EXISTENCE AND OPTIMALITY OF SOCIAL EQUILIBRIUM WITH MANY CONVEX AND NONCONVEX FIRMS*

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A B S T R A C T

This paper analyzes the existence of efficient equilibrium allocations in an economy with many convex and non-convex firms. A Social Equilibrium is defined as a price vector and an allocation satisfying the following properties: a) Consumers satisfy their preferences, subject to their budget constraints; b,1) Convex firms maximize profits at given prices; b,2) Non-convex firms maximize profits at given prices, subject to an input constraint; and c) All markets clear. We show that, under suitable assumptions, a Pareto Optimal Social Equilibrium does exist. This result can be interpreted as defining a regulation policy yielding efficient outcomes, which consists of imposing constraints on the availability of some inputs for the non-convex firms, and let them act in the market as (constrained) profit maximizers.

Keywords: Increasing Returns, Optimal Allocations, Social Equilibrium.

1.- INTRODUCTION

The literature on general equilibrium with non-convex firms presents a sharp contrast between the results on the existence of equilibria and the results on the optimality of such equilibria. Existence results are available under very weak assumptions, as in Bonnisseau & Cornet (1988) [see Brown (1991) for an excellent survey of this problem]. As for optimality, Guesnerie (1975) first provided an extension of the Second Welfare Theorem allowing for nonconvex production sets. He showed that any efficient allocation can be decentralized as a marginal pricing equilibrium, provided we are free to carry out any feasible lump-sum transfer which may be required. Furthermore, under very mild regularity conditions, marginal pricing is a necessary condition for achieving Pareto Optimality through a price mechanism. On this see Guesnerie (1990), Vohra (1991), Quinzii (1992).

It is not true, however, that Marginal Pricing implies optimality. Guesnerie (1975) and Brown & Heal (1979) provide examples of economies with a single non-convex firm, where no Marginal Pricing Equilibrium is Pareto efficient. Beato & Mas-Colell (1985) show that, in the presence of several firms, even production efficiency may fail in a Marginal Pricing Equilibrium. Vohra (1988), (1990) presents examples where Marginal Pricing is Pareto dominated by Average Cost Pricing (and thus it is not even second best efficient), and where it is shown that the partial equilibrium intuition about the efficiency of Two-Part Marginal Pricing also fails.

The robustness of these examples, and the necessity of marginal pricing for optimality constitute a sort of "impossibility result": increasing returns, optimal allocations and price mechanisms are not compatible in a general setting. Not surprisingly, few models provide sufficient conditions for that. Among them, let us mention the works of Scarf (1986), Dierker (1987) and Quinzii (1991). These models consider the existence of a single non-convex firm (in order to ensure production efficiency), and impose some conditions on agents' characteristics.

The models by Dierker and Quinzii use Marginal Pricing as the appropriate equilibrium notion, and establish conditions on the relative curvature of of the production frontier and of the community indifference curves, so that when the social indifference curve is tangent to the feasible set, it never cuts inside it. Scarf (whose main concern is the existence of a nonempty core) follows a different approach, by introducing the notion of Social Equilibrium instead. A Social Equilibrium consists of a price vector and a feasible allocation such that consumers maximize utility, the firm maximizes profits within the set of feasible productions (i.e., those using no more inputs than those available), and equilibrium profits are null. There are <u>two special assumptions</u> in Scarf's (1986) model which allow him to ensure that a Social Equilibrium exists and it is in the core. The first one is the distinction between "two types of commodities: consumer goods, which appear in consumers' utility functions, and producer goods or inputs to production, which do not" [Cf. Scarf

(1986, p. 403)]. The second one consists of assuming that there is a single firm whose production set is distributive[1].

This paper provides a contribution along Scarf's lines, but allowing for the presence of several non-convex firms, and dispensing with the distributivity requirement. It refers to the efficiency of equilibrium allocations in a market economy where non-convex firms may be regulated. For that we consider a general equilibrium model with a competitive sector (consisting of many convex firms) and a regulated sector (consisting of non-convex firms). Consumers own the firms in the competitive sector, and are entitled to a proportion of the regulated sector's profits. The equilibrium notion we adopt is an immediate extension of Scarf's (1986) Social Equilibrium. Here a Social Equilibrium is defined as a price vector and an allocation satisfying the following properties: a) Consumers satisfy their preferences, subject to their budget constraints; b,1) Convex firms maximize profits at given prices; b,2) Non-convex firms maximize profits at given prices, subject to an input constraint; and c) All markets clear.

We present a model where a Pareto Optimal Social Equilibrium is shown to exist. The key for the efficiency property is a suitable extension of Scarf's first special assumption: we singularize a group of commodities which are idiosyncratic inputs for the regulated sector, so that they

[1] A production set is said to be distributive when any nonnegative weighted sum of feasible production plans is feasible if it does not use fewer inputs than any of the original plans. Distributivity implies non-decreasing returns to scale, and that the iso-inputs sets are convex.

enter neither the preferences of consumers nor the production processes of competitive firms. We shall refer to these commodities as Capital Goods.

The following remarks will clarify the nature of this result:

(a) We do not impose zero profits as an equilibrium condition. This allows us to separate the efficiency and the core problems, and hence dispense with the requirement of a distributive aggregate production set.

(b) What the Theorem says is that there is at least one Social Equilibrium which is Pareto Optimal (it does not say that any Social Equilibrium is Pareto Optimal). Hence, this result can be interpreted as defining a regulation policy yielding efficient outcomes, which consists of imposing constraints on the availability of idiosyncratic inputs for the non-convex firms, and let them act then as (constrained) profit maximizers.

The rest of the paper is organized as follows. Section 2 describes the model, discusses the assumptions and states and comments on the main result. A few final comments are gathered in Section 3. The proof of the Theorem has been relegated to an Appendix.

2. THE MODEL AND THE RESULT

Consider a market economy with ℓ perfectly divisible commodities, m consumers and n firms. A point $\omega \in \mathbb{R}^{\ell}$ describes the aggregate vector of initial endowments, while $\mathbf{p} \in \mathbb{R}^{\ell}_{+}$ stands for a price vector.

For $j = 1, 2, \dots, n$, let $Y_j \subset \mathbb{R}^{\ell}$ be the jth firm's production set. A point $\mathbf{y}_j \in Y_j$ denotes a production plan for the jth firm, while a point $\bar{\mathbf{y}} = (\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_n)$ denotes an element of $\prod_{j=1}^{n} Y_j$. There are two types of firms: Competitive (convex) firms, and Regulated ones. Accordingly, the set $\mathcal{N} \equiv \{ 1, 2, \dots, n \}$ consists of the union of two different subsets, $\mathcal{C}$ and Q, such that $\mathcal{C} \cap Q = \varnothing$. The set $\mathcal{C}$ contains the indices corresponding to convex firms, while the set Q (its complement on $\mathcal{N}$), contains the indices corresponding to the regulated non-convex firms. We shall refer to firms in $\mathcal{C}$ as the <u>competitive sector</u>, and to firms in Q as the <u>regulated sector</u>. We assume that consumers own competitive firms, and are entitled to a given share of the regulated sector's profits.

Each consumer $i = 1, 2, \dots, m$, is thus characterized by a tuple,

$$[\ X_i,\ u_i,\ \omega_i,\ (\theta_{ij})_{j\in\mathcal{C}}\ ,\ \tau_i\ ]$$

where X_i, u_i, ω_i stand for the ith consumer's consumption set, utility function and initial endowments, respectively, θ_{ij} denotes the ith consumer's share of the jth firm's profits, for $j \in \mathcal{C}$, and τ_i represents the ith consumer's share in the aggregate profits of the regulated sector. By definition,

$$\sum_{i=1}^{m} \omega_i = \omega \quad ; \quad \sum_{i=1}^{m} \theta_{ij} = 1 \ , \ \forall\ j \in \mathcal{C} \quad ; \quad \sum_{i=1}^{m} \tau_i = 1$$

with $\theta_{ij} \geq 0$, $\tau_i \geq 0$, for every i, j.

Given a price vector $\mathbf{p} \in \mathbb{R}^{\ell}_{+}$, and a vector of production plans $\bar{\mathbf{y}}$ in $\prod_{j=1}^{n} Y_j$, the ith consumer's behaviour is described by a demand correspondence ξ_i, which is given by the set of solutions to the following program:

$$\text{Max.} \quad u_i(\mathbf{x}_i)$$

$$\text{s.t.}:$$

$$\mathbf{x}_i \in X_i$$

$$\mathbf{p}\,\mathbf{x}_i \le \mathbf{p}\omega_i + \sum_{j\in\mathscr{C}} \theta_{ij}\mathbf{p}\mathbf{y}_j + \tau_i \sum_{j\in Q} \mathbf{p}\mathbf{y}_j$$

Then, consumers' behaviour can be summarized by an aggregate net demand correspondence, that can be written as $\xi(\mathbf{p}, \bar{\mathbf{y}}) - \{\ \omega\ \}$, where $\xi(\mathbf{p}, \bar{\mathbf{y}}) \equiv \sum_{i=1}^{m} \xi_i(\mathbf{p}, \bar{\mathbf{y}})$.

The set of feasible allocations is given by[2]:

$$\mathscr{A}(\omega) \equiv \left\{ [(\mathbf{x}_i), \bar{\mathbf{y}}] \in \prod_{i=1}^{m} X_i \times \prod_{j=1}^{n} Y_j \ / \ \sum_{i=1}^{m} \mathbf{x}_i - \omega \le \sum_{j=1}^{n} \mathbf{y}_j \right\}$$

The projection of $\mathscr{A}(\omega)$ on the space containing Y_j gives us the jth firm's set of feasible productions.

Consider now the following assumptions:

A.0.- Commodities can be split into two groups: Capital goods (indexed $h = 1, 2, \ldots, k$) and ordinary commodities (indexed $h = k+1, \ldots, \ell$). Capital goods are idiosyncratic inputs for the regulated sector, so that they enter neither the preferences of consumers nor the

[2] The convention for vector inequalities is: $\ge$, $>$, $\gg$.

production processes of competitive firms. If $j \in Q$, $\mathbf{y}_j \in Y_j$ we can write production plans for nonconvex firms as $\mathbf{y}_j = (\mathbf{a}_j, \mathbf{b}_j)$, with $\mathbf{a}_j \in -\mathbb{R}^k_+$.

A.1.- For each firm $j = 1, 2, \ldots, n$,

(i) Y_j is a closed subset of $\mathbb{R}^\ell$ such that $0 \in Y_j$, and $Y_j - \mathbb{R}^\ell_+ \subset Y_j$.

(ii) The jth firm's set of feasible productions is compact.

A.2.- (i) For every $j \in \mathscr{C}$, Y_j is a convex set.

(ii) For every $j \in Q$, each $\mathbf{y}_j \in Y_j$:

(ii, a) $\mathbf{a}_j = 0$ implies $\mathbf{b}_j \leq 0$.

(ii, b) $(\mathbf{a}_j, \mathbf{b}_j) \in \partial Y_j$, $\mathbf{b}_{jt} > 0$ for some t, and $\mathbf{b}'_j > \mathbf{b}_j$ implies $(\mathbf{a}_j, \mathbf{b}'_j) \notin Y_j$ (where ∂Y_j denotes the boundary of the jth firm's production set).

A.3.- For each $i = 1, 2, \ldots, m$,

(i) X_i is a closed and convex subset of $\mathbb{R}^\ell$, bounded from below.

(ii) $u_i : X_i \longrightarrow \mathbb{R}$ is a continuous and quasi-concave function, which satisfies Local non-satiation.

(iii) $\omega_i \in X_i$, and there exists $\mathbf{x}_i$ in X_i such that $\mathbf{x}_i \ll \omega_i$.

Assumption (A.0) extends Scarf's special hypothesis about the nature of commodities (to see this simply set $\mathscr{C} = \emptyset$, and let Q consist of a single firm). It postulates the existence of a special group of input commodities in the regulated sector (note that this group may actually consist of a single commodity). Capital Goods are special because they enter neither the preferences of consumers nor the production processes of competitive firms.

Assumption (A.1) provides us with a suitable generalization of the axioms established in Debreu (1959). Besides the technical point on closedness, Part (i) requires that inactivity is possible and that there is free disposal. Part (ii) says that it is not possible to obtain an unlimited amount of production out of a limited amount of resources.

Assumption (A.2) establishes properties over competitive and regulated firms. Part (i) simply says that competitive firms have convex production sets. A monotonicity property is then assumed for non-convex firms. Part (ii, a) says that positive production in the regualted sector requires using up some Capital Goods, while Part (ii, b) postulates that higher output levels require using up more Capital Goods (note that Part (ii, b) implies Part (ii, a), when $\mathbf{0} \in \partial Y_j$).

Assumption (A.3) is standard and needs little comment. It contains all the elements required in order to guarantee an upper hemicontinuous and convex valued demand correspondence, in a private ownership market economy where firms' profits are nonnegative.

Consider now the following definition:

Definition.- We shall say that a point $[\mathbf{p}^*, (\mathbf{x}_i^*), \bar{\mathbf{y}}^*]$, is a **Social Equilibrium** if the following conditions are satisfied:

(α) For each $i = 1, 2, \ldots, m$, $\mathbf{x}_i^* \in \xi_i(\mathbf{p}^*, \bar{\mathbf{y}}^*)$

(β,1) For each $j \in \mathscr{C}$,

$$p^* y_j^* \geq p^* y_j \ , \quad \forall \ y_j \in Y_j$$

(β,2) For each $j \in Q$,

$$p^* y_j^* \geq p^* y_j \ , \quad \forall \ y_j \in Y_j \ / \ a_j \geq a_j^*$$

(γ) $\sum_{i=1}^{m} x_i^* - \sum_{j=1}^{n} y_j^* = \omega$

That is, a Social Equilibrium consists of a price vector and a feasible allocation where all agents are maximizing their payoff functions within their feasible sets. These feasible sets correspond to budget sets, for the case of consumers, production sets for the case of competitive firms, and production sets subject to a Capital Goods constraint, for the cse of non-convex firms. Note that, under assumption (A.1), profits are nonnegative in a Social Equilibrium. Hence equilibrium allocations are always individually rational.

The following result is obtained:

Theorem.- Let E be an economy satisfying assumptions (A.0), (A.1), (A.2) and (A.3). Then, there exists a Pareto Optimal Social Equilibrium.

This Theorem establishes that there exists a Social Equilibrium which is Pareto optimal (mind that it does not say that every Social Equilibrium is Pareto optimal). In such an equilibrium the allocation of those inputs which determine the non-convex firms' restrictions is generated in a way that none of these firms would find individually profitable to operate with fewer inputs.

This suggests that we can interpret the result in the Theorem as follows: For any economy satisfying assumptions (A.0) to (A.3), there exists an equilibrium policy consisting of choosing the "capacity" of regulated firms [i.e., an allocation of Capital Goods, $(\mathbf{a}^*_j)_{j \in Q}$], and a price vector $\mathbf{p}^*$, such that all agents behave as payoff maximizers at given prices within their choice sets, all markets clear, and the resulting allocation is Pareto optimal.

Note that if Q is empty, a Social Equilibrium corresponds to a standard competitive equilibrium. On the other hand, if the economy consists of a single firm, then the model turns out to be a slight variant of Scarf's one (the difference being that we are not assuming distributivity and hence we allow for positive equilibrium profits).

3.- FINAL REMARKS

We have presented a model which extends Scarf's (1986) one to a context where there are many convex and non-convex firms. Convex firms are assumed to behave competitively while non-convex firms may be thought of as public utilities (which can be privately owned but regulated). Assuming the existence of a special class of inputs (idiosyncratic Capital Goods), it has been shown that a Pareto Optimal Social Equilibrium does exist. This suggests that there is a regulation policy (consisting of imposing quantity constraints on the availability of those special inputs) which yields efficient outcomes in a pseudo-competitive environment.

In order to find out the appropriate distribution of these inputs, the planner has to solve the model and find out an efficient equilibrium allocation. Even though this is a nontrivial task, it requires much less information and control over the economy, than when it comes to decentralizing an efficient allocation as a marginal pricing equilibrium. The latter represents an extreme form of regulation which requires both the control over all resources of the economy, and the full knowledge of all agents' characteristics. In order to achieve a Pareto Optimal Social Equilibrium, the planner needs "only" to know the aggregate net demand and supply mappings, and the technologies of nonconvex firms, and have control over the endowments of Capital Goods.

The fact that all firms behave according to the principle of profit maximization (either constrained or unconstrained) implies that equilibrium profits are always nonnegative, so that the incentive aspect of the model looks better than in the case of marginal pricing (which does not necessarily mean that it looks good!).

The following remarks are intended to qualify the nature of the results in Section 2:

(i) Observe that there is no contradiction between the optimality of Social Equilibria in the Theorem, and the necessity of marginal pricing for efficiency. This is so because marginal conditions are not necessary for Capital Goods, since they have no alternative use [see Quinzii (1992, p. 143)]. Indeed equilibrium prices may be thought of as marginal with respect to the production sets truncated by the availability of Capital Goods.

(ii) The assumption that consumers own a given share of the regulated sector's aggregate profits is relevant for the efficiency results in the Theorem. When nonconvex firms are privately owned, this assumption can be interpreted as a postulating the existence of a tax scheme over the profits of the regulated sector and a system of transfers, so that for every $(\mathbf{p}, \bar{\mathbf{y}}) \in \mathbb{R}^{\ell}_{+} \times \prod_{j=1}^{n} Y_j$, each $i = 1, 2, \ldots, m$, we have:

$$\ell_i(\mathbf{p}, \bar{\mathbf{y}}) \equiv \tau_i \sum_{j\in Q} \mathbf{p}\mathbf{y}_j - \sum_{j\in Q} \alpha_{ij}\mathbf{p}\mathbf{y}_j$$

for some given τ_i's adding up to 1 (where α_{ij} denotes the *ith* consumer's share in the *jth* nonconvex firm). It is worth noticing within this context that: (a) without such a redistribution policy the optimality of Social Equilibria may fail, even under assumptions (A.0) to (A.3) (except in the case $n = 1$); and (b) in the presence of a system of taxes and transfers, equilibrium allocations will not generally be Individually Rational.

(iii) Suppose finally that we dispense with the idyosincratic character of Capital Goods (i.e., we simply assume that there are some inputs which limit the production possibilities of non-convex firms). It follows easily from the Theorem that a Social Equilibrium is Individually Rational and Second Best Efficient (in the sense that there is no feasible allocation in which consumers are better off, provided that nonconvex firms do not use more Capital Goods than those in the equilibrium allocation). This result does not depend on the assumption that consumers own a given share of the regulated sector's aggregate profits.

Appendix: Proof of the Theorem

A <u>Pricing Rule</u> for the *jth* firm is a (set-valued) mapping ϕ_j from the set of efficient production plans into $\mathbb{R}^{\ell}_{+}$. For any efficient production plan $\mathbf{y}_j \in Y_j$, $\phi_j(\mathbf{y}_j)$ should be interpreted as the set of price vectors found "acceptable" by the *jth* firm when producing $\mathbf{y}_j$. In other words, the *jth* firm is in equilibrium whenever the pair $(\mathbf{y}_j, \mathbf{p})$ is in the graph of ϕ_j. Observe that under assumption (A.1) the set of weakly efficient production plans consists exactly of those points in the boundary of Y_j, that we denote by ∂Y_j.

We say that $\phi_j : \partial Y_j \longrightarrow \mathbb{R}^{\ell}_{+}$ is a <u>Regular Pricing Rule</u>, if ϕ_j is an upper hemicontinuous correspondence, with nonempty, convex and compact values. We say that $\phi_j : \partial Y_j \longrightarrow \mathbb{R}^{\ell}_{+}$ is a <u>Loss-Free Pricing Rule</u> if $\mathbf{q}\mathbf{y}_j \geq 0$, for every $\mathbf{q} \in \phi_j(\mathbf{y}_j)$.

The following result is well established in the literature [see for instance Bonnisseau & Cornet (1988, Th. 2.1')]:

Lemma 1.- Let E stand for an economy satisfying assumptions (A.1) and (A.3), and suppose that ϕ_j is a regular and loss-free pricing rule, for all *j*. Then, there exists a price vector $\mathbf{p}^*$ and an allocation $[(\mathbf{x}^*_i), \bar{\mathbf{y}}^*]$ such that:

(a) $\mathbf{x}^*_i \in \xi_i(\mathbf{p}^*, \bar{\mathbf{y}}^*)$, $\forall\, i$

(b) $\mathbf{p}^* \in \bigcap_{j=1}^{n} \phi_j(\mathbf{y}^*_j)$

(c) $\sum_{i=1}^{m} \mathbf{x}^* - \omega = \sum_{j=1}^{n} \mathbf{y}^*$.

In order to prove the Theorem, let us define two particular pricing rules: Profit Maximization and Constrained Profit Maximization. When technologies are convex, Profit Maximization can be defined as follows:

$$\psi_j^{PM}(y_j) \equiv \{ q \in \mathbb{R}_+^{\ell} \ / \ q\, y_j \geq q\, y'_j \ , \ \forall\, y'_j \in Y_j \}$$

This pricing rule associates with each efficient production plan, the set of prices which support it as the most profitable one.

Constrained Profit Maximization (which does not require convexity), is given by:

$$\psi_j^{CPM}(y_j) \equiv \{ q \in \mathbb{R}_+^{\ell} \ / \ qy_j \geq qy'_j, \ \ \forall\, y'_j \in Y_j \text{ with } a'_j \geq a_j \}$$

where $y_j = (a_j, b_j)$, $y'_j = (a'_j, b'_j)$, according to (A.0).

Thus, ψ_j^{CPM} pictures the *jth* firm as selecting, for each given efficient production plan y_j, prices such that it is not possible to obtain higher profits within the set of efficient production plans which make use of equal or fewer Capital Goods.

Remark.- Observe that under assumptions (A.1) and (A.2) this pricing rule can equivalently be defined as the normal cone to $T_j(y_j)$ at y_j, where $T_j(y_j)$ denotes the comprehensive convex hull of the set:

$$Y_j(y_j) \equiv \{ y'_j \in Y_j \ / \ a'_j \geq a_j \}$$

This equivalence ensures that $\psi_j^{CPM}(y_j)$ is a non-degenerate convex cone for every $y_j \in Y_j$.

Lemma 2.- Under assumptions (A.1) and (A.2), both ψ_j^{PM} and ψ_j^{CPM} are closed correspondences, whose images are nondegenerate convex cones.

Proof.-

If $j \in \mathscr{C}$, Y_j is a closed convex set, and $\psi_j^{PM}(\mathbf{y}_j)$ is the normal cone to Y_j at $\mathbf{y}_j \in \partial Y_j$, which is a nondegenerate closed and convex cone. The graph of this mapping is known to be closed.

Let now $j \in Q$. Under assumptions (A.1) and (A.2), $\psi_j^{CPM}(\mathbf{y}_j)$ is a nondegenerate closed and convex cone (see the Remark above). To see that the graph is closed, let $[(\mathbf{a}^o, \mathbf{b}^o), \mathbf{p}^o]$ be an arbitrary point in $\partial Y_j \times \mathbb{R}_+^\ell$, and let $\{ (\mathbf{a}^\nu, \mathbf{b}^\nu), \mathbf{p}^\nu \}$ be a sequence converging to $[(\mathbf{a}^o, \mathbf{b}^o), \mathbf{p}^o]$, such that $[(\mathbf{a}^\nu, \mathbf{b}^\nu), \mathbf{p}^\nu] \in \partial Y_j \times \mathbb{R}_+^\ell$, and $\mathbf{p}^\nu \in \psi_j^{CPM}(\mathbf{a}^\nu, \mathbf{b}^\nu)$, for all ν. Suppose, by way of contradiction, that $\mathbf{p}^o \notin \psi_j^{CPM}(\mathbf{a}^o, \mathbf{b}^o)$. Then there exists $(\mathbf{a}', \mathbf{b}') \in Y_j$, with $\mathbf{a}' \geq \mathbf{a}^o$, such that:

$$\mathbf{p}^o(\mathbf{a}', \mathbf{b}') > \mathbf{p}^o(\mathbf{a}^o, \mathbf{b}^o) \qquad [1]$$

This implies that, for ν big enough ($\nu > \nu'$, say), we also have:

$$\mathbf{p}^\nu(\mathbf{a}', \mathbf{b}') > \mathbf{p}^\nu(\mathbf{a}^\nu, \mathbf{b}^\nu)$$

If $\mathbf{a}' \geq \mathbf{a}^\nu$ and $\nu > \nu'$, this contradicts the assumption that $\mathbf{p}^\nu$ belongs to $\psi_j^{CPM}(\mathbf{a}^\nu, \mathbf{b}^\nu)$. Suppose that this is not the case. We have now two possibilities. First, $\mathbf{a}^o = \mathbf{0}$, and consequently $\mathbf{a}' = \mathbf{0}$. Then it follows that $\mathbf{b}^o = \mathbf{0} \geq \mathbf{b}'$ [by (ii,a) of (A.2) and $\mathbf{0} \in Y_j$ in (i) if (A.1)], and hence the inequality [1] above cannot hold. Suppose then that $\mathbf{a}^o < \mathbf{0}$, and construct a new point $(\mathbf{a}'', \mathbf{b}'')$ in Y_j as follows: (i) $\mathbf{a}''_t = \mathbf{a}'_t + \varepsilon$, if $\mathbf{a}'_t < 0$ (where $\varepsilon > 0$ is a scalar arbitrarily small), and $\mathbf{a}''_t = 0$, otherwise; and (ii) $\mathbf{b}''_t = \mathbf{b}'_t - \delta_t$ (where $\delta_t \geq 0$ is a scalar arbitrarily small). Since Y_j is a closed and comprehensive set, these scalars can always be chosen so that $(\mathbf{a}'', \mathbf{b}'')$ lies in Y_j, and $\mathbf{p}^o(\mathbf{a}'', \mathbf{b}'') > \mathbf{p}^o(\mathbf{a}^o, \mathbf{b}^o)$. Note that, by construction, $\mathbf{a}'' > \mathbf{a}' \geq \mathbf{a}^o$.

Now observe that for ν big enough, there will be points $(\mathbf{a}^{\nu}, \mathbf{b}^{\nu})$ close to $(\mathbf{a}^{o}, \mathbf{b}^{o})$ such that $\mathbf{a}'' \geq \mathbf{a}^{\nu}$. For these points we have:

$$\mathbf{p}^{\nu}(\mathbf{a}'', \mathbf{b}'') > \mathbf{p}^{\nu}(\mathbf{a}^{\nu}, \mathbf{b}^{\nu})$$

while $\mathbf{p}^{\nu} \in \psi_j^{CPM}(\mathbf{a}^{\nu}, \mathbf{b}^{\nu})$, contradicting the hypothesis.

$\blacksquare$

Theorem.- Let E be an economy satisfying assumptions (A.0), (A.1), (A.2) and (A.3). Then, there exists a Pareto optimal Social Equilibrium.

Proof.-

We shall divide the proof into three steps.

(i) Consider an economy $\hat{E}$ identical to E in all respects except in that we substitute all regulated firms $j \in Q$ by a single aggregate one, with a production set denoted by Y_0, defined in the following way: Let $\hat{Y}_j$, $j \in Q$, denote the *jth* firm's feasible set. Then,

$$Y_0 \equiv ch\left[\sum_{j \in Q} \hat{Y}_j\right]$$

where ch (.) denotes the comprehensive hull of (.). Note that Y_0 inherits all properties assumed in (A.1) and (A.2). In particular, this way of constructing Y_0 ensures that it is a closed set since $\sum_{j \in Q} \hat{Y}_j$ is compact.

Let $\mathbb{P} \subset \mathbb{R}_+^{\ell}$ denote the standard price simplex, that is,

$$\mathbb{P} = \{\ \mathbf{p} \in \mathbb{R}_+^{\ell}\ /\ \sum_{t=1}^{\ell} p_t = 1\ \}.$$

Now, for each $j \in \mathscr{C}$, every $\mathbf{y}_j \in \partial Y_j$ define: $\phi_j(\mathbf{y}_j) \equiv \psi_j^{PM}(\mathbf{y}_j) \cap \mathbb{P}$; and for each $\mathbf{y}_0 \in \partial Y_0$, define: $\phi_0(\mathbf{y}_0) \equiv \psi_0^{CPM}(\mathbf{y}_0) \cap \mathbb{P}$. In view of Lemma 2, these mappings are upper hemicontinuous correspondences, with nonempty, convex and compact values. Hence, they belong to the family of regular and loss-free pricing rules. Thus, Lemma 1 ensures the existence of an equilibrium. By construction, this corresponds precisely to a Social Equilibrium for the $\hat{E}$ economy.

(ii) Let us show now that a Social Equilibrium for $\hat{E}$ actually corresponds to a Social Equilibrium for the original economy. First notice that, since every consumer owns a given share of the aggregate profits of the regulated sector, the aggregation of non-convex firms into a single one does not affect their wealth functions.

Now observe that, by construction, $\mathbf{y}_0^*$ can be expressed as $\sum_{j\in Q} \mathbf{y}_j^*$ with $\mathbf{y}_j^* \in Y_j$ for j in Q. Note that if $\mathbf{y}_0^*$ maximizes profits at prices $\mathbf{p}^*$ within the set:

$$Y_0(\mathbf{y}_0^*) \equiv \{ \mathbf{y}_0 \in Y_0 \ / \ \mathbf{a}_0 \geq \mathbf{a}_0^* \}$$

it must be the case that <u>every</u> $\mathbf{y}_j^*$ maximizes profits at $\mathbf{p}^*$ in the set

$$Y_j(\mathbf{y}_j^*) \equiv \{ \mathbf{y}_j \in Y_j \ / \ \mathbf{a}_j \geq \mathbf{a}_j^* \}$$

for $j \in Q$ (although the contrary need not be true). For suppose not, that is, suppose that for some k there exists $\mathbf{y}'_k \in Y_k(\mathbf{y}_k^*)$ such that $\mathbf{p}^* \mathbf{y}'_k > \mathbf{p}^* \mathbf{y}_k^*$; then, substituting $\mathbf{y}_k^*$ by $\mathbf{y}'_k$ in $\mathbf{y}_0^*$ we would get:

$$\mathbf{p}^* \left[\sum_{\substack{j\neq k \\ j\in Q}} \mathbf{y}_j^* + \mathbf{y}'_k \right] > \mathbf{p}^* \mathbf{y}_0^*$$

with $\left[\sum_{\substack{j\neq k \\ j\in Q}} \mathbf{y}_j^* + \mathbf{y}'_k \right] \in Y_0(\mathbf{y}_0^*)$, contradicting the hypothesis.

(iii) Finally, suppose that there is another feasible allocation $[(\mathbf{x}'_i), \bar{\mathbf{y}}']$ such that $u_i(\mathbf{x}'_i) \geq u_i(\mathbf{x}_i^*)$ for every i, with a strict inequality for some consumer. Since this allocation is feasible, it must be the case that

$$\sum_{i=1}^{m} x'_i \leq \omega + \sum_{j \in \mathscr{C}} y'_j + \sum_{j \in Q} y'_j$$

Now notice that non-satiation (together with the fact that every x^*_i is an equilibrium consumption plan) implies that

$$p^* \sum_{i=1}^{m} x'_i > p^* \sum_{i=1}^{m} x^*_i = p^* \omega + p^* \sum_{j \in \mathscr{C}} y^*_j + p^* \sum_{j \in Q} y^*_j$$

Therefore substituting we get

$$\sum_{j \in \mathscr{C}} p^* y'_j + \sum_{j \in Q} p^* y'_j > \sum_{j \in \mathscr{C}} p^* y^*_j + \sum_{j \in Q} p^* y^*_j$$

Now observe that $\sum_{j \in \mathscr{C}} p^* y'_j \leq \sum_{j \in \mathscr{C}} p^* y^*_j$ by definition of profit maximization. Hence it must be the case that $\sum_{j \in Q} p^* y'_j > \sum_{j \in Q} p^* y^*_j$. But this is not possible, since: (a) Feasibility and assumption (A.0) imply that $\sum_{j \in Q} a'_j \geq - \omega^K$ (where $\omega^K \in \mathbb{R}^k$ stands for the aggregate endowment of Capital Goods); and (b) $\sum_{j \in Q} y^*_j$ is a profit maximizing combination of production plans, subject to the restriction $\sum_{j \in Q} a'_j \geq \sum_{j \in Q} a^*_j = - \omega^K$ [in view of (A.0), the definition of Social Equilibrium, and the way in which $\bar{y}^*$ has been chosen].

The proof is in this way completed.

$\blacksquare$

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