same ability to make them more informative. The interrelation of all these factors together with the nature of market competition will drive the outcome of market experimentation.

# APPENDIX

*Proof of Lemma 1.* We have to prove here that $\frac{\partial\rho}{\partial Q_i} \lessgtr 0$ as $\bar{\varepsilon}_i \gtrless \underline{\varepsilon}_i$ where $\bar{\varepsilon}_i = Q_i - \gamma_i(P_1,P_2,\bar{\theta}) = Q_i - \bar{\gamma}_i$, $\underline{\varepsilon}_i = Q_i - \underline{\gamma}_i$.

Suppose that $\bar{\varepsilon}_i > \underline{\varepsilon}_i$ i=1,2 (<), then the MLRP in variable $\varepsilon_i$, i=1,2 implies that:

$$\frac{f'_i(\bar{\varepsilon}_1,\bar{\varepsilon}_2)}{f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)} \underset{(>)}{<} \frac{f'_i(\underline{\varepsilon}_1,\bar{\varepsilon}_2)}{f(\underline{\varepsilon}_1,\bar{\varepsilon}_2)} \underset{(>)}{<} \frac{f'_i(\underline{\varepsilon}_1,\underline{\varepsilon}_2)}{f(\underline{\varepsilon}_1,\underline{\varepsilon}_2)} \tag{1}$$

or $f'_i(\bar{\varepsilon}_1,\bar{\varepsilon}_2)f(\underline{\varepsilon}_1,\underline{\varepsilon}_2) \underset{(>)}{<} f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)f'_i(\underline{\varepsilon}_1,\underline{\varepsilon}_2)$

on the other hand, by the definition of $\rho$,

$$\frac{\partial\rho}{\partial Q_i} = \frac{[f'_i(\bar{\varepsilon}_1,\bar{\varepsilon}_2)f(\underline{\varepsilon}_1,\underline{\varepsilon}_2) - f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)f'_i(\underline{\varepsilon}_1,\underline{\varepsilon}_2)]\rho_0^2(1-\rho_0)^2}{D^2} \underset{(>)}{<} 0 \tag{2}$$

whenever $\bar{\varepsilon}_i > \underline{\varepsilon}_i$ (<), where $D = \rho_0^2 f(\bar{\varepsilon}_1,\bar{\varepsilon}_2) + (1-\rho_0)^2 f(\underline{\varepsilon}_1,\underline{\varepsilon}_2)$. ■

*Proof of lemma 2.* From the definition of $\rho$ and recalling that $\varepsilon_i = Q_i - \gamma_i(P_1,P_2,\theta)$,

$$\frac{\partial\rho}{\partial P_i} = \frac{\rho_0^2(1-\rho_0)^2}{D^2}\Big\{ (-\bar{\gamma}'_{i,P_i})f'_i(\bar{\varepsilon}_1,\bar{\varepsilon}_2)f(\underline{\varepsilon}_1,\underline{\varepsilon}_2) + \underline{\gamma}'_{i,P_i}f'_i(\underline{\varepsilon}_1,\underline{\varepsilon}_2)f(\bar{\varepsilon}_1,\bar{\varepsilon}_2) +$$

$$(-\bar{\gamma}'_{j,P_i})f'_j(\bar{\varepsilon}_1,\bar{\varepsilon}_2)f(\underline{\varepsilon}_1,\underline{\varepsilon}_2) + \underline{\gamma}'_{j,P_i}f'_j(\underline{\varepsilon}_1,\underline{\varepsilon}_2)f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)\Big\} \tag{3}$$

that may be expressed as

$$\frac{\partial\rho}{\partial P_i} = \frac{\rho_0^2(1-\rho_0)^2}{D^2}\Big\{ (-\bar{\gamma}'_{i,P_i})[f'_i(\bar{\varepsilon}_1,\bar{\varepsilon}_2)f(\underline{\varepsilon}_1,\underline{\varepsilon}_2) - f'_i(\underline{\varepsilon}_1,\underline{\varepsilon}_2)f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)]$$

$$-(\bar{\gamma}'_{i,P_i}-\underline{\gamma}'_{i,P_i})f'_i(\underline{\varepsilon}_1,\underline{\varepsilon}_2)f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)-(\bar{\gamma}'_{j,P_i})[f'_j(\bar{\varepsilon}_1,\bar{\varepsilon}_2)f(\underline{\varepsilon}_1,\underline{\varepsilon}_2)-f'_j(\underline{\varepsilon}_1,\underline{\varepsilon}_2)f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)]$$

$$-(\bar{\gamma}'_{j,P_i}-\underline{\gamma}'_{j,P_i})f'_j(\underline{\varepsilon}_1,\underline{\varepsilon}_2)f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)\Big\}$$

that by (2) and the definition of $\rho$ gives the first expression in lemma 2. Note that the second expression is obtained in the same way by manipulating (3) in the right way. ■

*Derivation of equation (10).*

(9) in the text is

$$\frac{\partial W_i}{\partial P_i} = \iint V'_i(\rho)\,\frac{\partial\rho}{\partial P_i}\,h(Q_1,Q_2)dQ_1dQ_2 + \iint V_i(\rho)\frac{\partial}{\partial P_i}\,h(Q_1,Q_2)dQ_1dQ_2 \tag{4}$$

Note that

$$\frac{\partial}{\partial P_i}\,h(Q_1,Q_2) = \frac{\partial}{\partial P_i}\,[\rho_0^2 f(\bar{\varepsilon}_1,\bar{\varepsilon}_2) + (1-\rho_0)^2 f(\underline{\varepsilon}_1,\underline{\varepsilon}_2)] = \rho_0^2[f'_i(\bar{\varepsilon}_1,\bar{\varepsilon}_2)(-\bar{\gamma}'_{i,P_i}) +$$
$$f'_j(\bar{\varepsilon}_1,\bar{\varepsilon}_2)(-\bar{\gamma}'_{j,P_i})] + (1-\rho_0)^2[f'_i(\underline{\varepsilon}_1,\underline{\varepsilon}_2)(-\underline{\gamma}'_{i,P_i}) + f'_j(\underline{\varepsilon}_1,\underline{\varepsilon}_2)(-\underline{\gamma}'_{j,P_i})]$$

then, the second term of the right hand side of (4) is

$$\iint V_i(\rho)\Big\{\rho_0^2[f'_i(\bar{\varepsilon}_1,\bar{\varepsilon}_2)(-\bar{\gamma}'_{i,P_i}) + f'_j(\bar{\varepsilon}_1,\bar{\varepsilon}_2)(-\bar{\gamma}'_{j,P_i})] + (1-\rho_0)^2[f'_i(\underline{\varepsilon}_1,\underline{\varepsilon}_2)(-\underline{\gamma}'_{i,P_i})$$
$$+ f'_j(\underline{\varepsilon}_1,\underline{\varepsilon}_2)(-\underline{\gamma}'_{j,P_i})]\Big\}dQ_1dQ_2 \tag{5}$$

Integration by parts of (5) and rearranging of terms gives:

$$\iint V_i'(\rho)\left[\bar{\gamma}'_{i,P_i}\frac{\partial\rho}{\partial Q_i} + \bar{\gamma}'_{j,P_i}\frac{\partial\rho}{\partial Q_j}\right]\rho_0^2 f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)dQ_1dQ_2 +$$

$$\iint V_i'(\rho)\left[\underline{\gamma}'_{i,P_i}\frac{\partial\rho}{\partial Q_i} + \underline{\gamma}'_{j,P_i}\frac{\partial\rho}{\partial Q_j}\right](1-\rho_0)^2 f(\underline{\varepsilon}_1,\underline{\varepsilon}_2)dQ_1dQ_2 \tag{6}$$

Substituing (6) in (4) yields:

$$\frac{\partial W_i}{\partial P_i} = \iint V_i'(\rho)\left[\frac{\partial\rho}{\partial P_i} + \bar{\gamma}'_{i,P_i}\frac{\partial\rho}{\partial Q_i} + \bar{\gamma}'_{j,P_i}\frac{\partial\rho}{\partial Q_j}\right]\rho_0^2 f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)dQ_1dQ_2 +$$

$$\iint V_i'(\rho)\left[\frac{\partial\rho}{\partial P_i} + \underline{\gamma}'_{i,P_i}\frac{\partial\rho}{\partial Q_i} + \underline{\gamma}'_{j,P_i}\frac{\partial\rho}{\partial Q_j}\right](1-\rho_0)^2 f(\underline{\varepsilon}_1,\underline{\varepsilon}_2)dQ_1dQ_2 \tag{7}$$

and by lemma 2, and by the definition of $\rho$

$$\frac{\partial W_i}{\partial P_i} = -\iint V_i'(\rho)\,\frac{\rho(1-\rho_0)^2}{D}\left[(\bar{\gamma}'_{i,P_i} - \underline{\gamma}'_{i,P_i})f_i'(\underline{\varepsilon}_1,\underline{\varepsilon}_2) + (\bar{\gamma}'_{j,P_i} - \underline{\gamma}'_{j,P_i})f_j'(\underline{\varepsilon}_1,\underline{\varepsilon}_2)\right]$$

$$\rho_0^2 f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)dQ_1dQ_2 - \iint V_i'(\rho)\,\frac{\rho_0^2(1-\rho)}{D}\left[(\bar{\gamma}'_{i,P_i} - \underline{\gamma}'_{i,P_i})f_i'(\bar{\varepsilon}_1,\bar{\varepsilon}_2) +\right.$$

$$\left.(\bar{\gamma}'_{j,P_i} - \underline{\gamma}'_{j,P_i})f_j'(\bar{\varepsilon}_1,\bar{\varepsilon}_2)\right](1-\rho_0)^2 f(\underline{\varepsilon}_1,\underline{\varepsilon}_2)dQ_1dQ_2 =$$

$$-\iint V_i'(\rho)\rho^2(1-\rho_0)^2\left[(\bar{\gamma}'_{i,P_i} - \underline{\gamma}'_{i,P_i})f_i'(\underline{\varepsilon}_1,\underline{\varepsilon}_2)+(\bar{\gamma}'_{j,P_i} - \underline{\gamma}'_{j,P_i})f_j'(\underline{\varepsilon}_1,\underline{\varepsilon}_2)\right]dQ_1dQ_2 -$$

$$\iint V_i'(\rho)(1-\rho)^2\rho_0^2\left[(\bar{\gamma}'_{i,P_i} - \underline{\gamma}'_{i,P_i})f_i'(\bar{\varepsilon}_1,\bar{\varepsilon}_2)+(\bar{\gamma}'_{j,P_i} - \underline{\gamma}'_{j,P_i})f_j'(\bar{\varepsilon}_1,\bar{\varepsilon}_2)\right]dQ_1dQ_2 \tag{8}$$

and rearranging (8) is

$$\frac{\partial W_i}{\partial P_i} = -\,(\bar{\gamma}'_{i,P_i} - \underline{\gamma}'_{i,P_i})\iint V'_i(\rho)\Big[\rho^2(1-\rho_0)^2 f'_i(\underline{\varepsilon}_1,\underline{\varepsilon}_2) + (1-\rho)^2\rho_0^2 f'_i(\bar{\varepsilon}_1,\bar{\varepsilon}_2)\Big]dQ_1 dQ_2$$

$$-\,(\bar{\gamma}'_{j,P_i} - \underline{\gamma}'_{j,P_i})\iint V'_i(\rho)\Big[\rho^2(1-\rho_0)^2 f'_j(\underline{\varepsilon}_1,\underline{\varepsilon}_2) + (1-\rho)^2\rho_0^2 f'_j(\bar{\varepsilon}_1,\bar{\varepsilon}_2)\Big]dQ_1 dQ_2 \tag{9}$$

Next, note the following facts:

$$(1-\rho)^2 = (1-\rho) - \rho(1-\rho) \tag{10}$$

$$\rho = \frac{\rho_0 f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)}{\rho^2 f(\bar{\varepsilon}_1,\bar{\varepsilon}_2) + (1-\rho_0)^2 f(\underline{\varepsilon}_1,\underline{\varepsilon}_2)} = \frac{\rho_0 f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)}{D}, \text{ so that}$$

$$\rho D = \rho_0^2 f(\bar{\varepsilon}_1,\bar{\varepsilon}_2) \tag{11}$$

and then derivating with respect to $Q_i$,

$$\frac{\partial\rho}{\partial Q_i} D + \rho[\rho_0^2 f'_i(\bar{\varepsilon}_1,\bar{\varepsilon}_2) + (1-\rho_0)^2 f'_i(\underline{\varepsilon}_1,\underline{\varepsilon}_2)] = \rho_0^2 f'_i(\bar{\varepsilon}_1,\bar{\varepsilon}_2), \text{ or}$$

$$\frac{\partial\rho}{\partial Q_i} D = (1-\rho)\rho_0^2 f'_i(\bar{\varepsilon}_1,\bar{\varepsilon}_2) - \rho(1-\rho_0)^2 f'_i(\underline{\varepsilon}_1,\underline{\varepsilon}_2) \tag{12}$$

for i=1,2, then (9) is by (10) and (12)

$$\frac{\partial W_i}{\partial P_i} = +\,(\bar{\gamma}'_{i,P_i} - \underline{\gamma}'_{i,P_i})\iint V'_i(\rho)\rho D\,\frac{\partial\rho}{\partial Q_i}\,dQ_1 dQ_2$$

$$-(\bar{\gamma}'_{i,P_i}-\underline{\gamma}'_{i,P_i})\iint V_i'(\rho)(1-\rho)\rho_0^2 f_i'(\bar{\varepsilon}_1,\bar{\varepsilon}_2)dQ_1dQ_2$$

$$+(\bar{\gamma}'_{j,P_i}-\underline{\gamma}'_{j,P_i})\iint V_i'(\rho)\rho D\,\frac{\partial\rho}{\partial Q_j}\,dQ_1dQ_2$$

$$-(\bar{\gamma}'_{j,P_i}-\underline{\gamma}'_{j,P_i})\iint V_i'(\rho)(1-\rho)\rho_0^2 f_j'(\bar{\varepsilon}_1,\bar{\varepsilon}_2)dQ_1dQ_2 \tag{13}$$

Next, integration by parts of the second and fourth terms of the right hand side of (13), for i=1,2

$$-\iint V_i'(\rho)(1-\rho)\rho_0^2 f_i'(\bar{\varepsilon}_1,\bar{\varepsilon}_2)dQ_1dQ_2=\iint\left[V_i''(\rho)\frac{\partial\rho}{\partial Q_i}(1-\rho)\quad-\quad V_i'(\rho)\frac{\partial\rho}{\partial Q_i}\right]\rho_0^2 f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)dQ_1dQ_2 \tag{14}$$

and inserting (14) in (13), and cancelling terms by (11),

$$\frac{\partial W_i}{\partial P_i}=\iint V_i''(\rho)\left[(\bar{\gamma}'_{i,P_i}-\underline{\gamma}'_{i,P_i})\frac{\partial\rho}{\partial Q_i}+(\bar{\gamma}'_{j,P_i}-\underline{\gamma}'_{j,P_i})\frac{\partial\rho}{\partial Q_j}\right](1-\rho)\rho_0^2 f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)dQ_1dQ_2 \tag{15}$$

(15) is equation (10) in the text. ■

<u>*Derivation of equation (23) in the text.*</u>

Note first that since $\rho(\mathcal{X},\mathcal{Y},P_A,P_B)=$

$$\frac{\rho_0^2 f\left(\mathcal{X}-\frac{(\bar{D}_A+\bar{D}_B)}{2},\ \mathcal{Y}-\frac{(\underline{D}_A-\underline{D}_B)}{2}\right)}{\rho_0^2 f\left(\mathcal{X}-\frac{(\bar{D}_A+\bar{D}_B)}{2},\mathcal{Y}-\frac{(\bar{D}_A-\bar{D}_B)}{2}\right)+(1-\rho_0)^2 f\left(\mathcal{X}-\frac{(\underline{D}_A+\underline{D}_B)}{2},\mathcal{Y}-\frac{(\underline{D}_A-\underline{D}_B)}{2}\right)} \tag{16}$$

then,

$$\frac{\partial\rho}{\partial P_A} = -\bar{D}'_{A,P_A}\left(\frac{\partial\rho}{\partial\mathcal{X}}+\frac{\partial\rho}{\partial\mathcal{Y}}\right) - \bar{D}'_{B,P_A}\left(\frac{\partial\rho}{\partial\mathcal{X}}-\frac{\partial\rho}{\partial\mathcal{Y}}\right)$$

$$-\frac{\rho(1-\rho_0)^2}{D}(\bar{D}'_{A,P_A}-\underline{D}'_{A,P_A})[\underline{f}'_1+\underline{f}'_2] - \frac{\rho(1-\rho_0)^2}{D}(\bar{D}'_{B,P_A}-\underline{D}'_{B,P_A})[\underline{f}'_1-\underline{f}'_2] \qquad (17)$$

and/or

$$\frac{\partial\rho}{\partial P_A} = -\underline{D}'_{A,P_A}\left(\frac{\partial\rho}{\partial\mathcal{X}}+\frac{\partial\rho}{\partial\mathcal{Y}}\right) -\underline{D}'_{B,P_A}\left(\frac{\partial\rho}{\partial\mathcal{X}}-\frac{\partial\rho}{\partial\mathcal{Y}}\right)$$

$$-\frac{(1-\rho)\rho_0^2}{D}(\bar{D}'_{A,P_A}-\underline{D}'_{A,P_A})[\bar{f}'_1+\bar{f}'_2] - \frac{(1-\rho)\rho_0^2}{D}(\bar{D}'_{B,P_A}-\underline{D}'_{B,P_A})[\bar{f}'_1-\bar{f}'_2] \qquad (18)$$

and

$$\frac{\partial W(P_A,P_B)}{\partial P_A} = \frac{\partial}{\partial P_A}\left\{\iint V(\rho(\mathcal{X},\mathcal{Y},P_A,P_B)h(\mathcal{X},\mathcal{Y})d\mathcal{X}d\mathcal{Y}\right\} \qquad (19)$$

where $h(\mathcal{X},\mathcal{Y})=\rho_0^2 f(\mathcal{X}-\frac{(\bar{D}_A+\bar{D}_B)}{2},\ \mathcal{Y}-\frac{(\bar{D}_A-\bar{D}_B)}{2}) + (1-\rho_0)^2 f(\mathcal{X}-\frac{(\underline{D}_A+\underline{D}_B)}{2},\ \mathcal{Y}-\frac{(\underline{D}_A-\underline{D}_B)}{2})$

then, (19) is:

$$\frac{\partial W(P_A,P_B)}{\partial P_A} = \iint V'(\rho)\frac{\partial\rho}{\partial P_A}\, h(\mathcal{X},\mathcal{Y})d\mathcal{X}d\mathcal{Y} + \iint V(\rho)\frac{\partial}{\partial P_A}\, h(\mathcal{X},\mathcal{Y})d\mathcal{X}d\mathcal{Y} \qquad (20)$$

Note that

$$\frac{\partial h(\mathcal{X},\mathcal{Y})}{\partial P_A} = -\bar{D}'_{A,P_A}[\bar{f}'_1+\bar{f}'_2]\rho_0^2 - \underline{D}'_{A,P_A}[\underline{f}'_1+\underline{f}'_2](1-\rho_0)^2$$

$$-\bar{D}'_{B,P_A}[\bar{f}'_1-\bar{f}'_2]\rho_0^2 - \underline{D}'_{B,P_A}[\underline{f}'_1-\underline{f}'_2](1-\rho_0)^2$$

so that integration by parts of the second term of (20), yields,

$$\iint V(\rho)\frac{\partial}{\partial P_A}\, h(\mathcal{X},\mathcal{Y})d\mathcal{X}d\mathcal{Y} = \iint V'(\rho)\Big\{\overline{D}'_{A,P_A}\Big(\frac{\partial\rho}{\partial\mathcal{X}} + \frac{\partial\rho}{\partial\mathcal{Y}}\Big)\overline{f}\rho_0^2 + \overline{D}'_{B,P_A}\Big(\frac{\partial\rho}{\partial\mathcal{X}} - \frac{\partial\rho}{\partial\mathcal{Y}}\Big)\overline{f}\rho_0^2$$

$$+\underline{D}'_{A,P_A}\Big(\frac{\partial\rho}{\partial\mathcal{X}} + \frac{\partial\rho}{\partial\mathcal{Y}}\Big)\underline{f}(1-\rho_0)^2 + \underline{D}'_{B,P_A}\Big(\frac{\partial\rho}{\partial\mathcal{X}} - \frac{\partial\rho}{\partial\mathcal{Y}}\Big)\underline{f}(1-\rho_0)^2\Big\}d\mathcal{X}d\mathcal{Y} \qquad (21)$$

Then, by substitution of (21) in (20), by (17)–(18), by cancellation of terms, and by the definition of $\rho$, (20) is

$$\frac{\partial W(P_A,P_B)}{\partial P_A} = -\iint V'(\rho)[\overline{D}'_{A,P_A} - \underline{D}'_{A,P_A}]\Big[\rho^2[\underline{f}'_1+\underline{f}'_2](1-\rho_0)^2 + (1-\rho)^2[\overline{f}'_1+\overline{f}'_2]\rho_0^2\Big]d\mathcal{X}d\mathcal{Y}$$

$$-\iint V'(\rho)[\overline{D}'_{B,P_A} - \underline{D}'_{B,P_A}]\Big[\rho^2[\underline{f}'_1-\underline{f}'_2](1-\rho_0)^2 + (1-\rho)^2[\overline{f}'_1-\overline{f}'_2]\rho_0^2\Big]d\mathcal{X}d\mathcal{Y}$$

$$(22)$$

Let $A = [\overline{D}'_{A,P_A} - \underline{D}'_{A,P_A}]<0$, and $B = [\overline{D}'_{B,P_A} - \underline{D}'_{B,P_A}]>0$.

Then, by (10) in the Appendix (22) is,

$$\frac{\partial W(P_A,P_B)}{\partial P_A} = \iint V'(\rho)(-A)\rho\Big[[\underline{f}'_1+\underline{f}'_2]\rho(1-\rho_0)^2 - [\overline{f}'_1+\overline{f}'_2]\rho_0^2(1-\rho)\Big]d\mathcal{X}d\mathcal{Y}$$

$$+\iint V'(\rho)(-A)(1-\rho)[\overline{f}'_1+\overline{f}'_2]\rho_0^2 d\mathcal{X}d\mathcal{Y}$$

$$+\iint V'(\rho)(-B)\rho\Big[[\underline{f}'_1-\underline{f}'_2]\rho(1-\rho_0)^2 - [\overline{f}'_1-\overline{f}'_2]\rho_0^2(1-\rho)\Big]d\mathcal{X}d\mathcal{Y}$$

$$+\iint V'(\rho)(-B)(1-\rho)[\overline{f}'_1-\overline{f}'_2]\rho_0^2 d\mathcal{X}d\mathcal{Y} \qquad (23)$$

By (16)

$$\rho D = \bar{f}\rho_0^2 \tag{24}$$

and the derivation with respect to $\mathcal{X}$ and $\mathcal{Y}$ yields respectively,

$$\frac{\partial\rho}{\partial\mathcal{X}} D = \bar{f}'_1\rho_0^2(1-\rho) - \rho \underline{f}'_1(1-\rho_0)^2$$
$$\frac{\partial\rho}{\partial\mathcal{Y}} D = \bar{f}'_2\rho_0^2(1-\rho) - \rho \underline{f}'_2(1-\rho_0)^2$$

so that (24) is

$$\frac{\partial W(P_A,P_B)}{\partial P_A} = \iint V'(\rho)A\left[\frac{\partial\rho}{\partial\mathcal{X}} + \frac{\partial\rho}{\partial\mathcal{Y}}\right]\rho D d\mathcal{X}d\mathcal{Y} + \iint V'(\rho)B\left[\frac{\partial\rho}{\partial\mathcal{X}} - \frac{\partial\rho}{\partial\mathcal{Y}}\right]\rho D d\mathcal{X}d\mathcal{Y}$$
$$+\iint V'(\rho)(-A)(1-\rho)[\bar{f}'_1+\bar{f}_2]\rho_0^2 d\mathcal{X}d\mathcal{Y} + \iint V'(\rho)(-B)(1-\rho)[\bar{f}'_1-\bar{f}'_2]\rho_0^2 d\mathcal{X}d\mathcal{Y} \tag{25}$$

and a new integration by parts of the two last terms of the right hand side of (25) and cancellation of terms, yields

$$\frac{\partial W(P_A,P_B)}{\partial P_A} = \iint V''(\rho)A\left[\frac{\partial\rho}{\partial\mathcal{X}} + \frac{\partial\rho}{\partial\mathcal{Y}}\right](1-\rho)\bar{f}\rho_0^2 d\mathcal{X}d\mathcal{Y} + \iint V''(\rho)B\left[\frac{\partial\rho}{\partial\mathcal{X}} - \frac{\partial\rho}{\partial\mathcal{Y}}\right](1-\rho)\bar{f}\rho_0^2 d\mathcal{X}d\mathcal{Y}$$
$$= \iint V''(\rho)\left[(A+B)\frac{\partial\rho}{\partial\mathcal{X}} + (A-B)\frac{\partial\rho}{\partial\mathcal{Y}}\right](1-\rho)\bar{f}\rho_0^2 d\mathcal{X}d\mathcal{Y}$$

This expression is (23) in the text. ■

*Proof of Proposition 5.* It suffices to prove that

$$E_{S_{P_1^{**}P_2^{**}}}[G(\rho(S_{P_1^{**}P_2^{**}}|P_1^{**},P_2^{**}))] > E_{S_{P_1^m}}[G(\rho(S_{P_1^m}|P_1^m))] \tag{26}$$

for any strictly convex function $G(\rho)$.

Consider $E_{S_{x,y}}[G(\rho(S_{x,y}|x,y))]$. To show (26), it suffices to show that price dispersion i.e. $x \neq y$ makes the signal $S_{x,y}$ more informative, i.e. that (27) in the text holds. This is equivalent to prove that $S_{x,y}$ attains a minimum at $x=y$.

Observe that, by the definitions of $S_{x,y}$ and $\rho$, the function $E_{S_{x,y}}[G(\rho(S_{x,y}|x,y))]$ can be expressed as,

$$\int G(\rho(s|x,y))[\rho_0 g(\bar{s}_{x,y},x,y) + (1-\rho_0)g(\underline{s}_{x,y},x,y)]ds =$$

$$\iint G(\rho(s^1,s^2|x,y))h(s^1,s^2)ds^1ds^2 = \iint G(\rho(Q_1,Q_2|x,y))h(Q_1,Q_2)dQ_1dQ_2 \qquad (27)$$

where $h(s^1,s^2) = h(Q_1,Q_2) = \rho_0^2\, f(\bar{\varepsilon}_1,\bar{\varepsilon}_2) + (1-\rho_0)^2 f(\underline{\varepsilon}_1,\underline{\varepsilon}_2)$, and

$$\bar{\varepsilon}_1 = s^1 - \gamma_1(x,y,\bar{\theta}) = Q_1 - \gamma_1(x,y,\bar{\theta})$$

$$\underline{\varepsilon}_2 = s^2 - \gamma_2(x,y,\underline{\theta}) = Q_2 - \gamma_2(x,y,\underline{\theta})$$

and $\underline{\varepsilon}_1$ and $\bar{\varepsilon}_2$ are defined similarly.

First note that (27) in the text holds if we replace V by G. Secondly, by the strict convexity of $G(\rho)$, and since by lemma 1, $\frac{\partial\rho}{\partial Q_i} \lesseqgtr 0$ depending on $x \gtrless y$,

$$\frac{\partial}{\partial x}\iint G(\rho(Q_1,Q_2|x,y))h(Q_1,Q_2)dQ_1dQ_2 =$$

$$\iint G''(\rho)\left((\bar{\gamma}'_{i,x} - \underline{\gamma}'_{i,x})\frac{\partial\rho}{\partial Q_i} + (\bar{\gamma}'_{j,x} - \underline{\gamma}'_{j,x})\frac{\partial\rho}{\partial Q_j}\right)(1-\rho)\rho_0^2 f(\bar{\varepsilon}_1,\bar{\varepsilon}_2)]dQ_{11}dQ_{21} \lesseqgtr 0$$

depending on $x \lesseqgtr y$. (28)

Hence by (27) and (28), $E_{S_{x,y}}[G(\rho(S_{x,y}|x,y))]$, attains a minimum at $x=y$, so that it is increasing in price dispersion, i.e. when $x \neq y$.

Now, by proposition 4, either $P_1^{**} > P_1^m$ and $P_2^{**} < P_1^m$, or $P_1^{**} < P_1^m$ and $P_2^{**} > P_1^m$, so that $P_1^{**} \neq P_2^{**}$. By the above argument $E_{S_{P_1P_2}}[G(\rho(S_{P_1P_2}|P_1,P_2))]$ increases in price dispersion and hence (26) is satisfied. ■

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