

**HONESTY VERSUS PROGRESSIVENESS  
IN INCOME TAX ENFORCEMENT PROBLEMS\***

**Francisco Marhuenda and Ignacio Ortuño-Ortín\*\***

WP-AD 94-06

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\* We would like to thank Parkash Chander, Luis Corchón, Carmen Herrero, José A. Silva and Fernando Vega-Redondo for helpful suggestions and discussion. We acknowledge financial assistance from the CICYT (Spanish Ministry of Education), Project No. PS90-0156 and PB 91-0756.

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**Editor: Instituto Valenciano de  
Investigaciones Económicas, S.A.**  
Primera Edición Mayo 1994.  
ISBN: 84-482-0544-8  
Depósito Legal: V-1477-1994  
Impreso por Copisteria Sanchis, S.L.,  
Quart, 121-bajo, 46008-Valencia.  
Printed in Spain.

# HONESTY VERSUS PROGRESSIVENESS IN INCOME TAX ENFORCEMENT PROBLEMS

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## ABSTRACT

We study an income tax enforcement problem using a principal agent model where the government sets the tax and inspection functions. These functions are announced to the agents and there is no commitment problem. The penalty function for dishonest taxpayers is given exogenously and must satisfy certain social norms. We give a partial characterization of the optimal policy and prove that for a large family of penalty functions this policy is such that honesty implies regressiveness. It is also shown that the result does not depend on the fact that agents know the true probability of inspection.

**KEYWORDS:** Principal agent, Tax Enforcement, Progressiveness.



# 1 INTRODUCTION

It is well accepted by now that the existence of private information by the agents in an economy sets important constraints in the kind of policies that a government can achieve. An example that has been widely studied is the optimal income taxation problem when agents can differ in their abilities to transform labor or effort in income. It is supposed that the government cannot observe that ability and in this case the tax paid by each agent is only a function of the income he obtains. This asymmetry of information would reduce the possibilities of getting, for example, certain egalitarian distributions of income. Thus, the effectiveness of the government policies decreases as compared with the ideal case where these information problems are not considered.

Recent works on the optimal income taxation area have studied the case in which the government does not have a precise knowledge of the income of any specific agent but has information about the distribution of income in the population (e.g. Reinganum and Wilde [5], Border and Sobel [1], Mookherjee and Png [4]). In this case the tax paid by an agent depends on his own income level report. The problem faced by the government is to find the optimal tax schedule, taking into account the enforcement costs associated with it. It is supposed that agents can be audited to find out their true income levels. Each of these audits is costly and perfect. In general, only a fraction of the agents will be audited and this creates the possibility of tax evasion. To induce agents to report their incomes honestly will require a penalty for the ones who are found to be dishonest. The enforcement problem becomes trivial if there are no bounds on the admissible penalties: the government would audit all agents with a very small probability and dishonest agents would be penalized with an extremely high penalty. This

scheme would approximate the complete information case. To avoid this kind of situation it is commonly assumed that there is a set of admissible penalties which are not too severe.

Thus, the tools of the government are a tax scheme, the probability of auditing each agent (which would depend on his report) and penalties for dishonest taxpayers. However, defining which is the set of admissible penalties is a delicate task since the results might depend very critically on that set. In a recent paper, P. Chander and L. Wilde ([2]) analyze a model in which the objective of the government is to collect a given revenue in an efficient way. They consider four cases each of them associated to a different kind of restriction on the set of admissible penalties (in two of these cases they also allow for the existence of bounded rewards). None of these penalties is too severe in the sense that the penalty imposed to a dishonest agent cannot be greater than his income. They find that in all the cases, and assuming that the government and the agents are risk neutral, the optimal tax scheme is a concave and nondecreasing function<sup>1</sup>. This result can be seen as another example of how the existence of asymmetric information between the government and the agents sets important restrictions on the nature of efficient policies: a progressive tax scheme (convex) is an inefficient tool to collect revenues from agents with private information about their incomes.

We regard this result as a very important one on the nature of the optimal tax policy. Given the “negative” character of it, a natural question arises: how general is it? or, put in another way, is this result robust to different specifications on the way penalties are chosen? The first model considered in [2] studies the optimal tax policy when a fixed penalty function is

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<sup>1</sup>For some of the cases, they prove that the “payment function”, and not the tax function, is concave. The reason of this distinction between tax and payment function is due to the fact that the revelation principle cannot be proven in these cases and then the optimal policy does not need to be truth-telling. This requires to analyze what agents in fact pay rather than the “nominal” tax

exogenously given to the government (principal), i.e., the problem of the government in this case is to find the optimal tax and inspection functions when the amounts paid by dishonest agents, if they are discovered, are determined by a given penalty function. The specific penalty function they consider is such that "...agents who underreport by a greater amount suffer a greater penalty." The other models analyzed are more general in the sense that the penalty function is not exogenously given. In these cases, the government has to find the optimal tax, inspection **and penalty** functions. Nevertheless, as it was said before, there are still certain bounds on these penalties, namely, no agent can be requested to paid an amount greater than his true income.

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One possible objection to the first case studied in [2], (when the penalty function is exogenously given to the government) is that they only consider **one** very specific penalty function. Moreover, that function is not always a penalty, i.e., it allows for cases in which an agent who is found to be dishonest ends up paying **less** taxes than an honest agent with the same initial income. It can be shown that this can only happen when the tax function is very convex, but in the model there is not any restriction on the admissible tax function besides the constrain that the tax cannot exceed income.

The rest of cases analyzed in [2] (when the penalty function is endogenously determined) may be seen as more general since they hold not for just one specific penalty function but for a large set of penalties. However, this point of view about the generality of their model is not completely clear. It is not difficult to see, as the authors show, that if the government is free to choose penalties under the only restriction that they cannot be greater than

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<sup>2</sup>It is worth noting that other authors (e.g. Sanchez and Sobel [6], Cremer, Marchand and Pestieau [3]) have already given a characterization of the optimal policy for very similar models in which a specific penalty function is given to the government. The problem, however, is that, as Chander and Wilde state, "they are subject to the criticism that the penalty could be more than the true income of a tax payer."

the true income, then the most severe one will always be chosen. Thus, in all these cases the function called the “draconian” penalty function in [2] will be adopted: whenever an agent is found to be dishonest he will have to pay his total income.

Observe that this function is such that the penalty assigned to a dishonest agent does not depend on the amount misreported. We believe that in most economies there exist social norms which are against this kind of draconian penalty function and this is not just because it is too “severe”<sup>3</sup>. More important, such penalty is not “continuous” and therefore, it is not in agreement with the social norm that “the punishment should fit the crime”.

Thus, our analysis will try to answer whether similar results to the ones in [2] hold for more **general** and **realistic** situations. In particular, we will characterize (partially) the optimal tax scheme when the penalty function is exogenously given to the principal but such function is drawn from a large admissible set. Moreover, all the functions in this set must satisfy three conditions or social norms that we believe are natural in most societies, namely, (i) there is a maximum penalty which is given by the true income of the agent, (ii) if a dishonest agent is audited he should pay at least the same amount as an honest agent of the same income level, i.e., the penalty function is really a “penalty”, and (iii), as said before, the punishment should fit the crime. (In [2] only condition (i) is required).

The main technical difficulty founded in our approach is that we cannot prove the revelation principle and in this case a general analysis becomes quite intractable. Thus, we concentrate our study in the characterization of the optimal policy among the ones which are truth-telling. In this sense

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<sup>3</sup>It is important to notice that their qualitative results do not depend on the fact that the maximum penalty is the true income of the agent, i.e., nothing essential would change if a different bound, for example a given percentage of the income, were adopted instead.

our results are weaker than the ones in [2]. However, if we believe that a desideratum of the society is to design policies that induce honest behavior, then the conclusion is also surprising: **an efficient policy cannot be both honest and progressive**, or to be more precise, if the optimal policy is truth-telling then the tax function must be such that the average tax is a decreasing function of the income.

This result is very much in the spirit of [2] and can be seen as an extension of their negative results for cases that we believe are more realistic. Some ideas used to prove the main results are also inspired in the proofs provided by them.

The approach taken here to study tax enforcement problems may be criticized on the grounds that in reality tax payers do not know with enough accuracy the probability of auditing each income level (or perhaps that the government never wants to communicate this probability). We show that, surprisingly, our main result still holds in models where agents have beliefs about the auditing probabilities which do not need to be correct: if the optimal policy induces honesty (and the government knows it) then it must be regressive (decreasing average tax). This result gives more robustness to our main conclusion, the tradeoff between progressiveness and honesty when the income level is private information of the agents.

## 2 THE MODEL

We consider a population of taxpayers (agents) whose income is distributed along an interval. Their distribution of income is modeled by a measure (which may be atomic or continuous) on that interval. The government (principal) collects taxes from the agents, according to their income, through

the following mechanism:

Taxpayers send to the government information about their income. The principal chooses which reports to inspect randomly. We assume that if the government carries an inspection the true income of the taxpayer inspected is revealed.

If a taxpayer is not inspected, or it turns out that she did report her own income the government collects from her the amount imposed by the tax scheme.

If a taxpayer is inspected and found guilty of misreporting she is imposed a punishment, given by the law, in addition to her own taxes.

**Assumption 2.1** *The distribution of agents is represented by a measure space  $([0, M], \mu)$ , where  $y \in [0, M] \subset \mathbb{R}$  represents income.*

We will identify the type of an agent with her income. Thus, we shall say agent  $y \in [0, M]$  to mean an agent with income  $y$ .

We do not make any further assumptions on  $\mu$  except that the functions considered are  $\mu$  – integrable. In particular,  $\mu$  could be non-atomic (continuous case) or a discrete measure, in which case it represents a discrete number of agents.

**Assumption 2.2** *Taxpayers observe their income costlessly. The principal knows  $\mu$  but can detect misreporting only if it inspects taxpayers at a cost per inspection of  $c > 0$ . The government does not have any more information about the agents.*

**Assumption 2.3** *The tax function  $t : [0, M] \rightarrow \mathbb{R}$  satisfies  $0 \leq t(x) \leq x$ .*

**Assumption 2.4** *Taxpayer  $y$  reports a certain income  $x \in [0, M]$  to the government. If she is not inspected she pays to the principal an amount  $t(x)$ . If she is inspected she must pay the amount*

$$f(x, y) = t(y) + \phi_t(x, y)$$

where  $\phi_t(x, y)$  is the punishment imposed to agent  $y$  for declaring herself as being of type  $x$ . Note that we are only fixing a functional form of  $f$ , i.e., the value of  $f(x, y)$  might depend on the particular tax function  $t$  as well as on  $x$  and  $y$ . To emphasize this dependency we write  $t$  explicitly, as a subscript in  $\phi$ .

**Assumption 2.5** *The function  $\phi_t(x, y)$  satisfies the following properties:*

- (i)  $\phi_t(x, y) \geq 0$ .
- (ii)  $f(x, y) = t(y) + \phi_t(x, y) \leq y$ .
- (iii) For  $x$  fixed,  $\phi_t(x, y)$  is increasing in  $y$ .
- (iv) For  $y$  fixed,  $\phi_t(x, y)$  is decreasing in  $x$ .
- (v)  $\phi_t(y, y) = 0$ .

Thus, there is no reward or punishment for declaring true own type. Parts (iii) and (iv) may appear unnatural if the tax function  $t(x)$  is decreasing. However, in the next section we will show that the optimal tax function must be nondecreasing, even for penalty functions which do not necessarily satisfy parts (iii) and (iv).

Also it is worth noting that (i) implies that  $f$  is *always* a penalty, i.e., the amount paid by a dishonest agent upon inspection is at least the same as she would have paid when reporting truthfully.

**Assumption 2.6** *The principal carries an inspection according to a certain probability function  $p(x) : [0, M] \rightarrow [0, 1]$ . It is supposed that the government can credibly commit to this policy.*

Thus, given  $u = (t, p, f)$ , if agent  $y$  reports income  $x$ , her expected taxes will be

$$s_u(x, y) = (1 - p(x))t(x) + p(x)f(x, y) = t(x) + p(x)(f(x, y) - t(x)). \quad (2.1)$$

**Assumption 2.7** *Given a tax system  $u = (t, p, f)$ , taxpayers choose their report  $x$  so as to minimize  $s_u(x, y)$ .*

This is equivalent to say that agents are risk neutral and know exactly the probability of carrying an inspection at each income level. Later on, we will discuss the case in which taxpayers may have only partial information about the probability of being inspected. Surprisingly, essentially the same proof shows that identical qualitative results hold for this case provided the principal knows the beliefs of the agents.

We will sometimes suppress the subindex and use  $s(x, y)$ , rather than  $s_u(x, y)$  when there is no possible confusion about the tax system  $u = (t, p, f)$ .

Let us define

$$\alpha(y) = \{z : \forall x \in [0, M], \quad s_u(z, y) \leq s_u(x, y)\}$$

**Assumption 2.8** *We only consider tax systems  $u = (t, p, f)$ , such that for each  $y \in [0, M]$ ,  $\alpha(y) \neq \emptyset$ .*

We remark that this assumption clearly holds whenever there is a finite number of agents. Note also that  $\alpha(y)$  may contain more than one element.

However, this will not be a problem in the analysis below. For simplicity we will assume it is single valued.

Hence, the expected transfer of money from agent  $y$  to the government will be given by

$$T(y) = s(\alpha(y), y).$$

The interpretation is that  $(1 - p(\alpha(y)))\mu(\alpha(y))$  agents will not be inspected and pay  $t(\alpha(y))$ , whereas  $p(\alpha(y))\mu(\alpha(y))$  will be inspected and face a punishment of  $\phi_t(x, y)$ , in addition to their true tax  $t(y)$ .

Assumption 2.7 implies that an agent with income  $y$  will not report  $z$  if  $t(z) > t(y)$ . Thus, a rational agent will never be required to pay more than her income regardless of whether she is inspected or not.

The next proposition shows that the expected payment is an increasing function of income.

**Proposition 2.9** *Let  $t$  be an increasing function. Then,  $T(y) = s(\alpha(y), y)$  is an increasing function.*

**Proof**

Let  $z \in [0, M]$  and suppose  $x \leq y$ . Since  $f(x, y)$  is increasing in  $y$ , then,

$$s(z, x) = t(z) + p(z)(f(z, x) - t(z)) \leq t(z) + p(z)(f(z, y) - t(z)) = s(z, y).$$

Taking the minimum over  $z \in [0, M]$  we obtain that  $T(x) \leq T(y)$ . □

Under smoothness assumptions, it is possible to characterize monetary transfer by a differential equation. Suppose for the next proposition that there is a continuum of agents.

**Proposition 2.10** *Let  $t$  be an increasing function. Then,  $T(y) = s(\alpha(y), y)$  is differentiable almost everywhere (with respect to Lebesgue measure). Further, if the mappings*

$$\theta \mapsto f(\alpha(x), \theta)$$

*and  $T$  are differentiable at  $x$ , then,*

$$T'(x) = p(\alpha(x)) \frac{\partial f}{\partial y} \Big|_{(\alpha(x), x)}$$

**Proof**

Since  $T(y) = s(\alpha(y), y)$  is increasing in  $y$ , it is differentiable almost everywhere. Further, it has right and left limits at every point and it may have, at most, jump discontinuities.

Assume  $\theta \mapsto f(\alpha(x), \theta)$  is differentiable at  $x \in [0, M]$ . Let  $z < x < y$ .

Consider now

$$T(y) = t(\alpha(y)) + p(\alpha(y))(f(\alpha(y), y) - t(\alpha(y))) \leq t(\alpha(x)) + p(\alpha(x))(f(\alpha(x), y) - t(\alpha(x))),$$

since  $\alpha(y)$  is the best response for agent  $y$ . Similarly,

$$T(x) = t(\alpha(x)) + p(\alpha(x))(f(\alpha(x), x) - t(\alpha(x))) \leq t(\alpha(y)) + p(\alpha(y))(f(\alpha(y), x) - t(\alpha(y))).$$

And

$$T(z) = t(\alpha(z)) + p(\alpha(z))(f(\alpha(z), z) - t(\alpha(z))) \leq t(\alpha(x)) + p(\alpha(x))(f(\alpha(x), z) - t(\alpha(x))).$$

Hence,

$$T(y) - T(x) \leq p(\alpha(x))(f(\alpha(x), y) - f(\alpha(x), x)),$$

and

$$p(\alpha(x))(f(\alpha(x), x) - f(\alpha(x), z)) \leq T(x) - T(z).$$

So

$$\frac{T(y) - T(x)}{y - x} \leq p(\alpha(x)) \frac{f(\alpha(x), y) - f(\alpha(x), x)}{y - x},$$

and

$$\frac{T(x) - T(z)}{x - z} \geq p(\alpha(x)) \frac{f(\alpha(x), x) - f(\alpha(x), z)}{x - z}.$$

Hence, letting  $z$  and  $y$  tend to  $x$ , we see that, since  $T$  is differentiable at  $x$ ,

$$T'_-(x) = T'(x) \geq p(\alpha(x)) \frac{\partial f}{\partial y} \Big|_{(\alpha(x), x)} \geq T'_+(x) = T'(x).$$

□

This is an extension of a result appearing in [2] where the above Proposition is obtained for a particular tax function.

One should notice that Proposition 2.10 does not apply to all reasonable penalty functions. In fact, Most of the penalty functions known to the authors in the literature have the following form (See Figure 1):

The penalty  $f(x, y)$  is continuous for all  $x$  and  $y$ , differentiable for  $x \neq y$  and has a kink along the diagonal, i.e., for  $x = y$ , although the directional derivatives in the direction of  $x$  and  $y$  exist. However  $\partial f / \partial y$ , which exists for  $y > x$ , can be extended continuously to the diagonal.

Thus, Proposition 2.10 may fail when  $x = \alpha(x)$ . In view of Proposition 3.18, which requires honest reporting policies, and examples 3.16 and 3.17 below it is worth studying what can be said in this case.

**Proposition 2.11** *Assume  $x = \alpha(x)$ , the mapping*

$$\theta \mapsto f(x, \theta)$$

is differentiable for  $\theta > x$  and  $\frac{\partial f(x,\theta)}{\partial y}$  is absolutely continuous in  $\theta$  for  $\theta > x$  and can be extended continuously to  $\theta = x$ . Then, if  $T'(x)$  exists, it satisfies

$$T'(x) \leq p(x) \frac{\partial f}{\partial y} \Big|_{(x,x)}$$

where the right hand side of the equation is to be understood as the continuous continuation of  $\frac{\partial f(x,\theta)}{\partial y}$  to  $\theta = x$ .

**Proof**

Let  $y > x$ . The Taylor expansion of  $f(x, \theta)$  for  $\theta < y$  is valid up to  $\theta = x$ .

Thus,

$$f(x, y) - f(x, x) = \frac{\partial f(x, c)}{\partial y} (y - x),$$

with  $x < c < y$ . As in Proposition 2.10, we have

$$\frac{T(y) - T(x)}{y - x} \leq p(x) \frac{f(x, y) - f(x, x)}{y - x} = p(x) \frac{\partial f(x, c)}{\partial y}.$$

Letting  $y$  tend to  $x$  and taking into account that  $c \in (x, y)$  and  $\frac{\partial f(x,z)}{\partial y}$  can be extended continuously up to  $x$ , the result follows. □

We remark that for the cases considered in [2], one obtains that  $\frac{\partial f}{\partial y} \Big|_{(x, x)} = 1$ . And, thus, under truthful reporting, there is a nice relationship between the marginal tax and the probability of inspection. In the present situation, this relation becomes more complicated.

Given a tax system  $u = (t, p, f)$ , net revenue of the government is given by the following expression

$$R(u) = \int_{[0, M]} (s(\alpha(y), y) - cp(\alpha(y))) d\mu$$

We will suppose that the penalty function  $f(x, y)$  is imposed exogenously on the government, for example, by the legal system, or some other

institution not contemplated in the model. However, the government has the capability to choose the tax function  $t$ , and the inspection policy  $p$ , subject to restriction 2.3. Thus, we only require that an agent will not be taxed more than her income. It is important to note that nothing is assumed on the particular shape of the tax function  $t$ .

**Assumption 2.12** *Given a fixed penalty function  $f$ , the government chooses a policy,  $t$  and  $p$ , satisfying the above requirements, and such that the tax system  $u = (t, p, f)$  maximizes  $R(u)$*

Hence, we are considering that the principal is also risk neutral and it is not concerned with distributional problems.

**Definition 2.13** *Given a fixed penalty function  $f$ , we say that the policy  $(t, p)$  is optimal for  $f$  if  $R(t, p, f) \geq R(\bar{t}, \bar{p}, f)$  for any other policy  $(\bar{t}, \bar{p})$*

One may think that a more appropriate objective for the government would be to minimize inspection costs subject to collect a given revenue <sup>4</sup>. As we will mention later on, our results apply also to this case.

### 3 MAIN RESULTS

In this section we give a partial characterization of the optimal policy. Unless it is explicitly stated otherwise, we will suppose assumptions 2.1, 2.2, 2.3, 2.4, 2.6, 2.7 and 2.12 hold.

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<sup>4</sup>Suppose, for example, that the government needs to collect a certain amount of money to finance a public good

Agent  $y$  will prefer to declare income  $y$  rather than  $x$  as long as

$$t(y) \leq t(x) + p(x)(f(x, y) - t(x)),$$

i.e., she will not declare income  $x$  if the government inspects type  $x$  with probability  $p(x)$  greater than

$$p(x) \geq \frac{t(y) - t(x)}{f(x, y) - t(x)}.$$

We will assume that if  $y \in \alpha(y)$ , then agent  $y$  will report honestly.

Given tax and penalty functions  $t$  and  $f$ , we define, for all  $x, y$  such that  $t(x) \leq t(y)$ ,

$$Q(x, y, t, f) = \frac{t(y) - t(x)}{f(x, y) - t(x)}.$$

The quantity  $Q(x, y, t, f)$  may be seen as a measure of the “incentives” of agent  $y$  to report income  $x$ . The numerator indicates the tax evaded by reporting  $x$ , if she is not inspected, whereas the denominator is the additional tax paid when she is caught.

The following result follows after a straightforward computation,

**Lemma 3.14**  $Q(x, z, t, f) \geq Q(x, y, t, f)$  if and only if

$$\frac{\phi_t(x, y)}{\phi_t(x, z)} \geq \frac{t(y) - t(x)}{t(z) - t(x)}.$$

Our objective now is to characterize some important features of the optimal policy. As mentioned in the introduction, other authors have studied this problem for some very special (and one may argue that not completely realistic) penalty functions. On the contrary, our analysis will cover the characterization of the optimal policy for *each*, arbitrary but fixed, penalty function drawn from a *large* set of admissible ones. Thus, our results enjoy of a “robustness” property as compared with previous ones.

The set of admissible penalty functions we study is given by those ones which satisfy assumption 2.5 and the following one

**Assumption 3.15** Consider penalty functions which satisfy the conditions:

(i) If

$$\frac{t(z)}{z} \geq \frac{t(y)}{y}$$

then, for all  $x$ ,

$$\frac{\phi_t(x, y)}{\phi_t(x, z)} \geq \frac{t(y) - t(x)}{t(z) - t(x)}.$$

(ii) If  $\tilde{t}(x) \geq t(x)$ , then, for all  $z$ ,

$$Q(x, z, t, f) \geq Q(x, z, \tilde{t}, f).$$

Since they seem to be rather ad hoc assumptions, we present next some examples of reasonable penalty functions which satisfy the hypotheses in 3.15.

**Example 3.16** Let  $0 < a \leq 1$ ,  $C \geq 0$ . Consider the following family of penalty functions,

$$f(x, z) = \begin{cases} t(z) + Ct^2(x) - Ct(x)t(z) + a(z - t(z))(1 - \frac{t(x)}{t(z)}) & x \leq z \\ t(z) & x > z \end{cases}$$

Note that this penalty functions include *all the ones which are linear and quadratic in  $t(x)$*  and, in addition to 3.15, satisfy,

$$f(0, y) = t(y) + \phi_t(0, y) = t(y) + a(y - t(y))$$

and

$$f(y, y) = t(y).$$

Hence,  $a(y - t(y))$  is the maximum penalty the government can impose on to agent  $y$ . Varying  $C \geq 0$  changes the convexity of the penalty  $\phi(x, y)$ .

For  $C = 0$ , the resulting penalty is linear in  $t(x)$ . Making  $C$  very large corresponds to having penalties  $\phi(x, y)$  which are very convex. Surprisingly, for all these cases one obtains that the optimal policy cannot be progressive (i.e. convex).

**Example 3.17** The following functions are particular cases of the example above.

$$f_1(x, y) = \begin{cases} t(y) + a(y - x - t(y) + \frac{x}{y}t(y)) & x \leq y \\ t(y) & x > y \end{cases}$$

and

$$f_2(x, y) = \begin{cases} t(y) + a(y - t(y))(1 - \frac{t(x)}{t(y)}) & x \leq y \\ t(y) & x > y \end{cases}$$

The punishment in the first penalty function (See Figure 2a) is linear in the amount of **income** misreported. When agent  $y$  claims no income and is inspected she is charged with a fraction  $a$  (with  $0 \leq a \leq 1$ ) of her income.

The second one (See Figure 2b) is similar except that the punishment is linear in the amount of **tax** the agent is trying to evade.

Observe that the above penalty functions are continuous at all points and the punishment  $\phi_t$  is increasing on the “size of the crime”. In particular, if a taxpayer makes a small mistake and is inspected, the punishment imposed on him will be likewise small.

One possible critique to this approach is that the penalty imposed by unit of evaded income does not have to be the same for all agents. Even worse, it might be the case that this ratio is decreasing in income.

The alternative (e.g. Sanchez and Sobel [6], Cremer, Marchand and Pestieau [3]) is to have a penalty which depends only on the amount evaded regardless of the income level of the agent. However, as we have already

noticed (see footnote 2), this leads to the situation in which an agent may be asked to pay more than her true income.

Remark that in the present context if the tax policy induces truthful reporting, agents do not pay any penalties and , therefore the possible regressiveness of the penalty function never realizes. Nevertheless, it is still a *credible* penalty since it never asks an agent to pay above her income. In contrast, the alternative penalty function just mentioned might not be credible, *even under a truthful policy* for it might not be feasible for a particular agent to pay the penalty required.

The following result has been already obtained in [2] for some particular penalty functions. Our proof here is very similar to the one provided by those authors, although we need to assume honest reporting. The reason for this additional requirement is that the Revelation Principle may not hold now.

**Proposition 3.18** *Let  $f$  be a penalty function satisfying hypotheses 2.5 (i), (ii), (v) and 3.15. Suppose the tax policy  $v = (t, p)$  is optimal and induces honest reporting. Then,  $t$  is nondecreasing and  $p$  is nonincreasing.*

**Proof**

Suppose  $t$  is not nondecreasing. Then, we can find  $y, z \in [0, M]$ , such that  $z < y$  and  $t(z) > t(y)$ . Then,

$$\frac{t(z)}{z} > \frac{t(y)}{y}.$$

Since the inequality is strict, it follows from 3.15 that  $Q(x, z, t, f) > Q(x, y, t, f)$ . From the fact that  $(t, p)$  induces truthful reporting we must have that for each  $x \in [0, M]$ ,

$$p(x) \geq \sup_{t(\theta) > t(x)} \frac{t(\theta) - t(x)}{f(x, \theta) - t(x)} = \sup_{t(\theta) > t(x)} Q(x, \theta, t, f).$$

In particular, for each  $x$  such that  $t(z) > t(x)$  we have that

$$p(x) \geq Q(x, z, t, f).$$

But, since  $t(z) > t(y)$ , we must have that for each  $x$  such that  $t(y) > t(x)$ ,

$$p(x) \geq Q(x, z, t, f) > Q(x, y, t, f).$$

Hence, the incentive constraints for agent  $y$  are not binding. We can raise her taxes by a small amount such that her incentives will continue to be nonbinding. This will not change her report and will not induce other agents to revise their reports either. Raising taxes for agent  $y$  will yield a higher revenue for the government at the same cost, contradicting the optimality of  $(t, p)$ .

Hence,  $t$  must be nondecreasing. We show next that from this it follows that  $p$  is nonincreasing. Indeed, since  $(t, p)$  is optimal we must have that

$$p(x) = \sup_{t(y) > t(x)} \frac{t(y) - t(x)}{f(x, y) - t(x)} = \sup_{t(y) > t(x)} Q(x, y, t, f)$$

which is nonincreasing if  $t(x)$  is nondecreasing. □

Next we will address the issue of regressivity of the optimal tax policy. Given a function  $t(x)$  we define

$$t_*(x) = x \sup\left\{\frac{t(z)}{z} : z \geq x\right\}.$$

Note that

$$\frac{t_*(x)}{x}$$

is decreasing and for each  $x$ ,  $t_*(x) \geq t(x)$ . We have

**Proposition 3.19** *If the tax function  $t$  is nondecreasing and the tax system  $(t, p, f)$  satisfies 2.1 through 3.15 and induces truthful reporting, then  $t_*$  is nondecreasing, continuous and  $(t_*, p, f_{t_*})$  induces truthful reporting.*

**Proof**

The proposition follows from the following

**Lemma 3.20** *Suppose that  $t$  is nondecreasing,  $(t, p, f)$  satisfies the hypotheses 2.1 through 3.15 above, induces truthful reporting and for  $x \leq x_0$  we have that  $\frac{t(x)}{x}$  is nondecreasing. Suppose that  $z = \sup\{\theta : \frac{t(\theta)}{\theta} = \frac{t(x_0)}{x_0}\} \neq x_0$ . Let*

$$t_*(y) = \begin{cases} t(y) & y \notin [x_0, z] \\ \frac{t(z)}{z}y & y \in [x_0, z] \end{cases}$$

*Then,  $t_*(x)$  is also nondecreasing, induces truthful reporting and for all  $x \in [0, M]$  satisfies  $x \geq t_*(x) \geq t(x)$ .*

(See Figure 3) **Proof**

Note first that for  $x_0 \leq x \leq z$  we have that

$$\frac{t(x)}{x} \leq \frac{t(z)}{z},$$

whereas for  $x \leq x_0$ ,

$$\frac{t(z)}{z} \leq \frac{t(x)}{x}$$

Clearly,  $x \geq t_*(x) \geq t(x)$ . Indeed  $t_*(x) = t(x)$  for  $x \notin [x_0, z]$ , whereas for  $x \in [x_0, z]$  we must have that

$$t_*(x) = x \frac{t(z)}{z} \leq x,$$

for  $t(z) \leq z$ . On the other hand for all  $y \in [x_0, z]$  we see that

$$\frac{t(y)}{y} \leq \frac{t(z)}{z}.$$

Hence,

$$t_*(x) = x \frac{t(z)}{z} \geq x \frac{t(x)}{x} = t(x).$$

In particular,  $t_*(x)$  does not change the incentives for  $m \notin [x_0, z]$ .

Let  $y \in [x_0, z]$ . Let  $x \leq y$ . We will show next that  $t_*(x)$  does not change the incentives for  $y$ . As remarked above it is enough to prove that

$$p(x) \geq Q(x, y, t_*).$$

Since  $(t, p)$  induces truthful reporting, we must have

$$p(x) \geq Q(x, z, t)$$

and from 3.15

$$Q(x, z, t) \geq Q(x, z, t_*)$$

But, since  $\frac{t_*(z)}{z} \geq \frac{t_*(y)}{y}$  we have that

$$Q(x, z, t_*) \geq Q(x, y, t_*)$$

Hence, we have that  $p(x) \geq Q(x, y, t_*)$  and it does not pay for agent  $y$  to declare herself as  $x$ . Since  $x \leq y$  was arbitrary, we see that  $(t_*, p)$  induces also truthful reporting.

We prove next that if  $t$  is nondecreasing, then  $t_*$  is nondecreasing as well. Let  $x < y$ , we will consider three possibilities: (1) Suppose  $x \leq x_0 \leq y$ . Then, since  $t$  is nondecreasing,  $t(x) \leq t(x_0)$  and hence,  $t_*(x) = t(x) \leq t(x_0) \leq t(y) \leq t_*(y)$ . (2) Let  $x_0 \leq x \leq y \leq z$ . Then,  $t_*(x) = \frac{t(z)}{z}x \leq \frac{t(z)}{z}y = t_*(y)$ . (3) Assume  $x_0 \leq x \leq z \leq y$ . Then,  $t_*(x) = \frac{t(z)}{z}x \leq t(z) \leq t(y) = t_*(y)$ .

It follows now easily from the facts that  $t_*$  is nondecreasing and  $t_*(x)$  is nonincreasing that  $t_*$  is continuous.  $\square$

The above result motivates the following definition.

**Definition 3.21** *A tax function  $t$  is said to be regressive if  $\frac{t(x)}{x}$  is decreasing.*

We can now state the main result,

**Theorem 3.22** *Let  $f$  be a penalty function and let  $(t, p)$  be an optimal tax policy for it such that hypotheses 2.1 through 3.15 hold. Then, either  $(t, p, f)$  does not induce truthful reporting, or else  $t$  is regressive.*

**Proof**

The theorem is a straightforward consequence of Propositions 3.18 and 3.19.

□

One possible criticism to this approach is that agents need to know the inspection rate  $p(x)$ . We address now the question of what happens when agents have only partial information on  $p(x)$ . As we shall see the same result as above holds, as long as the government knows the beliefs of the agents about  $p(x)$ .

Thus, we now modify our model in the following way: Taxpayers do not know, necessarily, the function  $p(x)$ . Rather, agent  $y$  thinks that the probability of being inspected when she reports her income to be  $x$ , is given by some probability function  $p^y(x)$ .

Consequently, given a tax system  $u = (t, p, f)$ , agent  $y$  will not seek to minimize  $s_u(x, y)$  given in 2.1, but she will rather attempt to minimize the following expression

$$s_u^y(x, y) = t(x) + p^y(x)(f(x, y) - t(x)). \quad (3.1)$$

Modulo these changes, our previous reasoning applies. This is so because the proof of Proposition 3.19 shows that replacing  $t(x)$  by  $t_*(x)$  will not change the incentives of the agents to misreport. Thus, we have

**Proposition 3.23** *Let  $f$  be a penalty function satisfying hypotheses 2.5 and 3.15. Suppose all the agents have the same beliefs on  $p(x)$  (i.e.  $p^y(x) = p^z(x)$  for all  $y$  and  $z$ ), the tax policy  $v = (t, p)$  is optimal and induces honest reporting. Then,  $t$  is nondecreasing.*

Of course, nothing could be said now about the true probability of inspection  $p(x)$ , unless there is some information relating  $p(x)$  and  $p^y(x)$  for each  $y \in [0, M]$ . We also have

**Proposition 3.24** *Let  $f$  be a penalty function and let  $(t, p)$  be an optimal tax policy for it such that hypotheses 2.1 through 3.15 hold. Suppose, in addition, that all the agents have the same beliefs on  $p(x)$ . Then, either  $(t, p, f)$  does not induce truthful reporting, or else  $t$  is regressive.*

One possible scenario in which the previous analysis may apply is when agents have some indirect information about the probability of inspection. For example, the number of tax inspectors is public and hence, the total number of inspections which can be carried out, say  $n$ , is known by the agents. Let  $M$  be the total number of agents. It is not unreasonable to assume that all agents believe that the probability of being inspected is constant and given by  $\bar{p} = n/M$ . Even though, this may not be the real inspection policy designed by the government, agents have no further information about it.

Finally we examine other alternatives for the government's objective. Recall that in the above consideration the goal of the principal is:

- (1) Given a penalty function  $f$ , find a tax policy  $u = (t, p)$  which maximizes net revenue given by

$$R(u) = I(u) - C(u),$$

where,

$$I(u) = \int_{[0,M]} (s(\alpha(y), y) d\mu, \quad C(u) = c \int_{[0,M]} p(\alpha(x)) d\mu.$$

An alternative approach would be:

(2) Given a penalty function  $f$ , find a tax policy  $u = (t, p)$  which collects a fixed target revenue,  $I(u)$ , at a minimum cost,  $C(u)$ .

We argue that if the government restricts itself to honest tax policies and  $\mu$  has a continuous density<sup>5</sup> on  $[0, M]$  then the optimal tax function for problem (2) must be regressive as well. Indeed, suppose  $f$  satisfies 2.1 through 3.15 and  $u = (t, p)$  is a solution to problem (2) which induces truthful reporting. If  $t$  is not regressive we can apply Proposition 3.19 to obtain a new tax function  $t_*(x)$  which yields a higher revenue at the same cost. Thus, letting  $u_* = (t_*, p)$  we must have that  $R(u_*) > R(u)$ . Since  $t_*$  is continuous (because  $t_*$  is nondecreasing and  $\frac{t_*(x)}{x}$  is decreasing), and  $\mu$  has a continuous density, there is  $x_0 \in [0, M]$  such that

$$\int_0^{x_0} t_*(y) d\mu + t(x_0) \mu([x_0, M]) = R(u).$$

Define, now, a new tax policy as follows (see Figure 4),

$$\bar{t}(x) = \begin{cases} t_*(x) & \text{if } x \leq x_0 \\ t_*(x_0) & \text{if } x \geq x_0 \end{cases}$$

$$\bar{p}(x) = \begin{cases} p(x) & \text{if } x \leq x_0 \\ 0 & \text{if } x \geq x_0 \end{cases}$$

It is straightforward to verify that  $\bar{u} = (\bar{t}, \bar{p})$  induces truthful reporting and produces the same revenue,  $R(\bar{u}) = R(u)$ , at a lower cost.

---

<sup>5</sup>A similar prove would apply to the discrete case.

## 4 FINAL COMMENTS

We have studied the income tax enforcement problem using a principal agent model where the government has the capability to set the tax and inspection functions. We have assumed that these functions are announced to the agents and there is not any commitment problem. The penalty function is given exogenously to the government and it must satisfy certain social norms. We give a partial characterization of the optimal policy and prove that for a very large family of penalty functions if the optimal policy is honest then,

- (1) The tax function is continuous and nondecreasing.
- (2) The inspection function is nonincreasing.
- (3) The tax policy is regressive.

It is also shown that (1) and (3) do not depend on the fact that agents know the true probability of inspection.

This result can be seen as an extension of the one in [2]. However, we think that this extension is not a straightforward one. The intuitive explanation of the results in [2] is based on two facts which depend very much on the specific penalty functions they consider: an optimal policy must be such that (i) “the audit probability at a point should be equal to the marginal tax rate for some agent,” (ii) “given that low income reports are the most attractive ones” the inspection probability function must be nonincreasing in income. Thus, (i) and (ii) are the key ideas used by those authors to show that the optimal tax must be concave.

In our case, on the contrary, in general neither (i) nor (ii) are necessarily true. The intuition now is as follows: Consider a penalty function  $f$

satisfying all our hypotheses. Let  $x < y < z$  and suppose the government has implemented a certain tax policy  $u = (t, p)$ . Our assumptions imply that if  $\frac{t(z)}{z} > \frac{t(y)}{y}$  (i.e.  $u$  is more progressive at  $z$  than at  $y$ ), then the incentives for  $z$  to report  $x$  are higher than the incentives for  $y$  to make the same report. Thus, if the government wishes to design a tax policy  $u$  which induces honest reporting, the probability of inspection for people who declare rent  $x$  cannot be binding for agents with rent  $y$ . (Since it has to induce agents with rent  $z$  to make a truthful report). It follows that the government can now increase the tax for agents with rent  $y$  and the tax policy we started with could not possibly be optimal.

The introduction of risk averse agents, additional information about the taxpayer, and the study of dishonest optimal policies are important topics for further research.

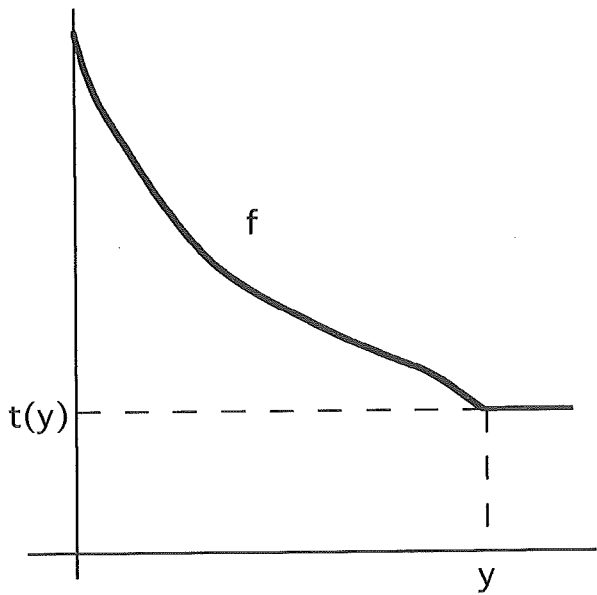


Figure 1

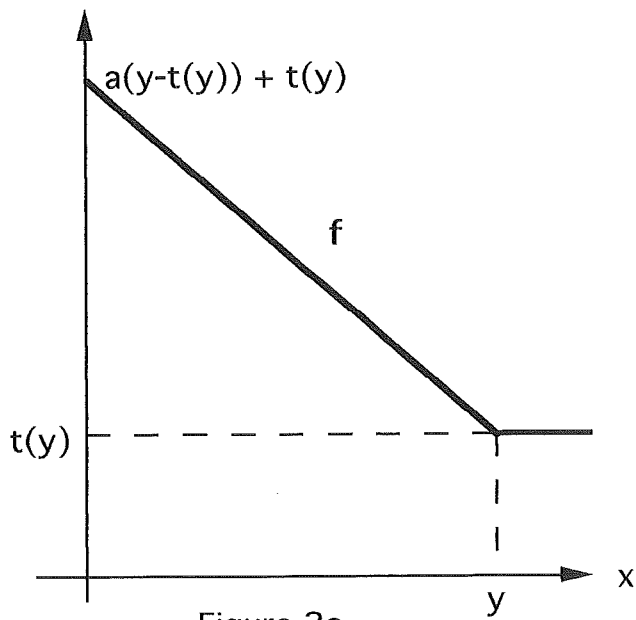


Figure 2a

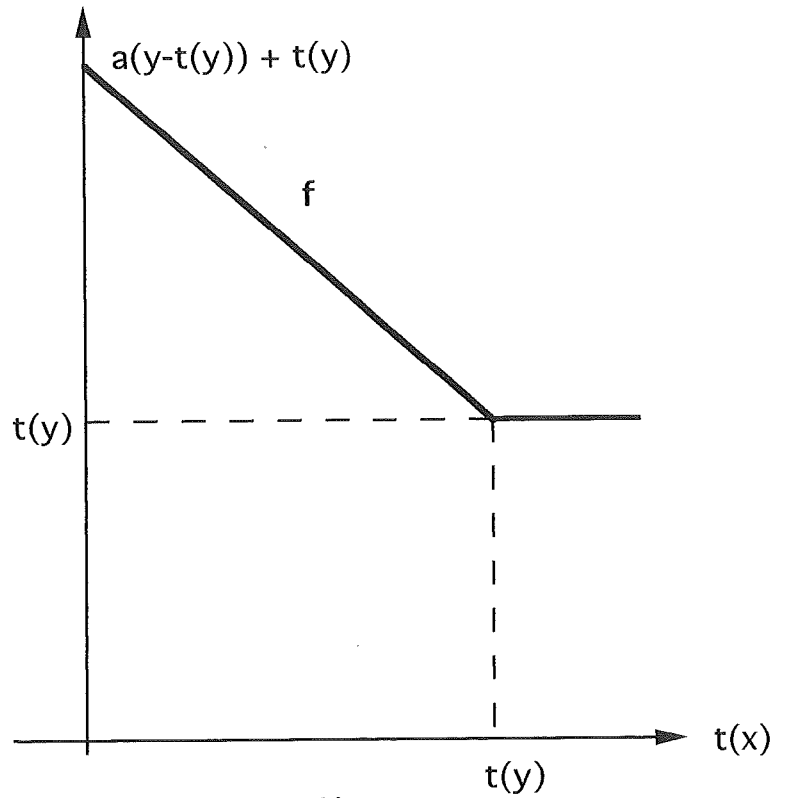


Figure 2b

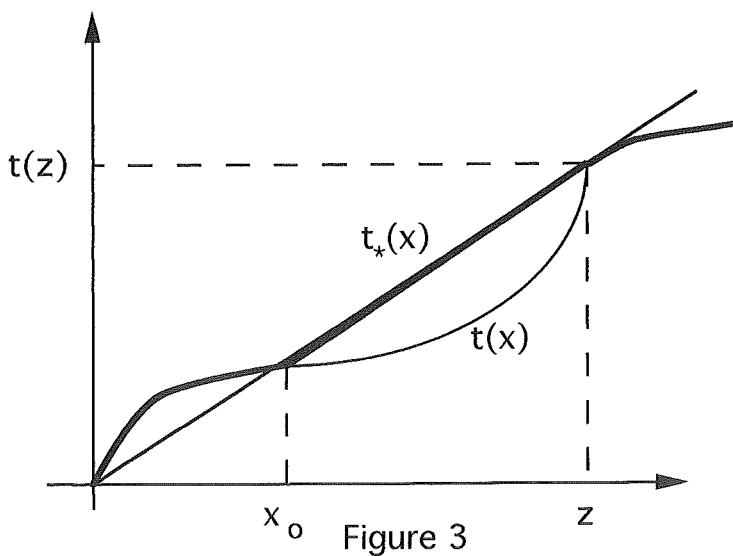


Figure 3

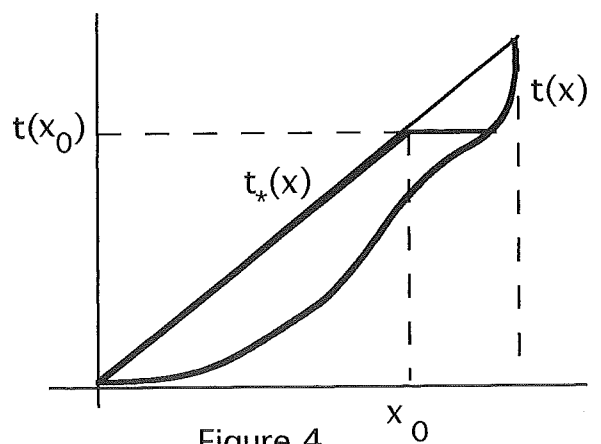


Figure 4

## References

- [1] Border, K. and Sobel, J. "Samurai Accountant: A Theory of Auditing and Plunder." *Review of Economic Studies* (1987) 54:525-540.
- [2] Chander, P. and Wilde, L. "A General Characterization of Optimal Income Taxation and Enforcement" *Caltech Social Science Working Paper* 796 (1993).
- [3] Cremer, H., Marchand, M. and Pestieau, P. "Evading Auditing and Taxing: The Equity-Compliance Tradeoff" *Journal of Public Economics* (1990) 43:67-92.
- [4] Mookherjee, D. and Png, I. "Optimal Auditing, Insurance and Redistribution" *Quarterly Journal of Economics* (1989) 104:399-415.
- [5] Reinganum, J. F. and Wilde, L. L. "Income Tax Compliance in a Principal-Agent Framework" *Journal of Public Economics* (1985) 1-18.
- [6] Sanchez, I. and Sobel, J. "Hierarchical Design and Enforcement of Income Tax Policies". *Southern European Discussion Series*, 86 (1991). Spain: Instituto de Economía Pública.



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