DIVISIONALIZATION IN MARKETS WITH HETEROGENEOUS GOODS*

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A B S T R A C T

This paper assumes that firms can create independent divisions which compete in a market where the product is spatially differentiated. Our first result is that if the number of firms is exogenous and higher than some critical level, then Perfect Competition is the only Subgame Perfect Equilibrium. On the other hand, if we assume free entry and a fixed cost per firm, depending on the relationship between fixed costs and the degree of product differentiation, the equilibrium outcome may be monopoly, oligopoly or monopolistic competition. Moreover, if the fixed cost is sufficiently small, the number of independent sellers will be lower than in the standard model where divisionalization is not possible.

I. INTRODUCTION

When firms have the possibility of creating independent divisions in an oligopolistic market, equilibrium outcomes are very different from those obtained when such a possibility does not exist, as has been shown in the case of a homogeneous product and quantity-setter divisions by Corchón (1991), Corchón & González-Maestre (1993), Schwartz & Thompson (1986) and Veendorp (1991). In this paper we present a model of heterogeneous goods in which there is spatial competition of the type considered by Salop (1979), divisions compete in prices and firms can create as many divisions as they like. As we will see, our conclusions differ from those obtained by Salop.

In the first part of the article, we analyze the effect divisionalization has on the equilibrium number of independent sellers when the number of firms is exogenous. Our first result is that if the number of firms is large enough, but finite, then in any Subgame Perfect Nash Equilibrium, they will create an infinite number of divisions and the outcome will thus be Perfect Competition (see Proposition 1, part i)). This result extends to the case of heterogeneous goods the result obtained in the context of homogeneous goods by Corchón & González Maestre (1993), which is a generalization of the one obtained previously by Corchón (1991). Also we find that if the number of firms is sufficiently small in relation to the degree of convexity of the transportation cost function, then firms do not divisionalize at all (see Proposition 1, part ii)).

In the second part of the article, we show that if there is free entry and a fixed cost per firm, the equilibrium outcome will be dramatically different from our first result. In fact, depending on the relationship between the fixed cost and the elasticity of the transportation cost function, market equilibrium may be oligopoly, monopolistic competition or pure monopoly (see Propositions 2 and 3). Also the number of divisions in equilibrium is bounded, regardless of the magnitude of the fixed cost. Moreover, in some cases, the degree of competition will be lower than in the case where there is no possibility of divisionalization (see Corollary 1). This result contradicts Schwartz & Thompson's (1986) conjecture about the social desirability of the possibility of divisionalization, since it makes clear that in many cases the relevant aspect of divisionalization is not its actual realization, but simply the credibility of its use in the case of entry.

The plan for the rest of the paper is as follows. In the next Section we analyze a model with a given number of firms and zero fixed costs. In Section III we analyze a model with a countably infinite number of potential firms and a positive fixed cost. Finally, Section IV gathers our main conclusions.

II. THE MODEL WITH NO ENTRY

In this Section we will assume that there are k firms in the first stage of the game, and each one can create as many divisions as it likes. In the second stage, divisions will compete in prices, including competition between divisions belonging to the same firm. That is, we will assume the following game, called G.1.

G.1:

Stage 1: Each firm decides simultaneously on the number of divisions.

Stage 2: The created divisions compete in prices.

Also, we consider a version of Salop's model (1979) given by the following assumptions.

A.1: The created divisions are distributed around a circumference with a length equal to one. The population is distributed uniformly around this circumference. Also, every consumer buys just one unit of one division's product.(1)

A.2: The transportation cost, which is paid by the consumer, is given by f(d), where d is the distance between the division and the consumer along the

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(1) This simplifying assumption is equivalent to assuming that in the Salop's original model, the parameters are such that the equilibrium configuration implies that no consumer will prefer the outside good, so that we will end up in the "competitive" region of every individual demand curve, according to Salop's terminology.

circumference and f'(d) > 0. Also, we will assume that f'(d).d tends to 0 when d tends to 0, while the function $g(d) \equiv \frac{f''(d)}{f'(d)}.d$ has both, an upper and a lower bound in the interval [0 , 1], denoted respectively by $\bar{g}$ and $\bar{\bar{g}}$.

A.3: The cost function for every division is given by

$C(x_i) = c.x_i$, where c is a constant and x_i is the production corresponding to division i.

Assumption A.2 is a generalization of the linear transportation cost considered in the Salop's original model. Notice that $f(d) = d^{\alpha}$ satisfies A.2. Also, A.2 is more general than assuming that f(d) is C^2 and $f' > \gamma > 0$ in the interval [0 , 1]. We can interpret the function g(d) as a measure of the degree of convexity of the transportation cost function. As we will see, the function g(d) is a crucial determinant of the degree of competition in the market.

As in the model studied by Corchón (1991), the justification of assumption A.3 is to isolate the Stackelberg leadership effect implied by divisionalization, from any other plausible explanation of the existence of decentralized firms. For instance, if we assume decreasing returns, then there is an extra incentive to decentralize in order to reduce costs.

Let us first consider the symmetric Nash equilibrium in the second stage under the assumptions that the number of divisions created by every firm is an integer and that they are located symmetrically around the circle.

Since the only source of differentiation comes from the transportation cost, a consumer located at distance $x \in (0 , 1/n)$ from division i will be indifferent about buying either from division i or from division i's nearest division if the following condition is satisfied:

$$p_i + f(x) = p + f(\frac{1}{n} - x)$$

where p is the price of the rest of the divisions, and n is the number of divisions. That is, we assume that the rest of the divisions charge the same price.

Therefore, the inverse demand function of the division i is given by

$$p_i = p - f(x) + f(\frac{1}{n} - x) \qquad (1)$$

Note that, according to (1), for a given level of p there is an inverse relationship between p_i and x, so that the decision on x is equivalent to the decision on p_i for a given p.

Also, the profits of division i are given by

$$\Pi_i = (p_i - c).2x \qquad (2)$$

since, by symmetry, the total amount sold by division i is 2x.

Therefore, profits of division i as a function of x are given by

$$\Pi_i = [p - c - f(x) + f(\frac{1}{n} - x)].2x$$

The first order condition of profit maximization implies

$$\frac{\partial \Pi_i}{\partial x} = -[f'(x) + f'(\frac{1}{n} - x)].2x + 2.[p - c - f(x) + f(\frac{1}{n} - x)] = 0$$

If we impose symmetry, then $p_i = p$, so that, according to expression (1) $2x = \frac{1}{n}$, which substituted in the first order condition yields

$$p - c = f'(\frac{1}{2n}).\frac{1}{n} \qquad (3)$$

Therefore, by substituting expression (3) into (2) we obtain that, for a given number of divisions, in a symmetric Nash equilibrium, the profits per division in the second stage of the game will be given by the following function

$$\Pi_i(n) = f'(\frac{1}{2n}).\frac{1}{n^2} \qquad (4)$$

Now we will consider the number of divisions as a continuous variable and we will provide an argument to justify such an assumption.

Let us suppose that every firm can impose a tariff on the output produced by its divisions. By means of this tariff, the firm can control the output produced by a division. We will define a "full division" as a division with no obligation to pay tariffs to the central firm. Also we will define a "fractional division" as a division which has to pay a tariff τ per unit of output to the central firm.

Let us consider the following assumptions

A.4: The number of divisions created by every active firm, say j, is a continuous variable denoted by m_j. If $m_j = [m_j]$, that is, if m_j is an integer,

then firm j creates m_j "full divisions". If $m_j - [m_j] = \delta_j > 0$ then it creates $[m_j]$ "full divisions" and an extra "fractional division".

A.5: The created divisions are located in the following way. The distance between two adjacent divisions, say i and j, is given by 1/n if both divisions are "full"; if both are "fractional" then they are separated by a semicircumference with length $(\frac{\delta_i + \delta_j}{n})$; and if i is "full" and j is "fractional", then this distance is $(\frac{1 + \delta_j}{n})$, where δ_j is obtained from the following condition

$$p^* - (1 + \tau_j)c = f'(\frac{\delta_j}{2n}).\frac{\delta_j}{n} \qquad (3')$$

where p^* is the equilibrium price obtained from equation (3) and τ_j is the tariff per unit of output paid by division j to the corresponding firm. We define δ_i in a similar way.

It is easy to show that equation (3') is obtained from the condition of profit-maximization for every "fractional division", when we impose the condition that $p_j = p^*$ and we fix δ_j. Therefore condition (3') yields the level of τ_j consistent with a symmetric Nash equilibrium in prices when every firm creates a number of divisions m_j such that $m_j = [m_j] + \delta_j$ and the total number of divisions is n. Thus, we can interpret δ_j/n as the "size" of the "fractional division created by firm j. Moreover, this "size" is also the level of output corresponding to this division in the symmetric equilibrium.

The main result of this section is summarized in the following

Proposition 1:

Under the assumptions A.1, A.2, A.3, A.4 and A.5, the following properties hold,

(i) if $k > \bar{g}+2$, *and a SPNE of the game G.1 exists, it implies Perfect Competition;*

(ii) if $k < \bar{\bar{g}}+2$, *and a SPNE of the game G.1 exists, it implies* $m^* = 1$, *and* $n^* = k$.

Proof:

Expression (4) gives the profits per division when the total number of divisions created in the first stage is n. Now, let consider how the equilibrium number of divisions is determined for a given number of firms.

In the first stage, the firm j decides on its number of divisions (which will be called m) given the total number of divisions created by the rest of the firms (which will be called t). That is, the problem of firm j is to maximize the function

$$\Pi_j = m.\Pi_i(n) = m.\Pi_i(m + t) = f'\left(\frac{1}{2[m + t]}\right).\frac{m}{(m + t)^2}$$

Where t is the number of divisions created by the rest of the firms.

As we assume that the game is symmetric, the above function is the same for $j = 1, \ldots, k$.

The derivative of the above function with respect to m yields the effect on firm i's profits of a change in its number of divisions keeping constant the number of divisions of the rest of the firms. The sign of $\frac{\partial \Pi_j}{\partial m}$ *is the same as the sign of the function* $F(m) \equiv (m + t)^4.\frac{\partial \Pi_j}{\partial m}$, *which is given by*

$$F(m) \equiv (m + t)^2.\left[f' - \frac{m.f''}{2(m + t)^2}\right] - 2(m + t)f'.m$$

or,

$$F(m) \equiv - \frac{m.f''}{2} + (m + t).(t - m).f'$$

Since $g(x) \equiv \frac{f''(x)}{f'(x)}.x$ *we can rewrite the last function as*

$$F(m) \equiv - m.g.(m + t).f' + (m + t).(t - m).f'$$

If we consider a symmetric subgame perfect Nash equilibrium,then it must be the case that $t = (k - 1)m$. *By substituting this condition in the above expression, we have*

$$\frac{F(m)}{f'} = m\ (k - g - 2) \qquad (5)$$

Obviously, the sign of the last expression is the same as F(m), since f' is positive. Since $m \geq 1$, *profits of firm j increase (respectively decrease) with an increase in its number of divisions if the number of firms is higher than* $\bar{g} + 2$ *(respectively lower than* $\bar{\bar{g}} + 2$*). This completes the proof, since, according to expression (3) and A.2,* $p = c$ *when n is infinity* ■

Proposition 1 states that if the number of firms is sufficiently small relative to $\bar{\bar{g}}$ we end up with the equilibrium analyzed by Salop, i.e. no divisionalization and firms charging a price above marginal cost. However if k is large enough but finite, all firms create an infinite number of divisions and price equals marginal cost. In particular, if the transportation cost is linear (as happens in Salop's original model), then g(d) = 0, thus, by Proposition 1, the equilibrium implies perfect competition with more than two firms.

It is easy to show that if the transportation cost function is of the form $f(d) = a.d^{\alpha}$, then $g(x) = \alpha - 1 = \bar{g} = \bar{\bar{g}}$. Also, in Appendix I, we show that in this case the SPNE exists, at least for some values of α. Note that in this case, α is a measure of the degree of differentiation of the product for a given number of firms. Moreover, since $x < 1$, this degree of diferentiation decreases with α. The less differentiated the product of every firm is, the tougher the price competition is in the second stage. Therefore, we can interpret the previous result as follows: the less competitive the price competition is (i.e, the lower α is), the more attractive it becomes to create new divisions and, consequently the more likely it is end up with perfect competition.

In the next Section we will see that if we allow k to be endogenous by considering the posibility of free entry, then the possibility of divisionalization might have a very anticompetitive effect, contrary to what happens when k is exogenous.

III. THE MODEL WITH FREE ENTRY

Now, we consider the model of Section II, but with free entry (i.e, with a countably infinite number of potential firms), and a fixed cost per firm associated with entry. Formally, we will consider the following game:

G.2

Stage 1: Firms decide on entry. If a firm decides to enter the market, then it has to pay a fixed cost $\varepsilon > 0$.

Stage 2: Every firm decides on its number of divisions.

Stage 3: The divisions compete in prices.

In order to illustrate in a simple way the main conclusions of our analysis, we will assume, initially, that the transportation cost is of the form $f(d) = a.d^{\alpha}$, where $\alpha > 0$. Thus, $g(d) = \alpha - 1$ is constant in this case.

Let k^* and m^* be respectively the number of firms and divisions per firm in a symmetric subgame perfect Nash equilibrium of game G.2.

Also, we can define n' as the maximum value of n such that profits per divisions are nonnegative. That is n' is the maximum value of n satisfying

$$\Pi_i(n) = A.\frac{1}{n^{\alpha+1}} \geq \varepsilon \qquad \text{where} \quad A = a.\alpha.2^{1-\alpha}$$

Note that n' is the equilibrium number of firms in the model where there is no possibility of divisionalization.

Let us introduce some notation. In the sequel [x] will stand for the integer part of x.

The following result characterizes the equilibrium number of firms and divisions in our model, under the assumption of symmetry.

Proposition 2:

Under A.1, A.3, A.4 and A.5, if $f(d) = a.d^{\alpha}$ *and* α *is not an integer, then if a symmetric SPNE of game G.2 exists, it will be given by* $(k^* = [\alpha]+1 , m^* = 1)$ *if* $\alpha+1 \leq n'$, *and by* $(k^* = n' , m^* = 1)$ *if* $\alpha+1 > n'$.

Proof:

First, note that $k^* \leq n'$, *since otherwise profits per firm will be negative. We have to consider two cases:*

Case (1): $\alpha+1 > n'$.

Suppose that $k^* < n'$. *Then* $\bar{\bar{g}} + 2 = \alpha + 1 > n' > k^*$. *Therefore by Proposition 1* $m^* = 1$, *but this implies positive profits for an extra entrant. Therefore, the only SPNE value of k is* $k^* = n'$ *and thus* $m^* = 1$.

Case (2): $\alpha+1 \leq n'$.

In this case if $k^* < [\alpha]+1 < \alpha + 1 = \bar{\bar{g}} + 2$, *then* $m^* = 1$ *according to Proposition 1, but this implies positive profits for an extra entrant. Also, by Proposition 1 if* $k^* > [\alpha]+1$ *then there would be perfect competition and negative profits for all the entrants. Thus, in the only symmetric SPNE,* $k^* = [\alpha]+1 < \alpha+1$, *which implies, according to Proposition 1 that* $m^* = 1$ ■

Corollary 1:

Under the assumptions A.1, A.3, A.4 and A.5, if $f(d) = a.x^{\alpha}$ *and* $\alpha+1 < n'$, *then any symmetric SPNE of the game G.2 implies a lower number of independent sellers than in the model where every firm can not create more than one division.*

Moreover, as in the case of game G.1, a SPNE exists, at least for some values of α, as it is shown in Appendix I.

Our previous results imply that the equilibrium price in game G.2 will never be lower than in the case in which divisionalization is impossible. Moreover, if the fixed cost is small enough, or equivalently, if the market size is large enough, the equilibrium price will be higher in the case of feasible divisionalization.

In fact, contrary to what happens in the standard locational models, price becomes independent of the fixed cost when this is small enough (note that the parameter α is related to preferences, but not to the fixed cost nor to market size). In the model with no possibility of divisionalization, n' tends to infinity when ε tends to zero, which implies that the price tends to the constant marginal cost c (see expression (3) in the previous section). However, in our model, the total number of independent divisions is given by $n^* = [\alpha] + 1$ when ε is small enough; consequently, the difference (price - marginal cost) tends to a finite positive number (see expression (3) again).

Therefore, the results obtained in Proposition 2, provide a taxonomy in which the equilibrium outcome of a particular market corresponds to oligopoly,

Also, we have shown above that $k^* \leq 2 + \bar{g}$ *; however in a symmetric SPNE of G.2 the equality* $k^* = 2 + \bar{g}$ *cannot be satisfied. In order to show this, note that if it were the case ,* $g^{-1}(k^* - 2) = g^{-1}(\bar{g})$*, but our assumption that* $g'(d) < 0$ *implies that* $g^{-1}(\bar{g}) = 0$*, since in this case,* $\bar{g}$ *is attained at* $x = 0$*; consequently, in this case the total number of divisions would be infinite, according to the above equation, which is inconsistent with a symmetric SPNE of G.2. Therefore, it must be the case that* $k^* < 2 + \bar{g}$*, which implies that* $g^{-1}(k^* - 2) > g^{-1}(\bar{g}) = 0$*. Therefore, if we define* k^0 *as the maximum k consistent with* $k < 2 + \bar{g}$*, then if* n^* *is the total number of divisions in equilibrium, the following inequality must be satisfied:*

$$n^* \leq \frac{1}{2.g^{-1}(k^0 - 2)} = \bar{n}$$

where $\bar{n}$ *is a finite number, independent of* ε*, and the proof is completed* ■

Therefore, according to Propositions 2 and 3, if we consider the possibility of divisionalization in a locational model of horizontal differentiation, then the standard limit theorems associated to this kind of models do not hold. In our model, if the fixed cost tends to zero, the number of independent sellers tends to a finite number and the price does not tend to the marginal cost. Thus, the "finiteness" property obtained by Shaked and Sutton (1983a) in the case of vertical differentiation, might arise also in a locational model of horizontal differentiation where the firms can create independent divisions.

Some of the implications with respect to international trade are also similar to those obtained in the referred cases of vertical differentiation

(see Shaked and Sutton (1983b)). In particular, if two identical countries, initially in autarky, remove all their trade restrictions, then the number of firms operating in the expanded market will be the same as in each of the two previously isolated economies. As in Shaked and Sutton's model of vertical differentiation, in this case there are welfare gains associated to a lower number of firms, since the total fixed cost is itself lower. In our model, all this welfare gains go to the industry as a whole, since the price remains the same. However, in the vertical differentiation case, there is an increase in consumers' surplus associated to both, a fall in the prices of the surviving firms and a higher average quality.

Finally, it is interesting to remark that, in many cases arising in our model, divisionalization is not carried out. However, the credibility of it being carried out in the case of entry, acts as an effective way of sustaining very anticompetitive equilibrium outcomes, as has been established in this Section. Indeed, our model suggests that if divisionalization is feasible, the incumbents might successfully deter entry by means of the credible threat implied by that same feasibility. Consequently, if the incumbent firms can decide on the number of divisions simultaneously with the potential entrants (as happens in our model), then the welfre effects associated with potential divisionalization might be different from those suggested by Shwartz & Thompson (1986) in a model where this kind of decisions are sequential.

IV. SUMMARY AND CONCLUSIONS

In this paper, we have analyzed a simple model of product differentiation in which firms can create independent divisions costlessly.

Our first result implies that if the number of firms is exogenous, then Perfect Competition is the only equilibrium in our model, provided that the number of firms is large enough (see Proposition 1).

However, if we allow free entry and there is a fixed cost per firm, then we get an equilibrium which implies a lower degree of competition than in the case where there is no possibility of divisionalization, provided that the fixed cost associated to entry is small enough (see Corollary 1 and Proposition 3).

Our final results suggest that the effect of divisionalization on welfare will be in general ambiguous: on the one hand, the lower number of firms implies a lower level of total sunk costs, but on the other hand the lower number of independent sellers lowers the degree of competition, which in turn, damages consumers' surplus.

Another implication of our results is that, under divisionalization, the equilibrium number of independent sellers might have an upper bound which is not related to the fixed entry cost. Therefore, according to Shaked and Sutton's terminology, a kind of "natural oligopoly", (usually associated to some models of vertical differentiation, see Shaked and Sutton (1983)), is also the equilibrium outcome in a locational model of horizontal differentiation.

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APPENDIX I

In this appendix we show that if the transportation cost is of the form $f(d) = d^{\alpha}$, then there exists a symmetric SPNE of the game G.1 for some values of α. We will do this in the following way:

First, we will show that, for some values of α, the profit function of every division is strictly concave in its output, given a common price p of the rest of the divisions, which implies that the first order conditions used in the proof of Proposition 1 yields a symmetric Nash Equilibrium in the second stage of G.1. Secondly, we will prove that the profit function of every firm associated to this symmetric Nash Equilibrium is quasi-concave in its number of divisions, given its competitors' number of divisions, which implies that a symmetric SPNE exists.

Formally, let be $\Pi_i(x, p)$ the profit function of division i in the second stage of G.1, and $\Pi_j(m, t)$ the profit function of firm j in the first stage, relative to a symmetric Nash Equilibrium in the first stage, where m and t are defined as in Section II. The main result of this Appendix can be summarized in the following

Lemma 1.

If $f(d) = d^{\alpha}$, *then the following properties hold:*

(i) $\Pi_i(x, p)$ *is concave in* x *if* $\alpha \in \{ (0 , 1] \cup \{2\} \cup [2.96 , 5.39] \}$;

(ii) $\Pi_j(m, t)$ *is quasi-concave in* m.

Proof.

Part (i): *By substituting the derivative of* $f(x) \equiv x^{\alpha}$ *in the calculations made in Section II, we obtain*

$$\frac{\partial \Pi_i}{\partial x} = -(\alpha + 1)x^{\alpha} + \left(\frac{1}{n} - x\right)^{\alpha} - \alpha\left(\frac{1}{n} - x\right)^{\alpha-1}.x + p - c$$

Thus the second derivative, yields

$$\frac{\partial^2 \Pi_i}{\partial x^2}.\frac{1}{\alpha} = -(\alpha + 1)x^{\alpha-1} - 2\left(\frac{1}{n} - x\right)^{\alpha-1} + (\alpha - 1).\left(\frac{1}{n} - x\right)^{\alpha-2}.x$$

Obviously, the above expression is negative if $\alpha \in (0\ ,\ 1]$ *or* $\alpha = 2$.

Now, let us consider the case $\alpha > 2$. *If we define* $\lambda \equiv \dfrac{x}{\frac{1}{n} - x}$, *then we can write*

$$sign\ \left\{\frac{\partial^2 \Pi_i}{\partial x^2}\right\} = sign\ F(\lambda)$$

where $F(\lambda) \equiv -(\alpha + 1)\lambda^{\alpha-1} - 2 + (\alpha - 1)\lambda$.

But $F(\lambda)$ *is strictly concave if* $\alpha > 2$, *thus the maximum of* $F(\lambda)$ *in this case is obtained from the following first order condition:*

$$F'(\lambda) \equiv -(\alpha - 1)(\alpha + 1)\lambda^{\alpha-2} + (\alpha - 1) = 0$$

which yields the solution $\lambda^{*} = \{\alpha + 1\}^{-(\alpha-2)^{-1}}$.

Therefore, the maximum value of $F(\lambda)$ *is given by*

$$F(\lambda^{*}) = (\alpha - 2)\{\alpha + 1\}^{-(\alpha-2)^{-1}} - 2$$

The above expression will be negative if and only if

$$ln(\alpha - 2) - (\alpha - 2)^{-1}.ln(\alpha + 1) < ln2$$

or,

$$H(\alpha) \equiv (\alpha - 2)\ln 2 + \ln(\alpha + 1) - (\alpha - 2)\ln(\alpha - 2) > 0$$

But the derivative of $H(\alpha)$ *is given by*

$$H'(\alpha) \equiv \ln 2 + \frac{1}{\alpha + 1} - \ln(\alpha - 2) - 1$$

It is easy to show that the above derivative is negative for all $\alpha \geq 2.95$. *But* $H(5.39) > 0$. *Thus,* $H(\alpha) > 0$ *for all* $\alpha \in [2.95 , 5.39]$, *which implies that for any* α *in that interval,* $F(\lambda) < 0$ *for all* $\lambda > 0$, *and this completes the proof of part (i)* ■

Part (ii): *In Section II we showed that the sign of* $\frac{\partial \Pi_i}{\partial x}$ *is the same as*

$$F(m) \equiv - m.(m + t).f'.g + (m + t).(t - m)f'$$

$$\equiv (m + t).f'.\{ t - m(g + 1) \}$$

But g *is a constant given by* $g \equiv \alpha - 1$ *in our case. Thus,* $F(m)$ *is positive for all* $m < t/\alpha$, *zero for* $m = t/\alpha$, *and negative for all* $m > t/\alpha$. *Therefore, the function* $\Pi_j(m , t)$ *is a "single-picked" function, and the proof is completed* ■

Finally, note that, according to the argument given in the proof of Proposition 2, if a symmetric SPNE of G.1 exists, a SPNE of the game G.2 also does so, if the transportation cost is of the form $f(d) = d^{\alpha}$. Thus the values of α considered in Lemma 1 ensures the existence of a SPNE in both games.

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