ON THE COMPETITIVE EFFECTS OF DIVISIONALIZATION*

Luis C. Corchón and Miguel González-Maestre**

WP-AD 94-10

* We thank J. Hamilton, X. Vives and seminar audiences in Bielefeld, University of Alicante, U. Autónoma of Barcelona and the 20th E.A.R.I.E. Conference held in Tel-Aviv for very useful comments. Authors acknowledge financial support from DGICYT under project PB 91-0756 and from the Instituto Valenciano de Investigaciones Económicas.

** L.C. Corchón: Instituto Valenciano de Investigaciones Económicas and University of Alicante; M. González-Maestre: University of Alicante.

Editor: **Instituto Valenciano de Investigaciones Económicas, S.A.**
Primera Edición Junio 1994.
ISBN: 84-482-0574-X
Depósito Legal: V-1961-1994
Impreso por Copisteria Sanchis, S.L.,
Quart, 121-bajo, 46008-Valencia.
Printed in Spain.

ON THE COMPETITIVE EFFECTS OF DIVISIONALIZATION

Luis C. Corchón & Miguel González-Maestre

A B S T R A C T

In this paper, we assume that firms can create independent divisions which compete in quantities in a homogeneous good market. Assuming complete information, identical firms and constant returns to scale, we prove the following: 1) Subgame Perfect Nash Equilibrium (SPNE) implies Perfect Competition if the number of firms is beyond some critical level. 2) This level is small (sometimes one) under reasonable circumstances. Assuming a fixed cost per firm, SPNE implies that even if this cost is arbitrarily small and the number of potential firms is arbitrarily large, 3) the number of active firms is small (sometimes a monopoly) and 4) the total number of divisions is bounded above. This implies that the market under consideration is a Natural Oligopoly. Next we study a model in which there is both a fixed cost and an upper bound on the maximum number of divisions which can be created. We show that 5) when this upper bound tends to infinity and the fixed cost tends to zero, SPNE may imply either Perfect Competition or a Natural Oligopoly. Finally 6) it is shown that the above results hold under incomplete information.

Keywords: Divisionalization, Limit Theorems, Perfect Competition.

I. INTRODUCTION

In this paper, we analyze a model of oligopolistic competition in which every firm can create independent divisions (see e.g. Milgrom & Roberts (1992) chp. 16)[1]. Once these divisions have been established they become independent sellers which compete in quantities in a homogeneous good market. In a previous paper (Corchón (1991)) it was found that under complete information and linear or unit-elastic demand functions, Subgame Perfect Nash Equilibrium (SPNE) of a two-stage game (in which firms decide on the number of divisions in the first stage and compete à la Cournot in the second stage) implies perfect competition *even if the number of firms is small*[2]. In this paper we study the question of the attainability of perfect competition under more general demand, information and cost structures.

In Section II we assume zero fixed costs. We show that if the number of firms is finite, but large enough in relation to the degree of convexity of the inverse demand function, the SPNE of the above game implies perfect competition (Proposition 1). If the inverse demand function is concave or the industry profit function is strictly concave (resp. concave) and there are more than one (resp. two) firms, SPNE implies perfect competition (Proposition 2). These results cover, as special cases, those obtained by Corchón (1991).

In Section III, we assume that in order to be active in the market, each firm has to incur in a positive fixed cost. We show that if the degree of convexity of the inverse demand function is small enough, the SPNE of a three-stage game (in which the first stage consist of the decision on entering

---

(1) The terminology used in this paper is different from that used in Corchón (1991). What he called a group (or a corporation) is called here a firm and what he called a firm is called here a division. We adopt here the terminology used in Schwartz & Thompson (1986) which is not only prior but it seem to us more descriptive.

(2) A similar result can be proved by using the Salop model in which firms are price-setters and the product is differentiated (see González-Maestre (1992)).

the market and the other two stages are the same as in the previous game) implies that there will be a small number of firms in the market *irrespectively of the size of the fixed cost* (Proposition 3). In fact, if the inverse demand function is concave, SPNE implies monopoly and if the profit function is concave (resp. strictly concave), then SPNE implies the existence of, at most, two (resp. one) active firms (Corollary 2). Also, under an additional assumption we obtain that the total number of divisions in equilibrium is independent of the size of the fixed cost (Proposition 4). These results imply that the market under consideration is a *Natural Oligopoly* in the terminology of Shaked & Sutton (1983). This contrasts widely with the Cournot model without divisionalization, in which equilibrium converges to perfect competition when the size of the fixed cost goes to zero, irrespectively of the degree of convexity of the inverse demand function (see for instance Novshek (1980) and the references therein). The explanation of our result lies in the fact that under constant returns to scale, perfect competition implies zero profit. Therefore, if the inverse demand function is, say, concave (or the profit function strictly concave) the results obtained in Section II imply that if two firms were to enter the market, they would obtain negative profits. Thus only one firm can be active and, under our assumptions, the need for divisionalization disappears. Therefore, the results obtained in Corchón (1991) are robust to the consideration of more general inverse demand functions, but they change dramatically when a fixed cost (no matter how small) per firm is assumed and an extra stage is added (this feature is shared -for the same reason- with the Bertrand model, see e.g. Sutton (1991 p. 32).

In Section IV we investigate the limit of SPNE outcomes when there is an upper bound on the number of divisions which can be created and a fixed cost per firm. The purpose of this Section is to analyze SPNE in the limit when frictions disappear, i.e. when the upper bound tends to infinity and the fixed cost tend to zero. We show that, for any positive fixed cost, the outcome of any SPNE coincides, for a sufficiently large upper bound on the number of firms with the SPNE analyzed in Section 3 (i.e. the natural oligopoly case). However, for a sufficiently large given upper bound on the number of firms, if the fixed cost tends to zero, SPNE is as close as we wish to the SPNE analyzed in Section 2 (i.e. perfect competition almost occurs). Therefore convergence either to perfect competition or to natural oligopoly depends on the rate of convergence of the frictions, i.e. both outcomes can be considered to be plausible in large economies. This contrasts widely with the standard limit theorems in markets with quantity-setter firms and no divisionalization in which convergence to perfect competition is always obtained.

Finally, Section V checks the robustness of the above results by considering incomplete information. It is shown that the main conclusions above still hold. We remark here that our results still hold under product heterogeneity or increasing average costs (see footnotes (2) and (5)).

Summing up, the main message of this paper is that markets in which the possibility of divisionalization exists, behave very differently to those in which this possibility does not arise. This may be due either to a genuine difference between both situations or to the wrong modelling. The robustness of our conclusions suggests that the former reason may be the right one.

Our results can be compared with those obtained by Schwartz & Thompson (1986) on the use of divisionalization to deter entry. They show that if the incumbent decides on the number of its divisions before potential entrants do, it will create a number of divisions such that entry is deterred[(3)]. Thus they conclude that "regarding welfare implications, the noninnovative entry that is always preempted by permanently divisionalized incumbents in our model, *would be Pareto nonoptimal entry* because of a pointless duplication of overhead promotion costs .... These results support a laissez-faire policy towards permanent divisionalization as an entry deterrent". (Schwartz & Thompson, p. 320). This comes from the fact that if the number of divisions equals the number of equilibrium firms under non-divisionalization, welfare is higher

---

(3) The argument behind their result is actually identical to the one used by Omori & Yarrow (1982), Corchón & Marcos (1988) and Vives (1988) to show that a Stackelberg equilibrium in quantities with the incumbent firm as the leader implies entry deterrence if the number of potential entrants is large enough. Actually divisionalization can be thought of as a natural way to make credible the commitment of the incumbent firm to its output.

under divisionalization since total fixed costs are smaller. However in our model, under concavity of the inverse demand function or strict concavity of the profit function, the incumbent does not need to create extra divisions in order to deter entry since the entry of an extra firm will credibly trigger negative profits for it. Therefore, the *possibility* of creating independent firms might yield a lower degree of competition than in the model where such a possibility does not exist[4]. This is because in our model, the incumbent can decide on the number of divisions simultaneously with the potential entrant. An extra implication of our model is that successful entry deterrence might be connected to the degree of convexity of the inverse demand function and not to the magnitude of sunk costs as suggested by traditional models.

The rest of the paper goes as follows. The model without a fixed cost is studied in Section II and the model with a positive fixed cost is analyzed in Section III. Section IV studies convergence matters when frictions vanish. Section V analyzes the case of incomplete information. Finally, Section VI gathers our final comments.

---

(4) In our case, the effect on welfare of the availability of divisionalization is ambiguous: on the one hand, total fixed cost decreases, since fixed costs are not replicated, but, on the other hand, the number of independent sellers falls, which implies a lower degree of competition.

II. THE MODEL WITH NO ENTRY COSTS

We will consider a market in which there is a given number (possibly infinite) of firms, denoted by k (> 1), which can create independent divisions that compete in quantities in a market for a homogeneous good[5].

We will assume that there is an inverse demand function $p(z)$, where z is total output and p is the price of the product. The cost function for each division given by $c.x_i$, where $c > 0$ and x_i is the output of division i.

If a given firm, say j , creates m divisions, and each of them produces the same amount x_i, then the total profits of firm j will be given by

$$\Pi_j = m.[p(z).x_i - c.x_i],$$

where m is the number of divisions of firm j. All our results hold if Π_j is only a fraction of the total profit obtained by the m divisions. In this case, each firm can be interpreted as holding a patent and deciding on the number of independent sellers who are allowed to use the patent. These sellers pay the firm a fraction of their profits (see Section V for a model in which divisions pay in proportion to their output or their sales).

The following assumptions will be maintained throughout the paper (when no confusion can arise derivatives will be denoted by primes, i.e. $dp/dz = p'$)

---

(5) If the product is heterogeneous, "prima facie", the incentive to divisionalize is larger than under product homogeneity. This can be seen by considering an inverse demand function corresponding to every good of the form $F(\sum_{j=1}^{n} x_j^{\alpha}).x_i^{\alpha-1}$, where $0 < \alpha < 1$ (see Spence (1976) or Dixit & Stiglitz (1977)). Letting $y_i = x_i^{\alpha}$, profits for division i become $F(\sum_{j=1}^{n} y_j)y_j - c.y_j^{1/\alpha}$. Thus, as noticed by Yarrow (1985), the model is identical to a homogeneous product model <u>with decreasing returns</u>. Therefore, in this framework there is an extra incentive to divisionalize and all our results hold a fortiori.

A.1: There is a real number y such that for any $z > y$, $p(z) < c$.

A.2: $p(z) \in C^2$.

A.3: There is a real number $\gamma < 0$ such that for any $z \leq y$, $p'(z) \leq \gamma$.

A.4: There is a real number $w > 0$ such that $p(w) > c$.

A.5: The number of divisions can be considered as a continuous variable.

Assumption 1 (A.1 in the sequel) means that for very large outputs, costs are never covered. A.3 says that the slope of the inverse demand function is negative and bounded away from zero in the relevant interval. A.4 is a feasibility condition. In conjunction with A.1 it implies that the analysis of the properties of SPNE can be restricted to a closed interval which will be denoted by $\mathfrak{Z}$. Finally A.5 can be interpreted as follows: if a firm decides to create, say, 3.25 divisions, then it endows divisions 1, 2 and 3 with the capital necessary to produce the Cournot equilibrium output. Take this magnitude to be one, normalizing if necessary. The fourth division is endowed with 0.25 units of capital and it will produce one fourth of the output of divisions 1, 2 and 3. If the reader is not completely happy with this interpretation we urge her/him to look at Appendix I where it is shown that our main results can be obtained assuming indivisible divisions.

Let us define $\beta \equiv \beta(z) \equiv \dfrac{p''.z}{p'}$, where $-\beta$ can be interpreted as a measure of the degree of convexity of the demand function. The following Lemma ensures that β is bounded below in $\mathfrak{Z}$.

Lemma 1: *Under A.1, A.2, A.3 and A.4, $\exists\ \beta'$ such that $\beta(z) \geq \beta'\ \forall\ z \in \mathfrak{Z}$.*

Proof: *The following minimization program*

$$\text{To min } \beta(z) \text{ s. t. } z \in \mathfrak{Z}.$$

has a solution since $\beta(z)$ is continuous (by A.2 and A.3) and $\mathfrak{Z}$ is compact (by A.1 and A.4). ■

Let us assume that information is complete, i.e. every player -division or firm- knows exactly all the relevant information, i.e. the inverse demand and the cost functions. We will now investigate the properties of Subgame Perfect Nash Equilibrium (SPNE) of the following game.

G.1: *Stage 1:* every firm decides (simultaneously) its number of divisions;
Stage 2: every division decides (simultaneously) its output.

Proposition 1: *Under A.1, A.2, A.3, A.4 and A.5, if $k > \max(-\beta', 1)$, Perfect Competition is the only Subgame Perfect Nash Equilibrium of the game G.1.*

Proof: *Since if k is infinite the Proposition is true, we will concentrate on the case of a finite k. In this case, it is clear that perfect competition is a SPNE, so it only remains to be shown that any SPNE yields perfect competition.*

Under symmetry in the second stage (see Appendix II), total profits of firm j are given by

$$\Pi_j = m.\Pi_i(x_i,z) = m.[p(z) - c].x_i,$$

where Π_i is the profit per division in the second stage, and m is the number of divisions created by firm j.

In the second stage, every division maximizes its own profits given the output produced by other divisions. Equilibrium at this stage, depends on the total number of divisions, which will be called n. The equilibrium value of n is determined by backward induction: every firm maximizes its total profits given the number of divisions created by its competitors. Thus we obtain that

$$\frac{\partial \Pi_j}{\partial m} = \Pi_i + m.\frac{\partial \Pi_i}{\partial x_i}.\frac{dx_i}{dn} + m.\frac{\partial \Pi_i}{\partial z}\frac{dz}{dn}$$

The first order condition in the second stage reads

$$\frac{\partial \Pi_i}{\partial x_i} + \frac{\partial \Pi_i}{\partial z} = 0,$$

By substituting this in the above derivative, we obtain that

$$\frac{\partial \Pi_j}{\partial m} = \Pi_i - m.\frac{\partial \Pi_i}{\partial x_i}.[\frac{dz}{dn} - \frac{dx_i}{dm}]$$

On the other hand, $\dfrac{\partial \Pi_i}{\partial x_i} = p - c$, and $\Pi_i = (p - c).x_i$, so that,

$$\frac{\partial \Pi_j}{\partial m} = (p - c).\{ x_i - m.[\frac{dz}{dn} - \frac{dx_i}{dm}] \} \tag{1}$$

If $p = c$ the allocation is perfectly competitive and thus the Proposition is proved. $p < c$ is impossible since in the second stage, the first order conditions of profit maximization are given by

$$p'.x_i + p - c = o \qquad \text{where, } i \ldots,n \tag{2}$$

and since x_i is positive for at least some i and p' is negative by A.3, $p > c$. Now we will show that if $k > \max(-\beta', 1)$, then $\dfrac{\partial \Pi_j}{\partial m} > 0$ and thus perfect competition is the only SPNE of the game G.1. To show this, let us argue by way of a contradiction. If (1) is not positive, it must be that

$$x_i \leq m.[\frac{dz}{dm} - \frac{dx_i}{dm}] \tag{3}$$

Let us define $z' = \dfrac{dz}{dm}$ and $x'_i = \dfrac{dx_i}{dm}$.
Since equilibrium is symmetrical in the second stage (see Appendix II)

$$z = n.x_i \tag{4}$$

By differentiating in (2) and (4), we obtain the following conditions,

$$p''z'x_i + p'x'_i + p'z' = 0 \tag{2'}$$

$$z' = x_i + n.x'_i \tag{4'}$$

From (2') and (4') we have,

$$x'_i = \frac{-x_i}{n(\frac{1}{\beta + n} + 1)}$$

$$z' = \frac{x_i}{\beta + n + 1} \tag{5'}$$

Therefore, we obtain that

$$z' - x_i' = \frac{x_i}{n(\beta + n + 1)} .(\beta + 2n)$$

Notice that $k > \max(-\beta', 1)$ *implies that* $\beta + n > 0$, *so the above expression is well defined. Thus, inequality (3) implies*

$$x_i \leq m. \frac{x_i}{n(\beta + n + 1)} .(\beta + 2n) \qquad (3')$$

which since $\beta + n + 1 > 0$, *is equivalent to*

$$\beta + n + 1 \leq m.(\frac{\beta}{n} + 2) \qquad (3'')$$

Equilibrium in the first stage can be shown to be symmetrical (see Appendix I), so $n = k.m$, *and the previous equation is equivalent to*

$$m(k - 2) \leq \frac{\beta}{k} - \beta - 1$$

Since $m \geq 1$ *(by A.4), the above inequality implies that*

$$k \leq - \beta(1 - \frac{1}{k}) + 1 \leq -\beta'(1 - \frac{1}{k}) + 1 \qquad (6)$$

since $0 < 1 - \frac{1}{k} < 1$ *for any* $k \geq 2$, *and* $\beta \geq \beta'$. *Now (6) can be written as*

$$k - 1 \leq - \beta'(\frac{k - 1}{k})$$

but, if $k \geq 2$, *the above inequality implies that* $k \leq - \beta'$, *which contradicts that* $k > \text{Max}(-\beta', 1)$. *Therefore* $\frac{\partial \Pi_j}{\partial m} > 0$, *so every firm can increase its profits by increasing its number of divisions and the Proposition is proved.* ■

Proposition 1 says that if k is beyond some critical level, determined by the degree of convexity of the inverse demand function, perfect competition is the unique SPNE of game G.1. The intuition behind this result is that the higher $-\beta'$ is, the more competitive the second stage is and therefore the lower the incentive is to create divisions in the first stage. Thus, a high value of $-\beta'$ makes it less likely that perfect competition is a SPNE. Notice that Proposition 1 includes, as special cases, examples 1 and 2 from Corchón (1991).

Corollary 1: a) *If $p = A/z$, then $\beta' = -2$. Therefore $k > \max(-\beta', 1)$ is equivalent to $k > 2$ (see Corchón (1991) p. 2, example 1).*

b) *If $p = a - z$, then $\beta' = 0$. Thus $k > \max(-\beta', 1)$ is equivalent to $k > 1$ (see Corchón (1991) p. 2, example 2).*

It may be worth pointing out that the condition $k > \max(-\beta', 1)$ implies (but it is not implied by) the following results. First, that the entry of a new firm increases total output and decreases the output of any incumbent (see equations (5) and (5') in the proof of Proposition 1). In other words, this condition implies that the effect of entry on equilibrium outputs agrees with our intuition about it. Secondly, that the Cournot equilibrium is unique. This follows easily from the fact that the condition above implies that the first order condition of profit maximization is strictly increasing on z.

We end this Section by identifying four alternative sufficient conditions for $k > \max(-\beta', 1)$ to hold. These conditions are reasonable and imply that a small number of groups may suffice for perfect competition to occur.

Proposition 2: *Under A.1, A.2, A.3, A.4 and A.5, any of the following conditions imply $k > \max(-\beta', 1)$.*

a) *$p(z)$ concave for all $z \in \mathfrak{Z}$.*

b) *$p(z)$ having constant elasticity -denoted by ε- and $k > -\varepsilon + 1$.*

c) *$\Pi(z)$ is concave for all $z \in \mathfrak{Z}$ and $k > 2$, where $\Pi(z)$ is the industry total profits as a function of z, i.e. $\Pi(z) = p(z).z - c.z$.*

d) *$\Pi''(z) < 0$ for any given $z \in \mathfrak{Z}$.*

Proof: *a) and b) are obvious. In order to prove c) notice that concavity of $\Pi(z)$ implies that $\beta(z).z \geq -2z \ \forall z \in \mathfrak{Z}$. Thus, $\beta' \geq -2$ and c) holds. d) is proved in a similar way.*■

III. THE MODEL WITH AN ENTRY COST

In this Section we will assume that firms have to pay a fixed cost in order to enter into the market. We will consider the following game, which is no more than G.1 with an extra stage in which the fixed cost is incurred.

G.2: *Stage 1:* every firm decides (simultaneously) on entry. A fixed cost $\varepsilon > 0$ is paid by a firm if it decides to enter into the market.

Stage 2: every firm decides (simultaneously) on the number of divisions.

Stage 3: every division decides (simultaneously) on its output.

Let k^* and m^* be the number of active firms and divisions respectively per firm in a SPNE of the game G.2. Then, we get the following

Proposition 3: *If A.1, A.2, A.3, A.4 and A.5 are satisfied, then any SPNE of the game G.2 implies* $k^* \leq \min(k, \max(-\beta', 1))$.

Proof: *Considering any SPNE of G.2, the number of firms* $k^* > \max(-\beta', 1)$ *can never be a SPNE since, if this were the case, by Proposition 1 any SPNE price would be equal to marginal cost and profits would be negative for every firm entering into the market* ■

An obvious conclusion from Propositions 2 and 3 is the following

Corollary 2: *Under A.1, A.2, A.3, A.4, and A.5 we have that*

a) *If* $p(z)$ *concave for all* $z \in \mathfrak{Z}$, $k^* = 1$ *and* $m^* = 1$.
b) *If* $p(z)$ *has constant elasticity -denoted by* ε*-,* $k^* \leq \min(k, -\varepsilon + 1)$.
c) *If* $\Pi(z)$ *is concave for any given* $z \in \mathfrak{Z}$, $k^* \leq 2$.
d) *If* $\Pi''(z) < 0$ *for any given* $z \in \mathfrak{Z}$, $k^* = 1$ *and* $m^* = 1$.

In other words: if the inverse demand function is concave, SPNE implies that there will be only one active firm in the market, with only one division. In the case in which the inverse demand function displays constant elasticity, the equilibrium number of firms is no greater than minus this elasticity plus one. An important implication is that if the elasticity of the inverse demand is smaller than one in absolute value, *monopoly will occur.* Finally, if the profit function is concave (strictly concave) there will be at most two (resp. one) active firms in the market. It is important to remark that this result is *independent of the relationship between the fixed cost and the market size*, as happens in the Cournot model withoutdivisionalization.

Another interesting implication of Corollary 2 is that, contrary to the conclusion obtained by Schwartz and Thompson (1986), in our case an incumbent firm does not need to create more than one division in order to deter entry. The intuition behind this result is the following: in our model, if $p'' \leq 0$ or if the profit function is strictly concave, then there is a credible threat that, if another firm enters into the market, the only SPNE for the rest of the game is perfect competition, which would imply negative profits for any active firm. This suggests that the relevant aspect of the divisionalization is not necessarily its effective realization, but simply the credibility of its use in the case of entry. Consequently, the welfare and policy implications on the existence of this strategic instrument may be different to those obtained by Schwartz and Thompson, when we consider that the decision on the number of firms is simultaneous rather than sequential.

The next result extends Proposition 3 since it shows, under an additional assumption, that there is an upper bound on the total number of active divisions at any SPNE, and that this upper bound is independent of the fixed cost as long as this is positive.

Proposition 4: *Under A.1, A.2, A.3, A.4 and A.5 and if either $\beta(\ )$ is strictly monotonic or $\beta(z) \neq -2\ \forall\ z \in \mathfrak{Z}$, the total number of active divisions in any SPNE of G.2 is bounded above by a finite number which is independent of the value of the fixed cost $\varepsilon > 0$.*

Proof: *First notice that in any possible SPNE of G2 the number of divisions must be finite, since an infinite number of divisions would yield an equilibrium price equal to the marginal cost and, therefore, losses for all* *active firms. From expression (3'), obtained in the proof of Proposition 1, a necessary condition in order to have a finite number of divisions in a SPNE of G.1, is given by*

$$m^*.(k^* - 2) \leq \beta/k^* - \beta - 1 \qquad (7)$$

since this is equivalent to $\dfrac{\partial \Pi_j}{\partial m} \leq 0$.

If the above condition is satisfied with an strict inequality, then $m^* = 1$ *so (7) reads* $k^*(k^* - 1) \leq -\beta(k^* - 1) \leq -\beta'(k^* - 1)$, *and* $k^* \leq -\beta'$. *Thus, the Proposition is proved in this case.*

Now, let consider the alternative case in which (7) is satisfied with equality, that is,

$$m^*.k^*.(k^* - 2) = \beta - \beta.k^* - k^* \qquad (8)$$

We have two possibilities:

1) If $k^* = 2$, *and we define* z^* *as the equilibrium level of z, then the previous equality implies* $\beta(z^*) = -2$. *If* $\beta(z) \neq -2\ \forall\ z \in \mathfrak{Z}$ *this case is impossible, so let us assume that* $\beta(\ )$ *is strictly monotonic. This implies that* $z^* = \beta^{-1}(-2)$. *Moreover, it must be the case that* $z^* < y$, *since otherwise* $p \leq c$, *which implies negative profits for any active firm. Also, symmetry and first order conditions in the third stage of G.2 implies that* $p(z^*) - c = -p'(z^*).z^*/n^* > 0$, *where* n^* *is the total number of divisions in equilibrium. Therefore,* $n^* = p'(z^*).z^*/[p(z^*) - c]$, *which is a finite number independent of* ε *since* $z^* = \beta^{-1}(2)$, *and the Proposition is proved in this case.*

2) If $k^* > 2$, *then from equation (8) and from the definition of* β', *we have*

$$m^*.k^*.(k^* - 2) = -\beta.(k^* - 1) - k^* \leq -\beta'.(k^* - 1) - k^*$$

Thus, in this case $n^* \leq \dfrac{-\beta'\ (k^* - 1) - k^*}{(k^* - 2)} \equiv F(k^*)$ *say. It is easy to show that sign* $dF(k^*)/dk^* = \text{sign}\ (\beta' + 2)$. *Then the maximum of* $F(k^*)$ *is achieved at the extrema, i.e. either at* $F(3) = -2\beta' - 3$ *or at* $F(\infty) = -\beta' - 1$. *Therefore* $n^* \leq \max\ (-2\beta' - 3), -\beta' - 1)$ *and the Proposition is proved.* ■

It may be worth pointing out that if the inverse demand function reads $p = A/x$ (and thus $\beta(z) = -2$ and $\beta(\ )$ is not invertible), it is possible to construct a counterexample to the above Proposition. This is due to the fact that in this case under no fixed costs and $k = 2$, any number of divisions is a SPNE (see Corchón (1991) p.2). Therefore, under a positive fixed cost, any number of divisions compatible with non negative profits is a SPNE. Thus, if the fixed cost tends to zero we can select a SPNE in which the number of active divisions becomes unbounded.

Proposition 4 shows that markets in which the above assumptions hold and in which there is a (possibly small) positive entry cost, are characterized by number of active divisions which is bounded above, i.e. by being natural oligopolies. This, of course, conflicts with the results obtained in the previous Section where it was shown that under zero entry cost if $k > \max(-\beta', 1)$ perfect competition is the unique outcome in any SPNE. At a first glance one would be tempted to say that Propositions 3-4 show that the result obtained in Section II is not robust since by introducing a small friction (i.e. a positive ε) this result disappears. However, in the next Section we will see that matters are more complex, since by introducing a second friction (a bound on the number of divisions which can be created) perfect competition might also be a possible SPNE outcome when frictions disappear.

IV. A MODEL WITH ENTRY COSTS AND AN UPPER BOUND ON THE NUMBER OF DIVISIONS

In this Section we will attempt to examine further the results obtained in Sections 2 and 3. In particular, we want to explain why for a very small entry cost, the SPNE outcomes of G.2 do not converge to those obtained in G.1. In order to do this we will maintain the structure of the game G.2 but we will introduce an upper bound on the number of divisions that can be created by each firm, denoted by d. The motivation for this assumption is that the creation of a new division by a firm takes time, and beyond a certain level, might be prohibitively costly. An alternative assumption, which would produce identical results, is that firms have to pay a cost for each created division.

Markets under consideration contain two kind of frictions, namely d and ε. Both frictions depend on a parameter $\rho \in [1,0]$ so we can write $d(\rho)$ and $\varepsilon(\rho)$. We will assume that $d' \leq 0$ and $\varepsilon' \geq 0$, i.e. the fixed cost (ε) is an increasing function of ρ and the upper bound to the number of divisions (d) is decreasing with the degree of friction. A frictionless market is one for which $\rho = 0$ (i.e. the one analyzed in game G.1). The purpose of the analysis will be to study the limit of SPNE in the game G.2 when $\rho \longrightarrow 0$.

In the rest of this Section, we will assume that for any value of ρ there exists a unique Cournot equilibrium. In relation to this Cournot equilibrium, we will define $\Pi(n)$ as the industry gross profits (i.e aggregate profits before subtracting the fixed costs), as a function of the total number of divisions. Also we will define $R(\rho,h)$ as follows,

$$R(\rho,h) = \frac{\Pi[h.d(\rho)]}{h.\varepsilon(\rho)}$$

where h is the number of firms entering the market. That is, $R(\rho,h)$ is the ratio (total industry gross profits)/(total industry fixed costs) for h firms and a given degree of friction, when every firm creates the maximum level of divisions. Also let $R_\rho \equiv \frac{\partial R}{\partial \rho}$ and $R_h \equiv \frac{\partial R}{\partial h}$. We obtain the following

Proposition 5: *Under A.1, A.2, A.3, A.4 and A.5, if* $sign\{R_\rho\}$ *is constant and* $R_h < 0$, *then*

a) *If* $\lim_{\rho \longrightarrow 0} R(\rho,h) < 1$ *for all* $h > max\{-\beta', 1\}$, *the only SPNE of the game G.2 when* ρ *is small enough implies* $k^* \leq max\{-\beta', 1\}$. *Moreover, under conditions a) or d) in Corollary 2,* $k^* = 1$ *and* $m^* = 1$.

b) *If* $\lim_{\rho \longrightarrow 0} R(\rho,h) > 1$ *for some* $h > max\{-\beta', 1\}$, *when* ρ *is small enough the only SPNE of the game G.2 implies* $k^* > max\{-\beta', 1\}$ *and* $m^* = d$.

Proof: *By using a similar argument as in Proposition 1, if* $h > max\{-\beta', 1\}$, *then SPNE of G.1 implies* $m^* = d$. *Now, let us consider parts (a) and (b):*

(a) The assumption made in this part and the monotonicity of $R(\rho,h)$ *with respect to* ρ, *implies that profits will be negative, for small enough* ρ, *if* $h > max\{-\beta', 1\}$. *Thus, SPNE of G.2 is inconsistent with* $k^* > max\{-\beta', 1\}$. *The rest of this part is proved in the same way as in Corollary 2.*

(b) Under our assumptions on $R(h,\rho)$, *in this case, if* ρ *is small enough, profits are positive for some* $h > max\{-\beta', 1\}$. *Also, the monotonicity of* $R(\rho,h)$ *with respect to h and* ρ, *ensures that* $k^* > max\{-\beta', 1\}$ *for small enough* ρ, *and this completes the proof.*■

Notice that if for all ρ in the sequence $d(\rho) = 1$, part b) in Proposition 4 is the standard limit theorem with quantity-setter firms. In contrast this Theorem, the main conclusion of Proposition 4 is that when frictions are removed, i.e. when $d \longrightarrow \infty$ and $\varepsilon \longrightarrow 0$, the outcome of a SPNE may be either very close to perfect competition (as in game G.1) or may be a natural oligopoly (as in game G.2). In this sense, matters here are quite different to standard limit theorems in which perfect competition is, under some regularity conditions, the unique limit outcome. Therefore an important lesson is that the possibility of divisionalization introduces subtle complications in such a way that almost frictionless worlds yield outcomes which might be far away from those perfectly competitive ones. Thus the SPNE outcomes of both G.1 and G.2 are perfectly reasonable in frictionless economies.

For instance, as an illustration, consider that both d and ε depend on the degree of complexity of the organization, i.e. a simple organization is one with a low set up cost and a very large reproductive power and conversely a complex organization is one with a high fixed cost and a low d. Then, Proposition 4 asserts that depending on the interplay between d and ε the limiting market structure of a sequence of organizations in which complexity tends to vanish might be perfect competition or duopoly or even monopoly. In order to obtain a more precise statement, the analyst needs very precise information about how these frictions vanish (see Güth & Ritzberger (1992) and Corchón & Ritzberger (1992) for a similar phenomenon in the context of non-cooperative models of bargaining). Therefore the convergence to perfect competition or to a natural oligopoly when divisionalization is available depends on how frictions vanish, and this also contrasts with the predictions of the standard Cournot model.

V. DIVISIONALIZATION UNDER INCOMPLETE INFORMATION

In previous Sections, we have assumed that firms possess complete information about costs and demand, so they know exactly how much profit isobtained by each division. However, if firms have incomplete information on demand or costs so that profits can not be monitored, they have to establish a payment scheme in which divisions pay according to an observable variable. In this Section we will deal with this problem by assuming the following structure of information. On the one hand firms have some probabilistic information on inverse demand and cost functions and they can only observe either, the quantities sold or revenues accrued by its divisions. On the other hand divisions are supposed to have complete information[6]. We will show that in the game G1 the qualitative feature of the result obtained in Section II holds in this framework, that is if the number of firms is larger than some finite number, then perfect competition is the unique SPNE. It is left as an exercise to prove that the flavor of the results obtained in Section III is also preserved in this framework.

Let us describe the informational framework. A state of the world, denoted by θ, is a pair (p( ), c), i.e. a specification of the inverse demand and the cost function. The set of states of the world, which by simplicity is assumed to be of finite cardinality, is denoted by Θ. Firms have common priors defined on Θ and denoted by ρ_θ. They are supposed to be expected profits maximizers. Under conditions stated in Propositions 6 and 7 below the equilibrium in the second stage (i.e. the quantity game among divisions) will be unique. So let us denote by $z(\theta, n)$, $x_i(\theta, n)$, etc the equilibrium values of z, x_i, etc in the second stage of the game when there are n firms and the state of the world is θ. Also let $\beta'' \equiv \min \beta'(\theta)$, $\theta \in \Theta$.

First, let us consider the case in which divisions pay in proportion to the output sold by them. Thus profits of, say, firm j, in state of the world θ

---

(6) It may be worth pointing out that both cases are identical to the problem of finding the optimal number of franchisees for an oligopolistic firm.

when there are n divisions are given by $\Pi_j = vmx_i(\theta, n)$, where v is an exogenous constant and m is the number of divisions created by firm j. Thus expected profits for firm j are given by $E_j = \sum_{\theta\in\Theta} p_\theta \; vmx_i(\theta, n) \equiv E_j(m, n)$. In this framework a symmetric SPNE (SSPNE) is a pair (m^*, n^*) such that $n^* = k.m^*$ and for all j $E_j(m^*, n^*) \geq E_j(m, m + (k - 1) m^*)$, i.e. m^* maximizes the expected payoff of j. Then we have our first result in this Section.

Proposition 6: *Under A.1, A.2, A.3, A.4, A.5 and the informational structure explained above if $k > \max(-\beta'' - 1, 1)$ Perfect Competition is the unique SSPNE of game G1 when divisions pay to the firm a fraction of its output.*

Proof: *First notice that profits of a typical division, say i, are given by*

$$\Pi_i = px_i - (c + v)x_i$$

Thus by interpreting c + v as a marginal cost, all results obtained in the Cournot model go thru in this case. In particular, since $k > \max(-\beta'' - 1, 1) \geq \max(-\beta' - 1, 1)$ equilibrium in the second stage is unique (since the first order condition of profit maximization is strictly decreasing on z) and symmetric (see Appendix II). Moreover z' and x_i' are identical to those found in the proof of Proposition 1.

Profits of firm j in state of the world θ $\Pi_j = mx_i(\theta, n)$. Thus the effect on profits in state θ of a change in m will be proportional to $x_i + m\, x_i'$ (we will omit the dependence of z, etc on the states of the world when this is clear from the context). Thus using symmetry ($t = (k - 1)m$) we obtain that

$$\text{sign}\,\frac{d\Pi_j}{dm} = \text{sign}\left(\frac{n(n + \beta + 1) - m(\beta + n)}{n + 1 + \beta}\right)$$

But $k > \max(-\beta'' - 1, 1) \geq \max(-\beta'(\theta) - 1, 1) \geq -\beta - 1$, so $n + 1 + \beta \geq k + 1 + \beta > 0$ and thus an increase in m increases profits in θ. Since this is true for every state of the world expected profits are strictly increasing on m and then any SPNE of the game G.1 implies an infinite number of divisions.■

It should be remarked that the condition $k > \max(-\beta' - 1, 1)$ is weaker than the condition used in the complete information case. In particular, if the set of states of the world includes only those with inverse demand and cost function fulfilling the conditions stated in Proposition 2 above, $k > \max(-\beta'' - 1, 1)$ holds.

Next, let us consider the case in which the payment made by each division is a fraction of sales[7]. Thus, profits of division i in state of the world $\theta = (p(\ ), c)$ are given by $p(z)x_i(1 - q) - cx_i$, where $0 < q < 1$. Expected profits for firm j are given by $E_j = \sum_{\theta\in\Theta} \rho_\theta\, qmp(z(\theta, n))x_i(\theta, n)$.

Equilibrium in the second stage of the game must be understood as a Nash equilibrium in quantities among divisions when payoff functions are as defined above. Again under our conditions this equilibrium will be shown to be unique. A SPNE is defined similarly to the previous case.

Let $e(\theta, z) \equiv p'(z)\, z / p(z)$ be the elasticity of demand valued at z if the state of the world is θ. Let $e'(\theta) = \min e(z, \theta)$, $z \in \mathfrak{Z}$. An argument similar to the one used in Lemma 1 shows that under assumptions 1-4 $e'(\theta)$ exists. Also let $e'' \equiv \min e'(\theta)$, $\theta \in \Theta$. Then we have the following result:

Proposition 7: *Under A.1, A.2, A.3, A.4, A.5 and the informational structure explained above if $k > \max(-\beta'', -e'', 1)$ Perfect Competition is the unique SSPNE of game G1 when divisions pay the firm a fraction of their sales.*
Proof: *The first order condition of equilibrium in the second stage is*

$$p(x) - \left(\frac{c}{1 - q}\right) + p'x_i = 0$$

Thus interpreting $c/(1 - q)$ as the marginal cost, all results obtained in the Cournot model go thru in this case. In particular since

---

(7) It is easy to show that given v, there is a payment based on sales which raises as much money as the one based on output. Thus if the market price is observable by the firm, the former is preferable.

$k > \max(-\beta'', -e'', 1) > \max(-\beta', 1)$ equilibrium in the second stage is unique (as we noticed at the end of Section II) and symmetric (see Appendix II). Also, the effects on x_j and z of changes in m are the same as in the previous model. In our case profits of firm j in state θ when there are n divisions are given by

$$\Pi_j(\theta, n) = qp(z(\theta, n))x_j(\theta, n)$$

And thus

$$\frac{d\Pi_j(\theta, n)}{dm} = q(p'(\frac{dz}{dm})x_j + p(x)(\frac{dx_j}{dm}))$$

By substituting the values of $\frac{dx_j}{dm}$ and $\frac{dz}{dm}$ in the expression above and taking into account that $x_j = zm/n$ and $t = (k - 1)m$ we obtain that

$$\frac{d\Pi_j(\theta, n)}{dm} = q(\frac{px_im}{n^2}[(\frac{1 + e}{n + 1 + \beta}) + (k - 1)]).$$

Since $n + \beta + 1 > 0$, the sign of the above derivative equals the sign of $k + e + (k - 1)(n + \beta)$ and this sign is positive if $k > \max(-\beta'', -e'', 1)$. Then a similar argument to the one used in the proof of Proposition 6 finishes the proof.■

Thus under incomplete information SPNE yields perfect competition under conditions similar to those used in the complete information case.

We end this Section by noticing that the results in this Section can be easily shown to hold in a complete information setting in which firms, forstrategic reasons, decide to raise money, not in proportion to profits, but in proportion to output or sales.

VI. SUMMARY AND CONCLUSIONS

In this paper we have shown that if the number of firms is large enough in relation to the degree of convexity of the inverse demand function and if there are constant returns to scale, the possibility of divisionalization implies that every SPNE yields perfect competition (see Proposition 1). In many cases, a small number of firms is enough to obtain perfect competition (see Proposition 2). These results generalize those obtained in Corchón (1991) (see Corollary 1).

However, if there is a positive fixed cost, only a small number of firms and divisions will be active in the market (see Propositions 3-4). In some cases, duopoly or even monopoly arises as SPNE irrespectively of the magnitude of the fixed cost (see Corollary 2). Also, it is important to remark that, in our model, the equilibrium number of independent sellers depends on the shape of the inverse demand function and not on the magnitude of fixed costs, as happens in the standard approach. Another important implication of our results is that, in the context of entry deterrence, the *possibility* of divisionalization might not be desirable from the social welfare point of view, and it may depend on the shape of the inverse demand function.

In Section IV we have considered a model in which there is an upper bound on the number of divisions that can be created. We have seen that when this upper bound tends to infinity and the fixed cost tends to zero the outcome of a SPNE might be characterized by (almost) perfect competition or a small number of active firms (sometimes monopoly). This shows that, when divisionalization is possible, the limiting outcome of almost frictionless markets depends on the rate at which frictions vanish, and therefore convergence to perfect competition is quite a delicate matter.

Finally in Section V we have shown that our basic insights survive in the consideration of asymmetric information between firms and divisions.

Summing up, it seems to us that models in which the possibility of divisionalization is explicitly modeled, bring fresh insights into oligopoly theory, in particular to questions like entry deterrence or conditions under which markets (large or not) are characterized by monopoly, duopoly or perfect competition. In this paper, we have studied divisionalization in the simple setting of a homogeneous product, even though, as we have remarked before, our results still hold in models of horizontal differentiation (i.e. Spence and Dixit & Stiglitz or Salop). It would be interesting to study divisionalization in models of vertical differentiation or in models in which horizontal and vertical differentiation occur simultaneously (see Faulí i Oller (1993) for a model of divisionalization with complementarities)[8]. In a broader context divisionalization means that agents control the number of their descendants. It would be nice to consider evolutive models in which this feature is incorporated.

---

(8) It would be very interesting to study acquisition games (see Kamien and Zang ((1990), (1991)) when divisionalization is available. We conjecture that in this case complete monopolization through acquisition is always possible.

A P P E N D I X I

In this Appendix, we extend the results obtained in Proposition 1 and 2 to the case in which the number of divisions is an integer.

Proposition 1': *Under A.1, A.2, A.3 and A.4, if* $-\beta' \le \dfrac{(k-2)(k+1)}{k-1}$ *Perfect Competition is the only SPNE of the game G.1.*

Proof: *Let* $\frac{\partial \Pi j}{\partial m}((k-1)m, m+i)$ *be the partial derivative of profits of firm j with respect to the number of divisions, evaluated at a point in which all other firms have (k - 1)m divisions and firm j has m + i divisions. We will show that under the assumptions above, this derivative is positive* $\forall\ i \in [0, 1]$*. This implies that profits of firm j are strictly increasing in this interval. Thus, given the number of divisions built by other firms, firm j has an incentive to built, at least, m + 1 firms. Thus, at any candidate equilibrium with finite m (which must be symmetric), firm j has incentive to deviate.*

Reasoning in the same way as in the proof of Proposition 1 we have that condition $\frac{\partial \Pi j}{\partial m}((k-1)m, m+i) > 0$ *implies that inequallity (3'') holds with m substituted by m + i, so that* $\beta + n + i + 1 > (m+i)\{(2 + \beta/(n+i)\}$*. If the Proposition were false then the previous inequality, the definition of β' and symmetry would imply* $\{m(k-2) + (1-i)\}(km+i)/m(k-1) \le -\beta'$*. Since the left hand side of this inequality is non-decreasing on m and m ≥ 1, then* $\{(k-2) + (1-i)\}(k+i)/(k-1) \le -\beta'$*. Again the left hand side of this inequality is decreasing on i so*

$$\{(k-2) + (1-1)\}(k+1)/(k-1) < (k-1-i))(k+i)/(k-1) \le$$
$$-\beta' \le (k-2)(k+1)/(k-1).$$

Therefore we arrive at a contradiction, and the proof is completed ■

Notice that the function $F(k) \equiv \dfrac{(k-2)(k+1)}{k-1}$ is increasing in k if k ≥ 2. Moreover, F(k) tends to infinity when k tends to infinity. That is, perfect competition is ensured if k is large enough in relation to the degree of convexity of the inverse demand function.

Proposition 2': *Under A.1, A.2, A.3 and A.4, any of the following conditions imply* $-\beta \leq \dfrac{(k-2)(k+1)}{k-1}$

a) *$p(z)$ concave for all $z \in \mathfrak{Z}$.*

b) *$p(z)$ having constant elasticity -denoted by ε- and $1 - \varepsilon \leq \dfrac{(k-2)(k+1)}{k-1}$*

c) *$\Pi(z)$ concave for all $z \in \mathfrak{Z}$ and $k > 3$.*

Proof: *Identical to the proof of Proposition 2, noticing that $F(2) = 0$ and $F(3) = 2$* ■

APPENDIX II

In this Appendix we will show that if equilibrium in the second stage is not perfectly competitive, SPNE of game G.1 is symmetrical. We will first show that given any number of divisions, the equilibrium in the second stage of G.1 is symmetrical. Indeed suppose it is not, and that there are two divisions, say, i and r such that their outputs are different. Without loss of generality let us assume that division i is active. Then

$$p'.x_i + p - c = o \quad \text{and} \quad p'.x_r + p - c \leq o$$

It is readily seen that if division r is not active $p - c \leq o$ which is impossible since $p' < 0$ and division i is active. Therefore both divisions are active and first order conditions hold with equality. Then $p'.x_i = p'.x_r$ and thus $x_i = x_r > 0$ (by A. 4). Now we will show that SPNE in the first stage is also symmetrical. If SPNE is not perfectly competitive we have that equation (1) in the main text must be non positive for all $j = 1,\dots, k$, that is,

$$\frac{\partial \Pi_j}{\partial m} = (p - c).\{ x_i - m.[\frac{dz}{dn} - \frac{dx_i}{dm}] \} \leq 0$$

Moreover, since $p > c$, all the firms will be active, which means that each one will set at least one firm. Now we have two possibilities. First, if the above condition is an strict inequality, $m^* = 1$ and the symmetry holds. Second, according to the calculations given in the proof of Proposition 1 if the previous condition holds with equality it is equivalent to

$$x_i = m. \frac{x_i}{n(\beta + n + 1)} .(\beta + 2n)$$

where x_i is the (symmetrical) output in the second stage. Given that this output is positive the above expression is simplified to

$$m = / \quad (\beta + n + 1) / (\frac{\beta}{n} + 2)$$

Since the right hand side of this expression is the same for every firm, symmetry in the first stage must hold.

R E F E R E N C E S

CORCHON, L.C., (1991). "Oligopolistic Competition Among Groups". *Economics Letters 36, 1-3.*

CORCHON, L.C. & MARCOS, F., (1988). "Entry, Stackelberg Equilibrium and Reasonable Conjectures". *International Journal of Industrial Organization 6.*

CORCHON, L.C. & RITZBERGER, K. (1992). "On the Non-Cooperative Foundations of Cooperative Bargaining". WP-AD 92-06. A Discusión, Universidad de Alicante.

DIXIT, A. & STIGLITZ, J. (1977). "Monopolistic Competition and Optimum Product Diversity". *American Economic Review, June, pp. 297-308.*

FAULI I OLLER, R. (1993)."Competition and Cooperation within a Multidivisional Firm". Mimeo. Universitat Pompeu i Fabra.

GONZALEZ-MAESTRE, M. (1992). "Divisionalization in Markets with Heterogeneous Goods". Mimeo, University of Alicante.

GÜTH, W. & RITZBERGER, K (1992). "On Durable Monopolies and the (Anti-) Coase Conjecture". CenTER Discussion Paper No. 9215.

KAMIEN, M.I. & ZANG I. (1990). "The Limits of Monopolization Through Acquisition". *Quarterly Journal of Economics May, pp. 465-499.*

KAMIEN, M.I. & ZANG I. (1991). "Competitively Cost Advantageous Mergers and Monopolization". *Games and Economic Behavior 3, pp. 323-338.*

MILGROM, P & ROBERTS, J. (1992). *Economics, Organization and Management.* Prentice Hall Inc.

NOVSHEK, W. (1980). "Cournot Equilibrium with Free Entry". *Review of Economic Studies XLVII, pp. 473-486.*

OMORI, T. & YARROW, G. (1982). "Product Diversification, Entry Prevention and Limit Pricing". *The Bell Journal of Economics pp. 242-248.*

SCHWARTZ, M., & THOMPSON, E.A., (1986). "Divisionalization and Entry Deterrence". *Quarterly Journal of Economics, CI, pp. 307-321.*

SHAKED, A. & SUTTON, J. (1983). "Natural Oligopolies". *Econometrica, 51, pp. 1469-1483.*

SPENCE, M. (1976). "Product Selection, Fixed Costs and Monopolistic Competition". *Review of Economic Studies, vol. XLIII, pp. 217-235.*

SUTTON, J. (1991). *Sunk Costs and Market Structure.* MIT Press.

VIVES, X. (1988). "Sequential Entry, Industry Structure and Welfare". *European Economic Review 32, pp. 1671-1687.*

YARROW, G. (1985). "Welfare Losses in Oligopoly and Monopolistic Competition" *The Journal of Industrial Economics, vol. XXXIII, pp. 515-529.*

PUBLISHED ISSUES

WP-AD 90-01 "Vector Mappings with Diagonal Images"
C. Herrero, A.Villar. December 1990.

WP-AD 90-02 "Langrangean Conditions for General Optimization Problems with Applications to Consumer Problems"
J.M. Gutierrez, C. Herrero. December 1990.

WP-AD 90-03 "Doubly Implementing the Ratio Correspondence with a 'Natural' Mechanism"
L.C. Corchón, S. Wilkie. December 1990.

WP-AD 90-04 "Monopoly Experimentation"
L. Samuelson, L.S. Mirman, A. Urbano. December 1990.

WP-AD 90-05 "Monopolistic Competition: Equilibrium and Optimality"
L.C. Corchón. December 1990.

WP-AD 91-01 "A Characterization of Acyclic Preferences on Countable Sets"
C. Herrero, B. Subiza. May 1991.

WP-AD 91-02 "First-Best, Second-Best and Principal-Agent Problems"
J. Lopez-Cuñat, J.A. Silva. May 1991.

WP-AD 91-03 "Market Equilibrium with Nonconvex Technologies"
A. Villar. May 1991.

WP-AD 91-04 "A Note on Tax Evasion"
L.C. Corchón. June 1991.

WP-AD 91-05 "Oligopolistic Competition Among Groups"
L.C. Corchón. June 1991.

WP-AD 91-06 "Mixed Pricing in Oligopoly with Consumer Switching Costs"
A.J. Padilla. June 1991.

WP-AD 91-07 "Duopoly Experimentation: Cournot and Bertrand Competition"
M.D. Alepuz, A. Urbano. December 1991.

WP-AD 91-08 "Competition and Culture in the Evolution of Economic Behavior: A Simple Example"
F. Vega-Redondo. December 1991.

WP-AD 91-09 "Fixed Price and Quality Signals"
L.C. Corchón. December 1991.

WP-AD 91-10 "Technological Change and Market Structure: An Evolutionary Approach"
F. Vega-Redondo. December 1991.

WP-AD 91-11 "A 'Classical' General Equilibrium Model"
A. Villar. December 1991.

WP-AD 91-12 "Robust Implementation under Alternative Information Structures"
L.C. Corchón, I. Ortuño. December 1991.

WP-AD 92-01 "Inspections in Models of Adverse Selection"
I. Ortuño. May 1992.

WP-AD 92-02 "A Note on the Equal-Loss Principle for Bargaining Problems"
C. Herrero, M.C. Marco. May 1992.

WP-AD 92-03 "Numerical Representation of Partial Orderings"
C. Herrero, B. Subiza. July 1992.

WP-AD 92-04 "Differentiability of the Value Function in Stochastic Models"
A.M. Gallego. July 1992.

WP-AD 92-05 "Individually Rational Equal Loss Principle for Bargaining Problems"
C. Herrero, M.C. Marco. November 1992.

WP-AD 92-06 "On the Non-Cooperative Foundations of Cooperative Bargaining"
L.C. Corchón, K. Ritzberger. November 1992.

WP-AD 92-07 "Maximal Elements of Non Necessarily Acyclic Binary Relations"
J.E. Peris, B. Subiza. December 1992.

WP-AD 92-08 "Non-Bayesian Learning Under Imprecise Perceptions"
F. Vega-Redondo. December 1992.

WP-AD 92-09 "Distribution of Income and Aggregation of Demand"
F. Marhuenda. December 1992.

WP-AD 92-10 "Multilevel Evolution in Games"
J. Canals, F. Vega-Redondo. December 1992.

WP-AD 93-01 "Introspection and Equilibrium Selection in 2x2 Matrix Games"
G. Olcina, A. Urbano. May 1993.

WP-AD 93-02 "Credible Implementation"
B. Chakravorti, L. Corchón, S. Wilkie. May 1993.

WP-AD 93-03 "A Characterization of the Extended Claim-Egalitarian Solution"
M.C. Marco. May 1993.

WP-AD 93-04 "Industrial Dynamics, Path-Dependence and Technological Change"
F. Vega-Redondo. July 1993.

WP-AD 93-05 "Shaping Long-Run Expectations in Problems of Coordination"
F. Vega-Redondo. July 1993.

WP-AD 93-06 "On the Generic Impossibility of Truthful Behavior: A Simple Approach"
C. Beviá, L.C. Corchón. July 1993.

WP-AD 93-07 "Cournot Oligopoly with 'Almost' Identical Convex Costs"
N.S. Kukushkin. July 1993.

WP-AD 93-08 "Comparative Statics for Market Games: The Strong Concavity Case"
L.C. Corchón. July 1993.

WP-AD 93-09 "Numerical Representation of Acyclic Preferences"
B. Subiza. October 1993.

WP-AD 93-10 "Dual Approaches to Utility"
M. Browning. October 1993.

WP-AD 93-11 "On the Evolution of Cooperation in General Games of Common Interest"
F. Vega-Redondo. December 1993.

WP-AD 93-12 "Divisionalization in Markets with Heterogeneous Goods"
M. González-Maestre. December 1993.

WP-AD 93-13 "Endogenous Reference Points and the Adjusted Proportional Solution for Bargaining Problems with Claims"
C. Herrero. December 1993.

WP-AD 94-01 "Equal Split Guarantee Solution in Economies with Indivisible Goods Consistency and Population Monotonicity"
C. Beviá. March 1994.

WP-AD 94-02 "Expectations, Drift and Volatility in Evolutionary Games"
F. Vega-Redondo. March 1994.

WP-AD 94-03 "Expectations, Institutions and Growth"
F. Vega-Redondo. March 1994.

WP-AD 94-04 "A Demand Function for Pseudotransitive Preferences"
J.E. Peris, B. Subiza. March 1994.

WP-AD 94-05 "Fair Allocation in a General Model with Indivisible Goods"
C. Beviá. May 1994.

WP-AD 94-06 "Honesty Versus Progressiveness in Income Tax Enforcement Problems"
F. Marhuenda, I. Ortuño-Ortín. May 1994.

WP-AD 94-07 "Existence and Efficiency of Equilibrium in Economies with Increasing Returns to Scale: An Exposition"
A. Villar. May 1994.

WP-AD 94-08 "Stability of Mixed Equilibria in Interactions Between Two Populations"
A. Vasin. May 1994.

WP-AD 94-09 "Imperfectly Competitive Markets, Trade Unions and Inflation: Do Imperfectly Competitive Markets Transmit More Inflation Than Perfectly Competitive Ones? A Theoretical Appraisal"
L. Corchón. June 1994.

WP-AD 94-10 "On the Competitive Effects of Divisionalization"
L. Corchón, M. González-Maestre. June 1994.

WP-AD 94-11 "Efficient Solutions for Bargaining Problems with Claims"
M.C. Marco-Gil. June 1994.