

**INDUSTRIAL DYNAMICS, PATH-DEPENDENCE
AND TECHNOLOGICAL CHANGE***

Fernando Vega-Redondo**

WP-AD 93-04

* I thank Rafael Rob and Luis Corchón for helpful conversations. Financial assistance by the Spanish Ministry of Education, CICYT project nos. PB-89/0294 and PS-90/0156 is gratefully acknowledged.

** F. Vega-Redondo: Instituto Valenciano de Investigaciones Económicas and University of Alicante.

**Editor: Instituto Valenciano de
Investigaciones Económicas, S.A.**
Primera Edición Julio 1993.
ISBN: 84-482-0263-5
Depósito Legal: V-2518-1993
Impreso por KEY, S.A., Valencia.
Cardenal Benlloch, 69, 46021-Valencia.
Printed in Spain.

**INDUSTRIAL DYNAMICS, PATH-DEPENDENCE
AND TECHNOLOGICAL CHANGE**

Fernando Vega-Redondo

A B S T R A C T

In this paper, I propose a game-theoretic, intertemporal model of industrial competition in which active firms innovate, imitate, enter or exit as it is optimal in some prevailing (Markov Perfect) equilibrium. The main novel feature of the approach is that technological change is modelled as advance along a directed graph of technologies. This permits a rigorous formalization of such key notions as technological distance, technological precedence, or switching costs, all of which play a crucial role in the model. In particular, they underlie a process of technological change which is highly path-dependent. The theoretical analysis of the paper centers on existence and ergodicity issues but also investigates the effect of different technological structures (digraphs) on the induced population dynamics.

EN BLANCO

1.- INTRODUCTION

The phenomenon of industrial competition is best viewed as a process of strategic interaction in an heterogeneous and ever-changing context. ⁽¹⁾ There are two main forces which underlie such process. On the one hand, the process of population turnover by which firms enter or exit depending on their particular fortunes; on the other hand, the various processes of know-how accumulation which continuously change the strategy spaces of incumbent firms (e.g., their "technological" choice sets). The objective of this paper is to propose a highly stylized model which displays these features and that could be tested against empirical evidence.

I now summarize very briefly the main components of the model. Firms are optimizing units which choose their actions (namely, their technologies and research expenditures) as part of an intertemporal equilibrium; specifically, a Markov Perfect Equilibrium. Exit of firms occur when they go bankrupt. New firms enter when there are profitable opportunities. As the process unfolds, firms change their respective sets of available technologies through innovation and imitation. Both are costly activities. On the one hand, innovation that goes beyond mere "learning by doing" requires the expenditure of resources. On the other hand, imitation of others (which can be done only with some lag) involves switching costs which depend on the magnitude of the technological shift.

A key feature of the approach resides in the way in which innovation and switching costs are modelled. The space of technologies is endowed with a directed-graph (digraph) structure which reflects both the notions of "technological precedence" and of "technological gap". The first is relevant for innovation (a firm learns or invents a new technology among the successors of currently available ones). The second is pertinent for imitation (as mentioned, switching costs are linked to the magnitude of the technological

¹ See, for example, Cubbin & Jeroski (1987), Dunne, Roberts & Samuelson (1988), or Cohen & Levin (1989) as a sample of the empirical research which shows that firm heterogeneity and a permanent "state of flux" is a prevalent feature of many important industries.

gap). A central objective of the paper will be to explore how the industrial dynamics is affected by alternative digraph structures. Specifically, the following such structures will be explored: a tree, a line, and a lattice.

I end with a review of related literature. The closest research I am acquainted with is reported by Ericson & Pakes (1989). These authors also propose a dynamic stochastic model where innovation and population turnover interact to produce the overall industrial dynamics. As in the present paper, their model induces a stochastic game, whose intertemporal equilibrium is analyzed. There are, however, three main differences between our approaches.

First, Pakes and Ericson abstract from technology diffusion (imitation), which is clearly an important component of the technological dynamics of many industries.

Second, they ignore the effect of switching or learning costs in processes of technological adjustments. This makes the model unable to reflect important path-dependency considerations which, in the real world, seem to underlie many firms' technological decisions (e.g. their often inertial behavior).

Third, their model implicitly restricts its attention to linear technological structures; that is, to contexts where more or less advanced technologies are all bound to be along a single development path. As indicated by numerous empirical and theoretical studies ⁽²⁾ this is hardly a good assumption to describe many important cases where there co-exist (at least, potentially) a number of different particular technological paths. Each of these paths generally embodies quite different idiosyncratic skills, capital requirements, or management structures. Under these circumstances, adjustments across different technological lines may well be very costly.

² See footnote 6 below for a brief reference to some empirical case studies. More generally, the reader may refer to the work of authors such as Brian Arthur or Paul David who have stressed the importance of path dependence in technological processes. In particular, I suggest reading David (1988) or Arthur (1989).

An additional important work in this area is Jovanovic (1982).³ In it, industrial competition is also modelled in an intertemporal context with firms entering, exiting, and learning along a dynamic equilibrium. It exhibits, however, the following two essential differences with the present approach. First, it contemplates a context with a continuum of firms and, therefore, no aggregate uncertainty. Second, firms are involved in learning about a fixed idiosyncratic parameter rather than exploring unbounded technological possibilities. This latter point was modified in Jovanovic & McDonald (1988), where genuine innovation is assumed to take place along the process. However, the maintained continuum assumption (first point above) makes it unsuitable to model some of the rich strategic considerations involved in technological interaction. To address them, one needs to focus on a small-number context where individual firm choices may have significant effects on competitors.

The study of intertemporal strategic processes of innovation and diffusion which has been recently developed for a "one-shot, single-project framework" also bears important relationship to the present work (see, for example, the recent handbook paper by Reinganum (1991) for a survey and basic references). While this paper inherits from this literature some of its theoretical apparatus, the main contrast with it resides in the fact that technological decisions are embedded in a continual industry-wide process of technological change.

Finally, the evolutionary approach of Nelson & Winter (1982) or Iwai (1984) also shares with this paper similar motivation. Their same concern is to study industrial dynamics as an interplay among all those components which are involved here: entry, exit, innovation, and imitation. However, in contrast to the present approach, their model incorporates no rational or equilibrium behavior, with firms being assumed to adjust behavior in a rule-of-thumb fashion.

The rest of the paper is organized as follows. In the next Section, Section 2, the model is presented and discussed. Section 3 contains the analysis. A summary closes the main body of the paper in Section 4. For the sake of smooth presentation, formal proofs are relegated to an Appendix.

³ See also the related more recent work of Hopenhayn (1992).

2.- THE MODEL

The presentation of the model is divided into the following subsections: The Firms (2.1); Technological Structure (2.2); Technology Sets (2.3); Stage Payoffs (2.4); Entry and Exit (2.5); Dynamic Game (2.6); Equilibrium (2.7).

2.1. The Firms

There is a set of potential firms which is countably infinite and indexed by $I = \mathbb{N}$. Time is measured discretely, $t = 0, 1, 2, \dots$. At any given t , a subset of firms, $P(t) \subseteq I$, represents the current participants in the industry. These are the firms which may choose to produce and sell their product with some of their currently available technologies. Such set $P(t)$ is partitioned into two further subsets, $Z(t)$ and $E(t)$. The former represents the incumbent firms (i.e., those which survive from the preceding period), the latter includes the current potential entrants.

At the start of the process the original set of incumbent firms $Z(0)$ is exogenously given. As time progresses, the sets $P(t)$, $Z(t)$ and $E(t)$ will evolve through entry, bankruptcy and exit, as described in 2.5 below.

2.2. Technological Structure

Let Θ denote the global technology space. It is assumed identical for all firms and endowed with a *directed graph (digraph) structure* (Θ, μ) . Here, μ represents a binary relation on Θ formalizing direct technological precedence, as presently explained.

When two technologies θ, θ' are consecutive points of Θ (i.e., $\theta \mu \theta'$) it is said that θ directly precedes θ' (technologically). Compositions of μ give rise to the notion of general (as opposed to direct or immediate) technological precedence. Specifically, when two technologies θ and θ' are joined by some μ -chain starting at θ and ending at θ' it will be simply said that θ technologically precedes θ' and we write $\theta \beta \theta'$. That is:

$$\theta \beta \theta' \Leftrightarrow \exists (\theta_1, \theta_2, \dots, \theta_n) \text{ s.t. } \theta_k \mu \theta_{k+1}, k = 1, 2, \dots, n, \theta_1 = \theta, \theta_n = \theta'.$$

The length of the shortest chain leading from θ to θ' will be denoted by $h(\theta, \theta')$. For the sake of formal convenience, I shall use the conventions $\theta \beta \theta$ and $h(\theta, \theta) = 0$ for all $\theta \in \Theta$, i.e., any technology θ "precedes" itself and defines a μ -chain of length zero.

Motivated by the interpretation of β as reflecting technological precedence, it will be assumed that it is an ordering (in general, a partial one) on Θ ; or, in the language of Graph Theory, that (Θ, μ) is an acyclic digraph. Such structure will be an essential component of both the innovation and imitation processes described in the next sections. In particular, it permits to make precise the notion of "technological gap" between two technologies which is presently introduced.

Consider any two different technologies θ' and θ'' and define the set of their common predecessors by:

$$Q(\theta', \theta'') = \{ \theta \in \Theta: \theta \beta \theta', \theta \beta \theta'' \}. \quad (1)$$

The technological gap from θ' to θ'' is defined as follows:

$$\gamma(\theta', \theta'') = \min \{ h(\theta, \theta''): \theta \in Q(\theta', \theta'') \}, \quad (2)$$

where it will be recalled that $h(\theta, \theta'')$ has been defined as the length of the shortest μ -chain leading from θ to θ'' . The formulated concept of technological gap is reminiscent of biological contexts where the genetic ("information") differences between two species can be linked to their separate divergent evolution from a common ancestor. Its present technological motivation is based on the interpretation of every μ -step in the digraph (Θ, μ) as an (homogeneous) "quantum" of knowledge. If any one of these knowledge quanta is not yet incorporated into the current technology, it needs to be learned (at a cost, as described below) in order to use any new technology that does include it.

2.3. Technology sets

At every period t , each firm $i \in I$ has a subset of Θ , denoted by $\Theta_i(t)$, as its current technological choice set. The law of motion for this set is defined as follows.

If firm $i \notin P(t)$, i.e., is not a current participant in the industry, it is simply written:

$$\Theta_i(t) = \{\#\}, \quad t = 0, 1, 2, \dots, \quad (3)$$

and interpreted to mean that firm i is forced to inaction, denoted by $\#$.

If firm $i \in P(t)$, i.e., it is a current participant in the industry, the following law of motion for its technology choice set is proposed:

$$\Theta_i(t) = (\Theta_i(t-1) \setminus D_i(t)) \cup M_i(t) \cup N_i(t), \quad t = 1, 2, \dots, \quad (4)$$

where, for the initial firms $i \in P(0)$, the set $\Theta_i(0) = \{\theta_0\}$ is exogenously given and:

- (i) $N_i(t)$ represents the set of firm i 's current inventions;
- (ii) $M_i(t)$ includes the technologies acquired through imitation of other firms;
- (iii) $D_i(t)$ stand for those technologies which are lost due to lack of usage; this phenomenon shall be called technological "dissipation".

A precise description of each of these sets presently follows.

(i) *Invention*

For each firm $i \in Z(t)$ which is an incumbent at t (i.e., had $\theta_i(t-1) \neq \#$), $N_i(t)$ is simply postulated to be some non-empty random sample from the set $S(\theta_i(t-1)) \cup \{\emptyset\}$, where:

(a) $S(\theta_i(t-1)) \equiv \{\theta \in \Theta: \theta_i(t-1) \mu \theta\}$ denotes the set of direct μ -successors of the technology $\theta_i(t-1)$, i.e., the previously adopted technology, and

(b) the element \circ is a fictitious term which stands for "failure".

The sample over $S(\theta) \cup \{\circ\}$ which formalizes "invention possibilities from θ " is assumed conducted according to some probability distribution $P_\theta(\cdot)$, exogenously given. This formulation reflects the idea that invention is uncertain, gradual and path-dependent. The cardinality of the set $N_i(t)$, denoted by $n_i(t-1)$, will be considered an object of decision during the preceding period (see subsection 2.6 below). It is assumed no smaller than one. Thus, just one invention draw is interpreted as the minimum degree of "learning by doing" associated to production itself during the preceding period. Any additional draw will reflect an R&D costly decision by the incumbent firm (c.f. assumption (A.3) in the next section). Note that, as formulated, R&D decisions only bear fruit with a one-period lag.

On the other hand, if firm $i \in E(t)$, i.e., it is a potential entrant at t (in particular, $\theta_i(t-1) = \#$), it is assumed that it cannot obtain any invention. This can be formalized by making $S(\#) = \emptyset$, i.e., "inaction" has no technological immediate successors. Admittedly, this is a very strong and unrealistic restriction. It is adopted just for simplicity and could be easily relaxed by assuming, for example, that any potential entrant may innovate from those technologies which it has available through imitation.

(ii) *Imitation*

As for the imitation possibilities reflected by $M_i(t)$ in (3), it is postulated that, for all $i \in P(t)$,

$$\begin{aligned} M(t) = M_i(t) &= \bigcup_{j \in I} \{\theta_j(\tau) : \tau \leq t - s_1\}, & \text{if } t \geq s_1 \\ &= \{\theta_0\}, & \text{if } t < s_1 \end{aligned} \quad (5)$$

That is, every firm $i \in P(t)$ has lagged access to all those actions available to any other firm in the sufficiently distant past. The time lag $s_1 \in \mathbb{N}$ is a parameter of the model. It may reflect considerations (exogenous to the model) which are related to, say, the speed of information transfer or the nature of the patent legislation.

(iii) *Dissipation*

Finally, technological information that is not being used somewhere in the industry by some viable firm is assumed lost ("dissipated") after some time. Denote by $V(t)$ the set of viable firms at t , i.e., those firms active at t which remain as incumbents at $t+1$. (The process of entry and exit is precisely described in Subsection 2.5 below.) The process of dissipation, which was identified in (3) with the sets $D_i(t)$, is now defined as follows:

$$D_i(t) = D(t) = \left\{ \theta \in \Theta: \theta \neq \theta_j(\tau), j \in V(\tau), t-s_2 \leq \tau < t \right\}, \quad (6)$$

where $s_2 \in \mathbb{N}$ is another parameter of the model, with $s_2 \geq s_1$. Note that, according to (6), in order for a technology to become integrated into the technological base of the industry, it must have been used by some viable firm (i.e., a firm that has lived through the whole period when the technology was used). Otherwise, it is assumed that such technology has not been used long enough to become assimilated into the "technological pool" of the industry.

2.4. Stage Payoffs

Stage payoffs are decomposed into three parts: gross payoffs, adjustment costs, and R&D expenditures. I describe each of them in turn.

(i) Gross payoffs are given by a function:

$$\varphi_i: \Theta^I \longrightarrow \mathbb{R}_+, \quad (7)$$

where we adopt the convention that:

$$\forall \underline{\theta} = (\theta_1, \theta_2, \dots) \in \Theta^I, \varphi_i(\underline{\theta}) = 0 \text{ if } \theta_i = \#. \quad (8)$$

Each function $\varphi_i(\cdot)$ gives firm i 's gross payoffs in "reduced form" with technologies as their sole arguments. It is easy to expand the model to include explicit market demands and production decisions. In Vega-Redondo (1991) this was done by contemplating a two-stage decision process in which the technology was first chosen simultaneously by all active firms, then followed by usual Cournot competition (see also Section 3.1 below for an illustration along these lines).

(ii) Adjustment costs for each firm i are given by a function:

$$C_i: (\Theta \cup \{\#\}) \times (\Theta \cup \{\#\}) \longrightarrow \mathbb{R}_+, \quad (9)$$

with the interpretation that $C_i(a,b)$ represents the adjustment cost in shifting from a to b , where any of these could either be some technology in Θ or $\#$ (inaction).

When both arguments are different from $\#$, it will be assumed that the function C_i admits the representation:

$$C_i(\theta, \theta') = \hat{C}_i(\gamma(\theta, \theta')), \quad (10)$$

i.e., it only depends on the technological gap involved in the shift from θ to θ' . (See (A.2) below.)

If $\#$ is involved, it is postulated that:

$$C_i(\#, \theta) = \eta, \quad (11)$$

where $\eta > 0$ is then interpreted as a fixed entry cost. Finally, the particular assumption made on the cost of switching to "inaction" $\#$ will be inessential for our purposes, as long as, naturally, we have:

$$C_i(\#, \#) = 0. \quad (12)$$

(iii) Finally, R&D expenditures for each firm i are given by a function

$$R_i: \mathbb{N} \longrightarrow \mathbb{R}_+, \quad (13)$$

where $R_i(\cdot)$ is some function of the number of invention draws enjoyed by firm i in the next period. (See (A.3) below.)

Given gross payoffs, adjustment costs, and research expenditures as described above, (net) payoffs result from subtracting the two latter from the former. Combining (8), (9) and (13), they are given for each firm $i \in I$ by a function

$$\pi_i: \Theta \times \Theta^I \times \mathbb{N} \longrightarrow \mathbb{R}, \quad (14)$$

which for every tuple $(\theta_i(t-1), \underline{\theta}(t), n_i(t))$ of preceding technology choice, current technology profile and invention level induces the net payoff:

$$\pi_i(\theta_i(t-1), \underline{\theta}(t), n_i(t)) = \varphi_i(\underline{\theta}(t)) - C_i(\theta_i(t-1), \theta_i(t)) - R_i(n_i(t)). \quad (15)$$

2.5. Entry and Exit

The rule of entry and exit in the industry is written as follows:

$$P(t) \equiv Z(t) \cup E(t) = [P(t-1) \setminus X(t-1)] \cup E(t), \quad t = 0, 1, 2, \dots, \quad (16)$$

where $Z(0) = \{1, 2, \dots, m\}$ is given, $X(t-1)$ stands for those firms in $P(t-1)$ which meet a certain exit condition in $t-1$, and $E(t)$ represents the set of firms which may consider entry at t . In each of these two latter respects, entry and exit, I shall specifically postulate the following.

As for firms' exit, it is assumed that there is some $v > 0$ such that:⁽⁴⁾

$$X(t) = \{ i \in P(t-1): \pi_i(\theta_i(t-1), \underline{\theta}(t), n_i(t)) < v \}. \quad (17)$$

That is, firms which are unable to obtain a stage payoff of at least v in any given period are forced to exit the game (irreversibly, given the entry formulation presented next). Of course, the implicit idea that capital markets are very imperfect must underlie the assumption that exit is exclusively linked to current payoffs. This contrasts sharply with the polar opposite (utterly perfect capital markets) considered in most of the literature (e.g. Ericson & Pakes (1989).) I find hard to say which of both extreme assumptions is more realistic or otherwise appropriate.

Entry, on the other hand, is formalized as follows:

$$E(t) = \{ t+m+1 \}, \quad (18)$$

that is, each firm $i \notin Z(0)$ is indexed according to its corresponding future time of entry. Such very stylized (and rigid) entry rule is chosen for its special simplicity. Other more elaborate ones could be considered with similar implications.⁽⁵⁾

⁴ Since gross payoffs have been assumed non-negative, "bankruptcy" is defined in terms of a positive V . This is simply done for formal convenience, since adjusting V to any level is just a matter of payoff scaling.

⁵ For example, firms could be "put in line", any of them being given the opportunity to enter when, and only when, all the preceding ones have already

2.6. The Dynamic Game

The items described in Sections 2.1 to 2.5 define most of the components of the model. A complete description of the induced dynamic stochastic game still requires the following complementary specifications.

(i) Order of play and available information

At each $t = 1, 2, \dots$, every firm $i \in P(t)$ chooses simultaneously its respective current actions $(\theta_i(t), n_i(t))$ knowing the sequence of all past actions:

$$\left\{ (\theta_j(\tau), n_j(\tau))_{j \in I} \right\}_{\tau < t}$$

and the current technology choice sets: ⁽⁶⁾

$$\left\{ \Theta_j(t) \right\}_{j \in I}.$$

Of course, as it is standard in Game Theory, it is also implicitly assumed that the rules of the game and the laws of the environment, i.e., every one of the items described in Sections 2.1 to 2.5, are common knowledge.

(ii) Intertemporal payoffs

Let $x^t \in (\Theta^I \times \mathbb{N}^I)^\infty$ denote a typical path of action profiles starting at any time t . That is:

$$x^t = [(\theta(t), \underline{n}(t)), (\theta(t+1), \underline{n}(t+1)), (\theta(t+2), \underline{n}(t+2)), \dots]. \quad (19)$$

Every firm $i \in I$ has at each time t an intertemporal payoff function defined over such paths:

$$\Phi_i: (\Theta^I \times \mathbb{N}^I)^\infty \longrightarrow \mathbb{R}. \quad (20)$$

⁶ The alternative assumption could be made that the realizations of the current invention draws of a firm are not observed by other firms. Or it could be postulated that the decision on the number of invention draws is only taken by a firm after it has adopted its technological decision, so that the play at every period involves two stages. These or other similar modifications could be contemplated without affecting the nature of the results.

which, given some discount rate $1 > \delta \geq 0$ (common to all firms), will be assumed to associate to every path x^t the discounted sum of the corresponding stream of stage payoffs (c.f. (15)). That is:

$$\Phi_i(x^t) = \sum_{\tau \geq t} \delta^{\tau-t} \pi_i(\theta_i(\tau-1), \underline{\theta}(\tau), n_i(\tau)). \quad (21)$$

2.7. Equilibrium Concept

The analysis of the game can be greatly simplified if we rely on its Markov structure and study it as a *Markov stochastic game*. A Markov description of the game requires the specification of a state space Ω , whose elements $\omega \in \Omega$ embody a sufficient description of all the payoff-relevant aspects of any game situation (i.e., of any history).⁽⁷⁾ In fact, for equilibrium analysis, it is enough to require that states discriminate only among situations which are payoff-relevant for strategies that are not strictly dominated.

Among alternative state spaces which are sufficient in this sense, it is of special interest that which is minimal. Let Ω^* be such state space. (It is easy to see that, by the minimality requirement, Ω^* must be unique.) For each firm $i \in I$, those strategies of the form:

$$\sigma_i: \Omega^* \longrightarrow \Delta(\Theta \times \mathbb{N}), \quad (22)$$

are called *Markovian strategies*, where $\Delta(\Theta \times \mathbb{N})$ denotes the set of probability measures on $\Theta \times \mathbb{N}$. At every period t , they induce a corresponding (possibly mixed) action $\sigma_i(\omega(t))$ which only depends on the current (minimally sufficient) state $\omega(t)$.

The game form proposed is totally symmetric among firms, except for the order of entrance in the industry. Or in other words, if such order were chosen randomly and anonymously *ex ante*, the rules of the game would treat all firms identically. Reflecting this and the fact that payoff functions will also be assumed symmetric (c.f. Section 3.1), it is natural to require that

⁷ (Definition 2.1 below).

firms' strategies reflect such "anonymity". To make this idea precise, let $p: I \rightarrow I$ be some given permutation of the set of firm indices. Denote by

$$\chi_p: \Omega^* \rightarrow \Omega^* \quad (23)$$

a mapping which, for each $i \in I$, interchanges the positions of firms i and $p(i)$ in all respects (including the order of entry). The following definition introduces the equilibrium concept which shall underlie the game-theoretic analysis of the paper.

2.1. Definition: A Markov Perfect Equilibrium (MPE) of the game is a profile of Markovian strategies $(\sigma_i^*)_{i \in I}$ such that, at every t and every prevailing state $\omega(t)$, it defines a Nash equilibrium of the continuation game. The MPE is called symmetric (SMPE) if for any permutation $p(\cdot)$ of the set of firm indices, for all $i \in I$ and $\omega \in \Omega^*$, $\sigma_i^*(\omega) = \sigma_{p(i)}^*(\chi_p(\omega))$.

2.2. Remark: It is immediate to see that every MPE is a subgame perfect equilibrium, and that every SMPE is itself a MPE. In other words, if a firm's competitors decide to ignore considerations which are either payoff-irrelevant or non-symmetric it is (weakly) optimal for this firm to ignore them also.

3.- ANALYSIS

This Section includes the analytical results of the paper. Its content is divided into three subsections. Subsection 3.1 introduces some required assumptions on gross-payoff and cost functions. Subsection 3.2. presents the particular families of digraph (technological) structures which shall be object of analysis. Finally, in Subsection 3.3, the results of the paper are formally presented and discussed.

3.1. Assumptions

The first assumption postulates a numerical representation of the "value" of different technologies and assumes that only their relative magnitude enters (symmetrically) in the gross payoff functions of firms.

(A.1) *There is some non-negative function $\rho: \Theta \rightarrow \mathbb{R}_+$ ($\rho(\#) = 0$) such that, for each firm $i \in I$, $\varphi_i(\underline{\theta}) = f_i(\underline{\rho}(\underline{\theta}))$ where $\underline{\rho}(\underline{\theta}) = (\rho(\theta_1), \rho(\theta_2), \dots)$ and the set of functions*

$$f_i: \mathbb{R}^I \rightarrow \mathbb{R}, i \in I,$$

are "anonymous" (i.e., invariant under index permutation), homogeneous of degree zero, continuous, and increasing in their respective $\rho_i \equiv \rho(\theta_i)$.

Moreover, $\forall \varepsilon > 0, \exists \delta > 0$ such that:

$$\forall \rho \in \mathbb{R}^I, \frac{\rho_i}{\sum_{j \neq i} \rho_j} \leq \delta \implies f_i(\rho) \leq \varepsilon.$$

The set of functions $(f_i)_{i \in I}$ should be conceived as a reduced-form construct representing some underlying process of market competition. A simple context which satisfies the preceding assumption is provided by a Chamberlinian market in which:

(a) Each firm i confronts a specific inverse-demand function for its "type of product" θ_i of the following form:

$$P_i(\underline{q}, \underline{\theta}) = \frac{\rho(\theta_i)}{\sum_{j \in I} \rho(\theta_j)} \Phi \left(\sum_{j \in I} q_j \right), \quad (24)$$

where q_i is the quantity produced of firm i , and $\Phi(\cdot)$ is some real function.

(b) There are no fixed (nor sunk) costs.

(c) Firms take their decisions in two steps: first, simultaneously, on technologies; second, also simultaneously, on quantities produced.

The next two assumptions deal with the nature of adjustment costs and R&D expenditures.

(A.2) For all $i \in I$, $\hat{C}_i(\cdot) = \hat{C}(\cdot)$ is monotonically non-decreasing with $\hat{C}(1) = 0$. Moreover, $\forall M \exists \gamma_0$ such that if $\gamma > \gamma_0$, $\hat{C}(\gamma) > M$.

(A.3) For all $i \in I$, $R_i(\cdot) = R(\cdot)$ is monotonically non-decreasing with $R(1) = 0$. Moreover, $\forall M \exists n_0$ such that if $n > n_0$, $R(n) > M$.

The preceding assumptions contemplate symmetric cost functions for R&D activities and technological switching which are identical for all firms, monotonically non-decreasing, and unbounded in their respective arguments. It is also assumed that both gradual adjustment (one μ -step at a time) and gradual invention (one sample draw at a time) are costless. The justification of this latter assumption derives from the idea that some learning-by-doing on the job is simply "automatic" and requires no specific resources devoted to it. In any case, the analytical purpose of this assumption is simply to ensure that the process of technological invention (if anything, at such slow pace) will continue forever in full probability.

3.2. Alternative Digraphs

The analysis shall focus on three alternative types of digraphs.

(i) A linear structure where every option belongs to the same "technological line".

(ii) A branching structure (a directed tree) where every bifurcation gives rise to "irreversible divergence" of their respective successors.

(iii) A reticular structure where bifurcations are not necessarily irreversible but have some successor paths which eventually merge.

The above three structures represent very stylized formulations of three different qualitative kinds of technological scenarios. Their starkness aims at highlighting in the most clear-cut way the main issues involved.

Before discussing each of these structures in detail, I introduce the following simplifying assumption which will be considered in every case.

(A.4) (i) *There is some fixed $\xi > 1$ such that $\forall \theta \in \Theta, \forall \theta' \in S(\theta), \rho(\theta') = \xi \rho(\theta)$.*

(ii) *$\forall \theta, \theta' \in \Theta, P_{\theta}(S(\theta)) = P_{\theta}(S(\theta')) < 1$ with $P_{\theta}(\cdot | S(\theta))$ uniform on $S(\theta)$.*

The previous assumption requires that: (i) every technological "quantum" involves the same proportional advance in value (c.f. (A.1)); (ii) invention from every technology θ is always equally uncertain and symmetric across successors in $S(\theta)$. As explained below, this assumption is a way of ensuring that the technological structure is stationary, i.e., isomorphic to its substructures.

(i) A Linear Structure

The digraph (Θ, μ) is a *linear technological structure* if:

$$(L) \quad \forall \theta, \theta' \in \Theta, \theta \beta \theta' \text{ or } \theta' \beta \theta.$$

Under (L), all technologies can be viewed as more or less advanced points along a common technological ladder. This is the implicit assumption in most of the literature on technological change (see, for example, Helpman & Grossman (1991) or the aforementioned Ericson & Pakes (1989)). One of the essential objectives of the paper is to contrast it with the following two alternatives.

(ii) A Branching Structure

To define it formally, some additional notation is needed. A finite sequence $\Gamma = (\theta_1, \theta_2, \dots, \theta_m)$ is called a path of (Θ, μ) if $\forall k=1, 2, \dots, m-1, \theta_k \mu \theta_{k+1}$. Given such a path, denote by $s(\Gamma) \equiv \theta_1$ and $e(\Gamma) \equiv \theta_m$ its first and last elements, respectively. The digraph (Θ, μ) is a *branching technological structure* (or a tree) if:

(B)(i) $\forall \theta, \theta' \in \Theta, \theta \beta \theta'$, there is a unique path Γ s.t. $s(\Gamma) = \theta, e(\Gamma) = \theta'$.

(ii) $\forall \theta, \theta' \in \Theta, \text{card } S(\theta) = \text{card } S(\theta') \geq 2$.

Under (B), every technology gives rise to at least two disconnected families of succeeding technologies. This is in sharp contrast with (L) above. Some sort of "compromise" between (B) and (L) is provided by the third type of technological structure which is considered next.

(iii) A Reticular Structure

Informally speaking, the idea captured by a reticular structure is the following. Consider any given technology θ and the set of its technological successors $S(\theta)$. Suppose that, besides having a higher value (recall (A.4)), the technologies in $S(\theta)$ may also differ from θ in terms of a number of different relevant dimensions (or characteristics).⁽⁸⁾ Further assume that (a) these characteristics can be appropriately quantified in terms of integer numbers, and (b) any technology is uniquely defined by its value and a particular specification of its characteristics. Then, for any two technologies θ and θ' with $\theta \beta \theta'$ there will generally be many different paths

⁸ Consider, for example, the case of the computer manufacturing industry. In it, each particular technology may differ in the number of bits per chip used (keeping, say, the number of chips fixed) or differ in the number of chips used with the same of number of bits per chips. Swan (1991) carries out an empirical analysis of the technological competition in the computer industry exactly along these lines. In fact, he uses directed graphs similar to those employed here to describe recent technological developments in this industry. In Foray & Grübler (1989), such digraphs are also used to understand the technological evolution of the casting industry.

linking them. In other words, even though technological bifurcations are possible, they are not irreversible.

I now formalize these matters. Consider a number of different dimensions $D = D_1 \times D_2 \times \dots \times D_m$, where $D_j = \{-r_j, -r_j+1, \dots, -1, 0, 1, \dots, r_j-1, r_j\}$, $r_j \in \mathbb{N}$,⁽⁹⁾ represent the $2r_j+1$ different values of the j th dimension. A digraph (Θ, μ) is a *reticular technological structure* (or a lattice) if:

(R) (i) Every $\theta \in \Theta$ is uniquely characterized by its $\rho(\theta)$ and the vector of characteristics $d(\theta) \in D$.

(ii) There is some $k = (k_1, k_2, \dots, k_m) \in \mathbb{N}^m$ such that $\forall \theta \in \Theta$,
 $\{d(\theta') : \theta' \in S(\theta)\} = \{\hat{d} \in D : |\hat{d}_j - d_j(\theta)| \leq k_j\}$.

Part (i) requires no further explanation. Part (ii) postulates that, for any given θ , the vectors of characteristics spanned by its successors are all those which, in each dimension j , are not farther away from $d_j(\theta)$ than k_j . The following diagram illustrates a digraph satisfying (R) in which there is only one dimension ($D = D_1$), $r_1=2$, and $k_1=1$.

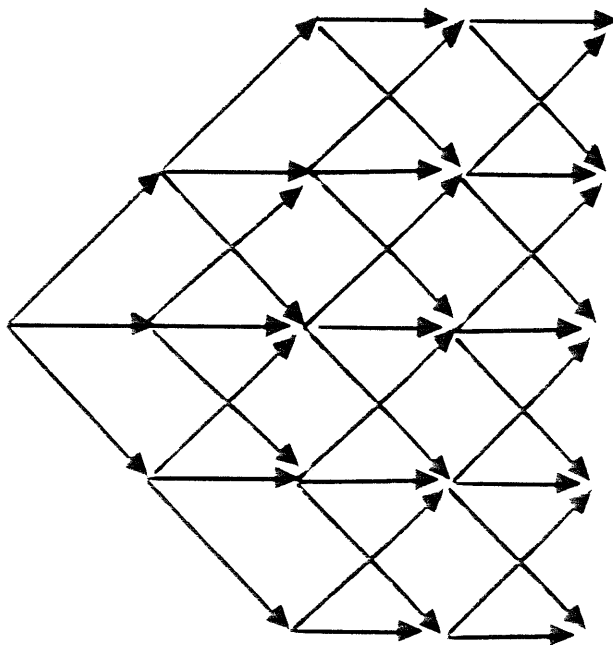


FIGURE 1

A reticular structure: $D_1 = D$ and $r_1 = 2, k_1 = 1$

⁹ Note that if r_j is zero for each $j = 1, 2, \dots, m$, then a linear technological structure obtains.

3.3. Results

(i) Existence and Ergodicity of an SMPE

I start with the following existence result.

3.3.1. **Theorem:** Assume (A.1) to (A.4), and (L), (B), or (R). A Symmetric Markov Perfect Equilibrium (SMPE) always exists.

Let $\underline{\sigma}^* = (\sigma_i^*)_{i \in I}$ be a SMPE. Given an initial state ω_0 , $\underline{\sigma}^*$ defines a stochastic process on Ω^* , the minimally sufficient state space for the game (c.f. Subsection 2.6). Consider now any real function

$$\zeta: (\Omega^*)^q \rightarrow \mathbb{R}, \quad (25)$$

for some $q \in \mathbb{N}$. For every state path $\underline{\omega} \equiv (\omega(0), \omega(1), \omega(2), \dots) \in (\Omega^*)^\infty$, a real path $\zeta(\underline{\omega}) \equiv (y(q), y(q+1), y(q+2), \dots)$ can be associated through $y(t) = \zeta(\omega(t-q+1), \omega(t-q), \dots, \omega(t))$ for each $t \geq q$. This induces, given $\underline{\sigma}^*$ and ω_0 , a real stochastic process in the obvious fashion. Such process shall be denoted by $\mathcal{P}(\underline{\sigma}^*, \zeta, \omega_0)$ in order to express its dependence of the strategy profile $\underline{\sigma}^*$, the function ζ , and the initial condition $\omega(0) = \omega_0$.

The case where the function ζ is "anonymous" is of special interest. In line with our preceding formalization of symmetry (or anonymity) of a MPE, the function ζ is said to be *anonymous* if for any $(\omega_1, \omega_2, \dots, \omega_q) \in (\Omega^*)^q$ and any set of index permutations $p_j: I \rightarrow I, j=1, 2, \dots, q$,

$$\zeta(\omega_1, \omega_2, \dots, \omega_q) = \zeta(\chi_{p_1}(\omega_1), \chi_{p_2}(\omega_2), \dots, \chi_{p_q}(\omega_q)). \quad (26)$$

Natural examples of real variables specified by such anonymous functions in our context are the following: number of active firms every period, average profitability or research intensity, the rates of change or variances of any of these variables, etc.

Given the stochastic process describing a certain aspect of industrial dynamics, a natural question to ask is whether one can obtain from it any expected long run behavior. When this is the case, such behavior is summarized by the so-called *invariant distribution*. More ambitiously, it can be inquired whether such invariant distribution is unique (i.e., independent of initial

conditions). If so, the corresponding stochastic process is called *ergodic*. In this case, one can show that the unique invariant distribution is also the unique limit distribution in the following strong sense: any path of the process induces a realized distribution which converges with probability one to the invariant distribution in the long run. Thus, in this case, such distribution may be viewed as a compact description of the actual (not just expected) long-run behavior of the process. (See Karlin & Taylor (1975) for a standard reference on these issues.)

The analysis that follows addresses these issues. I start with the following result.

3.3.2. Theorem: Assume (A.1) to (A.4) and (L), or (B), or (R). Let $\underline{\sigma}^* = (\sigma_i^*)_{i \in I}$ be an SMPE and ζ an anonymous real function as in (25). For any $\omega_0 \in \Omega^*$, the stochastic process $\mathcal{P}(\underline{\sigma}^*, \zeta, \omega_0)$ induces a well-defined invariant distribution.

The next issue to deal with is that of ergodicity. In order to establish it, I shall need to restrict to equilibria which avoid situations of technological "impasse". A formalization of this idea is presently introduced.

Denote by $\hat{p}(t) \equiv \max \{p_i(t), i \in I\}$, i.e., the "technological frontier" of the industry at t . Let $n^*(t)$ denote the aggregate set of successful inventions (i.e., invention draws which are different from \emptyset) occurred at t . Given some SMPE $\underline{\sigma}^*$, I postulate:

(*) $\forall \hat{\omega} \in \Omega^*, \forall B > 0, \forall t, t', \exists m \in \mathbb{N}$ such that:

$$\text{Prob}_{\underline{\sigma}^*} \left\{ \frac{\hat{p}(t')}{\hat{p}(t)} \geq B \mid \sum_{\tau=t}^{t'} n^*(\tau) \geq m, \omega(t) = \hat{\omega} \right\} = 1.$$

Condition (*) precludes situations of "strategically-enforced" technological stagnation. It asserts that if the number of successful invention draws becomes large enough between two points in time, the technological frontier of the industry should grow to any arbitrary level.⁽¹⁰⁾

¹⁰ Note that since, in equilibrium, the number of innovation draws per firm i bounded above (c.f. (A.4)), the condition could have been equivalently stated in terms of a sufficient number of successful "inventors".

In the Appendix, I present a simple two-firm example where Condition (+) is violated for an SMPE. It describes a situation where each firm decides to remain technologically stationary under the (credible) threat that, if it were to deviate, the technological competition that would follow would bring about its eventual bankruptcy with positive probability. Under either the assumption that firms are sufficiently patient or that the potential "monopoly rents" are small, this is enough to deter the adoption of any innovation.⁽¹¹⁾ Building upon this idea, it is easy to construct equilibria which induce non-ergodic stochastic processes.

In general, it seems clear that the sort of "implicit collusion" which must underlie a violation of Condition (+) can only prevail if firms are sufficiently patient. This conjecture is confirmed by the next Proposition, which also provides a scenario where Condition (+) always holds. An alternative justification of this condition is discussed in Remark 3.3.5 below.

3.3.3. Proposition: *Assume (A.1) to (A.4) and either (L), or (B), or (R). There exists some $\delta_0 > 0$ such that, if $\delta \leq \delta_0$, condition (+) is satisfied for every SMPE.*

Under Condition (+), the following ergodicity result is now established.

3.3.4. Theorem: *Assume (A.1) to (A.4) and (L), or (B), or (R). Let ζ be an anonymous real function as in (25) and $\underline{\sigma}^* = (\sigma_i^*)_{i \in I}$ an SMPE which satisfies condition (+). There exists some $\eta_0 > 0$ such that if $\eta \leq \eta_0$, the stochastic process $\mathcal{P}(\underline{\sigma}^*, \zeta, \omega_0)$ is ergodic, i.e., its invariant distribution is independent of ω_0 .*

3.3.5. Remark: In Ericson & Pakes (1989), the industrial dynamics induced by their model is shown to be ergodic under a condition such as (+) derived endogenously from the model. The main reason why this is possible in their context is that they contemplate the existence of an "upward competitive

¹¹ This idea is strongly reminiscent of the argument used in the classical Fol Theorems of repeated games. The essential difference here is that the threat which supports equilibrium behavior is restricted to depend only on payoff relevant aspects of the situation.

drift" imposed on the industry from outside (e.g. the development of better competing products elsewhere in the economy). Since potential "threatening" entrants are supposed to enjoy the benefits of this outside drift in their ever improving entering conditions, this external pressure precludes any equilibrium strategy which may lead to technological stagnation. It seems very plausible that the consideration of a similar external "environment" in the present model would allow to dispense with the exogenous need of Condition (*) for establishing ergodicity. If this were the case, this condition could be interpreted as an indirect way of incorporating constraints on equilibrium behavior imposed on the industry by external, un-modelled trends.

(ii) *Population dynamics*

This section provides some further results which illustrate more specific implications of the model. They will focus on the extent of turnover induced by the alternative types of technological structures considered: (L), (B), or (R). Very schematically, they establish that, if the discount factor and entry cost are small enough:

(a) Under (L), there will be no firm turnover when the technological change is sufficiently gradual. In this case, if entry is not too costly, the limit distribution over population sizes is concentrated in its maximum. (Proposition 3.3.6);

(b) Under (B), the process of turnover will continue indefinitely. More specifically, every firm which enters will also exit later on in full probability. Furthermore, the limit distribution over population sizes has full support over all feasible values (Proposition 3.3.7);

(c) Under (R), the conclusions of (a) or (b) will essentially obtain if, essentially, the "technological span" of Θ , $\sum_{j=1}^m r_j$, is, respectively, small or large enough (Proposition 3.3.8).

Denote by $z(t) \equiv |Z(t)|$ the number of incumbents at t . Let \bar{z} represent the maximum number of viable incumbents which may simultaneously survive in the industry. By (A.1) and (17), this number is finite (see expression (36) in the Appendix for its precise determination). Denote by $\zeta_z(\cdot)$ the function which assigns to every state ω in Ω^* the corresponding number of incumbents. As above, let $\mathcal{P}(\Omega^*, \zeta_z, \omega_0)$ represent the induced stochastic process and

$\lambda(\underline{\sigma}^*, \zeta_z, \omega_0)$ the density function of its invariant distribution. The previous informal statements are now precisely stated.

3.3.6. **Proposition:** Assume (A.1) to (A.4) and (L). There is some $\delta_0 > 0$, $\xi_0 > 1$, such that if $\delta \leq \delta_0$ and $\xi \leq \xi_0$, then $Z(t) \subseteq Z(t')$, $\forall t' > t$, in every SMPE $\underline{\sigma}^*$. Moreover, there is some $\eta_0 > 0$ such that if $\eta \leq \eta_0$, the ergodic distribution has $\lambda(\underline{\sigma}^*, \zeta_z, \cdot)(\bar{z}) = 1$.

3.3.7. **Proposition:** Assume (A.1) to (A.4), (B) and $\bar{z} \geq 2$. There is some $\delta_0 > 0$ and $\eta_0 > 0$, such that if $\delta \leq \delta_0$ and $\eta \leq \eta_0$, $\forall \varepsilon > 0$, $\exists \Delta > 0$ such that if $t' - t \geq \Delta$ then $Z(t) \cap Z(t') = \emptyset$ with probability of at least $1 - \varepsilon$ in every SMPE $\underline{\sigma}^*$. Moreover, the ergodic distribution $\lambda(\underline{\sigma}^*, \zeta_z, \cdot)$ has support $\{1, 2, \dots, \bar{z}\}$.

3.3.8. **Proposition:** Assume (A.1) to (A.4), (R) and $\bar{z} \geq 2$. There are some $\delta_0 > 0$, $\xi_0 > 1$, $\eta_0 > 0$, such that if $\delta \leq \delta_0$, $\xi \leq \xi_0$, $\eta \leq \eta_0$, then:

(i) There is some $\bar{r} > 0$, $\bar{c} > 0$ such that if $\sum_{j=1}^m r_j \leq \bar{r}$, $\hat{C}(2) \leq \bar{c}$, then $Z(t) \subseteq Z(t')$, $\forall t' > t$, in every SMPE $\underline{\sigma}^*$. Moreover, the ergodic distribution has $\lambda(\underline{\sigma}^*, \zeta_z, \cdot)(\bar{z}) = 1$.

(ii) Given $k \in \mathbb{N}^m$, there is some $\underline{r} > 0$ such that if $\sum_{j=1}^m r_j \geq \underline{r}$, $\forall \varepsilon > 0$, $\exists \Delta > 0$, such that if $t' - t \geq \Delta$ then $Z(t) \cap Z(t') = \emptyset$ with probability of at least $1 - \varepsilon$ in every SMPE $\underline{\sigma}^*$. Moreover, the ergodic distribution $\lambda(\underline{\sigma}^*, \zeta_z, \cdot)$ has support $\{1, 2, \dots, \bar{z}\}$.

The intuition lying behind the preceding results is easy to describe.

Under (L), all firms' technologies belong to the same "line" and remain relatively close throughout. Thus, if technological innovation is sufficiently gradual no firm becomes backwards enough to be forced to exit the industry.

Under (B), and if firms are not too far-sighted, they will end up "betting" on different technological lines. When their technological paths become sufficiently far apart, large relative advances along any one of them will lead to the bankruptcy of those firms which are already too "committed"

to alternative ones. In the limit, this produces an industry in continuous flux, with its invariant distribution having full support.

Finally, scenario (R) displays qualitative features of both (L) and (B): there is technological branching, but not irreversible one. Thus, it is not surprising that the induced behavior reproduces that of either (L) or (B), depending on whether the technological range is small or large. In the former case, the branching possibilities remain quite limited and are overcome by the possibility of technological reversibility (provided both technological change and switching costs are sufficiently gradual). In the latter case, technological divergence may (and will, from time to time) become sufficiently large to make the eventual bankruptcy of firms unavoidable.

(iii) Suggestions for further research and the role of simulations

A variety of other interesting results could be derived within the set-up described. For example, it follows easily from the preceding analysis that, if firms are impatient, the alternative technological structures (L), (B), or (R) have significantly different effects on the long-run profitability variance of the industry. Thus, while under (L) this variance remains bounded within relatively narrow limits, under (B) - or (R) with a large technological range - the process will witness wide differences in firms' profits.

Another important concern of future research should be to explore the effect of different parameters of the model on the equilibrium incentives to innovate (i.e. to "buy innovation draws"). In this respect, the diffusion lag s_1 would represent a natural candidate, whose interpretation, for example, as the regulated life-span of patents could permit extracting from the resulting conclusions some valuable insights on technological policy.

The technological structure itself should also have interesting implications on the incentives to innovate. For example, it seems intuitive that, under technological scenario (L), quite backwards technological followers should never spend resources on innovation and rely instead exclusively on imitation. This would be in agreement with the empirical regularity which indicates that only firms which are close to the technological frontier carry out genuine innovation activities (c.f. Dosi,

Pavitt & Soete (1990)). However, under alternative technological assumptions such as (B) or (R), I would conjecture that path-dependence considerations would probably add some interesting caveats to the previous statement.

The above comments simply illustrate some of the pending questions which could be addressed in future research. In general, of course, a complete analysis of the industrial dynamics should ideally focus on the full knowledge of the limit or invariant distribution of the corresponding stochastic process. However, the complexity of the framework proposed makes it a formidable task to attempt a general analytical approach to this problem. Thus, a sensible alternative is to have dynamic simulations "solve" indirectly for the stationary distribution in particular contexts. This is a particularly sound option when the dynamic process is known to be ergodic (c.f. Theorem 3.3.4). By conducting such simulations for a wide range of different parameters and alternative technological scenarios, such simulation exercises should yield some light on interesting "comparative-dynamics" issues. This task is also left for future research.

4.- SUMMARY AND CONCLUSIONS

This paper has proposed an intertemporal model of industrial technological competition in which firms' choices and possibilities are subject to crucial path-dependent considerations. The technological space has been endowed with a digraph structure. This formalizes, in a stylized fashion, such notions as technological precedence, technological gaps, or switching costs, all of which play a key role in the model.

At each point in the process, active firms adopt a technology within their choice set, itself the result of their own past innovating activities and a (lagged) process of technological diffusion across the whole industry. New firms enter into the industry only gradually as allowed by current technological availability. On the other hand, incumbent firms exit when forced by bankruptcy. This context induces a dynamic game. In it, firms have been assumed to play a Symmetric Markov Perfect Equilibria (SMPE). That is, a symmetric equilibrium where each firm's strategy only depends on payoff-relevant considerations.

The analysis has focused on three alternative types of technological structures: linear (L), branching (B), or reticular (R). First, it was proven that an SMPE always exists. Second, it was shown that every SMPE induces a well-defined limit (or invariant) distribution for any associated real and anonymous function of the prevailing situation. Third, conditions have been proposed which guarantee that such limit distributions are ergodic, i.e., independent of initial conditions.

The second part of the paper has investigated the effect of different technological structures on the "population dynamics" of the industry. It was shown, in particular, that if firms are relatively impatient the extent of population turnover may be highly sensitive to the underlying technological structure. Thus, the situation will range from no turnover under (L) to a never-ending process of firm renewal under (B). Scenario (R) was seen to yield either one conclusion or the other depending on the magnitude of the technological range.

In the last part of the paper, some heuristic comments have been proposed which suggest further issues for future research. It was also argued that, in view of the complexity of the theoretical framework, a fuller analysis of long-run behavior may require the use of computer simulations. The above mentioned ergodicity results support this option.

A P P E N D I X

Proof of Theorem 3.3.1:

The proof could proceed, in a more straightforward way, by conceiving the game as the limit of essentially finite "truncated" games, as in Fudenberg & Levine (1983). Because of its future use, however, the existence proof given here will rely on the fact that, under the postulated assumptions, every SMPE of the game may be defined upon a finite state space. To establish this fact, a Markov description of the game will be proposed which is then shown to admit a finite but sufficient "symmetric" coarsening.

Consider the basic state space:

$$\Omega^0 \equiv \left(\mathcal{H}_x \ 2^\Theta \right)^I, \quad (27)$$

where a typical state $\omega^0 \in \Omega^0$ specifies, for each firm $i \in I$, the pair $\omega_i^0 \equiv (h_i, N_i)$ where the first component is firm i 's technological history $h_i \in \mathcal{H} \equiv (\Theta \cup \{\emptyset\})^\infty$ and the second one is the set of firm i 's current invention draws. Here, the symbol \emptyset represents a "dummy" constant which fills unneeded (or inexistent) dimensions. (For example, finite histories are represented by sequences which are constantly equal to \emptyset beyond a certain point.)

Such an "exhaustive" state space is obviously sufficient for all strategies which are not strictly dominated and are only dependent on payoff-relevant aspects of the game history. First, it is artificially enlarged as follows:

$$\Omega^1 \equiv \left(\mathcal{H}_x \ 2^\Theta \right)^I \times \mathcal{H}^I, \quad (28)$$

where $\omega^1 = (\omega^0, h)$, $h \in \mathcal{H}^I$. If strategies are symmetric, it is possible to abstract from the firm indices and restrict attention to participating firms. (Firms only differ in their order of entry. Thus, once they have entered, payoff relevant strategies should abstract fully from their particular index. On the other hand, a non-participating firm, has the only trivial strategy of selecting the unique element of its choice set, namely #).

By (A.1) and (17), there are at most a finite number of simultaneously participating firms, say v , in any given period. On the other hand, by (A.3), the number of invention draws chosen by any firm in any period is bounded throughout by some given natural number, say d . Finally the postulated process of technological dissipation, permits ignoring histories going back longer than s_2 periods. These three facts combined render it sufficient to contemplate a state space

$$\Omega^2 \equiv (\Theta \cup \{\emptyset\})^u \quad (29)$$

where there are at most $u = 2(v \cdot s_2) + d$ "dimensions", appropriately codified.

Addressing each of the different technological scenarios in turn -- (L), (B), and (R) -- it is shown next that, under (A.1) to (A.4), any SMPE can be defined on a finite coarsening of Ω^2 .

(i) Scenario (L)

By (A.1) and (L), each technology θ is uniquely characterized by its corresponding value $\rho(\theta)$. For each $x \in \Omega^2$, let

$$V_j \equiv \{\tilde{\rho} \in \mathbb{R}: \tilde{\rho} = \rho(\theta), \theta \in \Theta_j\}, \quad (30)$$

for each of the participating firms, $j = 1, 2, \dots, v'$, $v' \leq v$. The homogeneity of degree zero of the payoff functions allows the values in

$$V = \bigcup_{j=1}^{v'} V_j$$

to be normalized, making $\min \{\rho: \rho \in V\} = 1$. Then, associated to any $x \in \Omega^2$, a real vector $y(x) \in \mathbb{R}^u$ may be associated with the convention that $\rho(\emptyset) = 0$. Moreover, by (A.4(i)), for any positive components y_i and y_j ($i, j = 1, 2, \dots, u$) the relation $y_i = \xi^k y_j$ must hold for some $k \in \mathbb{Z}$, an integer number. I now show that such integers k must remain bounded across all reachable states (i.e., those states which have any possibility of occurring along the process).

Denote by $\bar{\gamma} \in \mathbb{N}$ a sufficiently large number of " μ -steps" such that $\hat{C}(\bar{\gamma}) > f_1(1, 0, 0, \dots)$. Such $\bar{\gamma}$ is ensured by (A.2) and represents a bound on the number

of μ -steps which any incumbent firm will adjust its technology in any given period. Given any t and the state $x(t) \in \Omega^2$ prevailing in it, consider $y(x(t)) \in \mathbb{R}^u$, as described above. Let $h[y(x(t))] \in \mathbb{R}$ be the maximum component of $y(x(t))$. The fact that innovation is gradual and "dissipated" when not used in s_2 periods implies, in view of the preceding considerations, that:

$$\forall \tau, \tau': t-s_2 \leq \tau, \tau' \leq t, h[y(x(\tau))] \geq h[y(x(\tau'))] \xi^{-s_2 \cdot \bar{\gamma}}. \quad (31)$$

By (A.1), there is a minimum ratio ε between the technological values of two firms which is consistent with the survival of the laggard. Let k_0 satisfy $\xi^{-k_0} < \varepsilon$. Expression (31) implies that any firm $i \in P(\tau)$, $t-s_2 \leq \tau < t$, which is active and survives at τ , i.e., $i \in Z(\tau+1)$, must use a technology $\theta_i(\tau)$ with a value

$$\begin{aligned} \rho(\theta_i(\tau)) &\geq \xi^{-k_0} h[y(x(\tau))] \\ &\geq \xi^{-k_0} h[y(x(\tau'))] \xi^{-s_2 \cdot \bar{\gamma}} \geq \xi^{-(s_2 \cdot \bar{\gamma} + k_0)} \rho(\theta_i(\tau')) \end{aligned} \quad (32)$$

for all τ' , $t-s_2 \leq \tau' < t$. Denote $b \equiv s_2 \cdot \bar{\gamma} + k_0 + 1$. The previous argument permits replacing the state space Ω^2 in (29) by:

$$\Omega_L^3 \equiv \left\{ y \in \mathbb{R}^u: \exists y_i=1 \ \& \ y_j, y_{j'} > 0 \implies (\exists k \in \mathbb{Z}, |k| \leq b, y_j = \xi^k y_{j'}) \right\}, \quad (33)$$

which is obviously a finite set, as desired.

Scenario (B)

The considerations underlying (31) and (32) which permit the restriction to a finite set of value vectors also apply in this case. Under (B), however, a technology is no longer characterized by its value alone but requires the specification of its position in the graph. The key idea in the argument that follows is that, given the fact that no technological shift will ever be considered which involves a gap larger than $\bar{\gamma}$ above, any configuration in the graph may be described without any loss of relevant information through a finite "truncated tree" which never involves branches joining bifurcation points longer than $\bar{\gamma}$.

Let $x \in \Omega^2$, as described above. Let $\hat{\Theta}$ be the (finite) set of non-dummy components of x and let $\hat{\rho}: \hat{\Theta} \rightarrow \mathbb{R}$ be uniquely defined such that $\min_{\theta \in \hat{\Theta}} \{ \hat{\rho}(\theta) \} = 1$ and $\forall \theta, \theta' \in \hat{\Theta}$,

$$\frac{\hat{\rho}(\theta)}{\hat{\rho}(\theta')} = \frac{\rho(\theta)}{\rho(\theta')}. \quad (34)$$

As above, it can be guaranteed that $\forall \theta, \theta' \in \hat{\Theta}$, $\hat{\rho}(\theta) = \xi^k \hat{\rho}(\theta')$ with $|k| \leq b$.

Complement now $\hat{\Theta}$ with ad-hoc "technologies" to form a set $\tilde{\Theta} \supseteq \hat{\Theta}$ such that $(\tilde{\Theta}, \tilde{\mu})$ is a tree ($\tilde{\mu}$, $\tilde{\beta}$, and $\tilde{\gamma}$ will denote the counterparts of μ , β , and γ) with the following characteristics. First, it preserves the ordering given by β in the underlying tree (Θ, μ) :

$$(i) \quad \forall \theta, \theta' \in \hat{\Theta}, \theta \tilde{\beta} \theta' \Leftrightarrow \theta \beta \theta'.$$

Second, the function $\hat{\rho}$ is extended to $\tilde{\Theta}$, giving rise to a function $\tilde{\rho}: \tilde{\Theta} \rightarrow \mathbb{R}$ meeting the following requirement:

$$(ii) \quad \forall \theta, \theta' \in \tilde{\Theta}, \theta \tilde{\mu} \theta' \Rightarrow \tilde{\rho}(\theta') = \xi \hat{\rho}(\theta).$$

Third, the $\tilde{\gamma}$ -distance between technologies in $\hat{\Theta}$ whose γ -distance does not exceed $\bar{\gamma}$ is preserved:

$$(iii) \quad \forall \theta, \theta' \in \hat{\Theta}, \gamma(\theta, \theta') \leq \bar{\gamma} \Rightarrow \tilde{\gamma}(\theta, \theta') = \gamma(\theta, \theta').$$

Finally, the $\tilde{\gamma}$ -distance between consecutive "branching points" of $(\tilde{\Theta}, \tilde{\mu})$ is bound by $\bar{\gamma}$. Denote by $B(\tilde{\Theta}, \tilde{\mu}) = \{ \theta \in \tilde{\Theta} : \exists \theta', \theta'', \theta' \neq \theta'', \theta \tilde{\mu} \theta', \theta \tilde{\mu} \theta'' \}$, i.e., the set of branching points of $(\tilde{\Theta}, \tilde{\mu})$.

(iv) Let $\theta, \theta' \in B(\tilde{\Theta}, \tilde{\mu})$, $\theta \neq \theta'$, such that $\nexists \theta'' \in B(\tilde{\Theta}, \tilde{\mu})$, different from θ and θ' with $\theta \tilde{\beta} \theta'' \tilde{\beta} \theta'$. Then $\tilde{\gamma}(\theta, \theta') \leq \bar{\gamma}$.

Clearly, given any state $x \in \Omega_2$, a tree satisfying (i)-(iv) can always be associated to it. Under (B), it represents a sufficient description of the game situation. Moreover, given $\bar{\gamma}$ and b , the set of minimal such trees is

finite. They define, therefore, the elements of a sufficient and finite state space, which will be denoted by Ω_B^3 .

Scenario (R)

As before, for any state $x \in \Omega^2$ and every pair of technologies θ, θ' which are components of x , it can be ensured that $\rho(\theta) = \xi^k \rho(\theta')$ with $|k| \leq b$ for some pre-specified b . Thus, by normalizing technological values appropriately, any current state may be sufficiently represented within a reticular structure of the same dimension range as the original one and a finite depth which is only dependent on b and $\sum_{j=1}^m r_j$. Being analogous to the preceding arguments, I do not dwell into further details. The finite state space which results which results will be denoted by Ω_R^3 .

It has been shown that for any of the three technological structures contemplated, (L), (B), and (R), there is a finite state space Ω_h^3 , $h = L, B, R$, which, for any given firm, is able to reflect all the pay-off relevant and symmetric considerations. Let Ω_L^s, Ω_B^s , and Ω_R^s be maximal coarsenings of their respective former counterparts. Since each Ω_h^3 is finite so is Ω_h^s for each $h = L, B, R$. Denote by $\Theta_h^s(\omega)$ the (finite) number of actions that the given firm has in state $\omega \in \Omega_h^s$. The existence of a SMPE then immediately follows from the fact that if the firms restrict to strategies of the form

$$\sigma: \Omega_h^s \longrightarrow \Delta \left(\bigcup_{\omega \in \Omega_h^s} \Theta^s(\omega) \right), \quad h = L, B, R, \quad (35)$$

the game can be viewed as a finite game. This completes the proof. ■

Proof of Theorem 3.3.2:

Let Ω^* be the minimally sufficient joint state space of the game, as introduced in Subsection 6.2 (iii). Denote by Ω^{**} the quotient space Ω^*/ψ , where the equivalence relation ψ is defined by:

$\forall \omega, \omega' \in \Omega^*, \omega \psi \omega' \Leftrightarrow$ There exists a permutation $p(\cdot)$ of I s.t.
 $\omega = \chi_p(\omega')$

If firms play a symmetric MPE $\underline{\sigma}^*$ (i.e., strategies as in (35)), the stochastic process on Ω^* induced by $\underline{\sigma}^*$ and any initial state $\omega_0 \in \Omega^*$ will only depend on (i.e., its transition probabilities will be a function of) the element $[\omega] \in \Omega^{**}$ to which the current ω belongs. Thus, it may be viewed as a stochastic process on Ω^{**} . On the other hand, any anonymous function ζ as in (25) is constant on any element of Ω^{**} . Therefore, it may be also regarded as a function on Ω^{**} .

From the considerations explained in the preceding proof, it is clear that the set Ω^{**} is finite. By a well-known result on Markov Chains (see, for example, Karlin & Taylor (1975)), every Markov process defined on a finite state space has a well-defined invariant distribution from any initial state. The existence of such invariant distribution on Ω^{**} for any initial state $\omega_0 \in \Omega^*$ induces a corresponding invariant distribution for the process $\mathcal{P}(\underline{\sigma}^*, \zeta, \omega_0)$. This completes the proof. ■

Example: Condition (*) can be violated in a SMPE.

Consider scenario (B), assumptions (A.1)-(A.4), and payoff conditions which allow at most two viable firms in the market (c.f. (17) and (36) below). Let:

- (i) $\hat{C}(2) = R(2) > f_1(1,0,0,\dots)$.
- (ii) $f_1(\rho, \xi\rho, 0, 0, \dots) < \eta < f_1(\rho, \rho, 0, 0, \dots)$;
- (iii) $\forall i, j \in I, \rho_i \geq \xi^2 \rho_j \Rightarrow f_i(\underline{\rho}) < v$;

Thus, by (i), no firm survives if it performs a technological shift involving a gap of more than one quantum or choose more than one invention draw; by (ii), every new firm which enters the industry will do so with technological-value parity with the incumbent; by (iii), any firm that lags two ξ -steps in technological value will not survive.

As in the proof of Theorem 3.3.1, let $\omega^0 = (\omega_i^0)_{i \in I} \in \Omega^0$, where $\omega_i^0 \equiv (h_i, N_i)$. Consider a state with two incumbents, say 1 and 2, and the following partial prescriptions:

(a) Let $h = (h_1, h_2)$ be such that both firms are at the same status-quo technology and enjoy the same technology choice set prior to invention. Then, neither firm adopts any invention, staying with the prior choice.

(b) Let $h = (h_1, h_2)$ be such that the status-quo technologies of each firm, θ_1 and θ_2 , display $\gamma(\theta_i, \theta_j) \geq 3$ for each $i, j = 1, 2, i \neq j$. Then each firm adopts any successful invention if one arises; otherwise, it stays with the preceding choice.

I shall now argue that, under certain conditions on payoffs and parameter values, the previous prescriptions form part of a SMPE. I simply sketch the argument. Denote:

$$\begin{aligned} f_1(1,0,0,\dots) - f_1(1,1,0,0,\dots) &= \Delta_1 > 0; \\ f_1(1,1,0,\dots) - f_1(1,\xi,0,0,\dots) &= \Delta_2 > 0. \end{aligned}$$

The key point to check is that deviations from (a) can be deterred. Given the parameters of the model, and in particular the discount factor δ , choose the ratio

$$\alpha_1 \equiv \frac{\Delta_1}{f_1(1,0,0,\dots)}$$

sufficiently small. Then, any deviation from (a) can entail at most an arbitrarily small gain in total (discounted) payoff. Any positive probability of such gain would be more than offset if there is also positive probability (independent of α_1) that the firms will enter into a situation where (b) applies. For, in this case, there will be positive probability (again, independent of α_1) that the firm goes bankrupt in finite time. This will deter the deviation if α_1 is small enough.

To see that after any deviation from (a), there is a given positive probability that (in finite time) (b) applies, focus on the following chain of events. After, say, firm 1 adopts an invention deviating from (a), firm 2 obtains one successful invention different from that of firm 1, followed by three subsequent periods when both firms obtain successful inventions. Suppose that the diffusion lag $s_1 \geq 4$. Then, to adopt its respective inventions by each firm is a mutual best response if, given

$$\alpha_2 \equiv \frac{\Delta_2}{f_1(1,1,0,\dots)},$$

δ is not too large. Once the circumstances contemplated in (b) apply, they remain in place until one of the firms goes bankrupt, with probabilities over different paths which are only dependent of invention probabilities, not on the rest of parameters of the model. Once a firm goes bankrupt, the other one enjoys a "temporary monopoly" which, in view of (ii), ends with the conditions contemplated by (a) and restores a stationary situation. Given this fact, to follow the prescriptions of (b) is also a mutual best response for each firm, again provided that α_1 is sufficiently small.

From the above discussion, it follows that, under the conditions described, any situation displaying the features contemplated by (a) is an absorbing state of the process. This violates condition (+). Relying on the idea embodied in this example, it should be clear how to construct equilibria which, by having more than one absorbing set of states, yield non-ergodic processes. ■

Proof of Proposition 3.3.3:

Consider first the case with $\delta = 0$. Then, any SMPE defines a Nash equilibrium of the one-shot game induced at any t by $\Theta_i(t)$, the technology choice set of firm i after invention, and the previous technological choice $\theta_i(t-1)$. By (A.1) and (A.2), it is a strictly dominated strategy for any firm $i \in Z(t)$ not to increase the value of its technology if it has obtained a successful invention draw. (This does not mean that it must necessarily adopt one such invention when arises. However, if it does not, it must be because it shifts to some other technology available through imitation that itself must have a higher technological value than the status quo.) Analogously, it is a dominated strategy to choose a technology of lower value than the current status quo for any incumbent, or one of lower value than the maximum available in the industry by an entrant. Therefore, none of these strategies can belong to any one-shot Nash equilibrium.

Let now $\delta > 0$. If $\delta \leq \delta_0$ for sufficiently small δ_0 , the preceding restrictions on equilibrium strategies also applies to any SMPE. Given any

$m, t, t' \in \mathbb{N}$, suppose $\sum_{\tau=t}^{t'} n^*(\tau) \geq m$. Then, since, by (A.3), the equilibrium number of invention draws of any incumbent firm is bounded above, say by d , this implies that there have been at least $q \equiv m/d$ successful inventing firms from t to t' . By making m sufficiently large, Condition (\dagger) must be satisfied for any $B > 0$. This is a consequence of the following considerations: (a) any successful inventing firm will raise its technological value; (b) a firm which lags by some maximum pre-specified (relative) amount behind any firm of the industry will not survive; (c) any new entrant will choose the technology available with the highest value. Points (a) and (c) were explained above. Point (b) is a consequence of assumption (A.1) and (17). This completes the proof of the Proposition. ■

Proof of Theorem 3.3.4:

By Theorem 3.3.2, the stochastic process $\mathcal{P}(\underline{\sigma}^*, \zeta, \omega_0)$ induces a well-defined invariant distribution for any anonymous function ζ and initial state $\omega_0 \in \Omega^*$. To show that such invariant distribution is independent of initial conditions, it is enough to show that, for every state $\omega \in \Omega^*$, there is positive probability of reaching some state in a given equivalence class $[\hat{\omega}] \in \Omega^{**}$ (c.f. the proof of Theorem 3.3.2). Given the anonymity of ζ , this implies that the process $\mathcal{P}(\underline{\sigma}^*, \zeta, \omega_0)$ has a single ergodic class. Or, equivalently, that it is ergodic. (See, for example, Karlin & Taylor (1975, Theorem 1.3)).

Let $e_z \equiv (1, 1, \dots, 1, 0, 0, \dots)$ denote the vector in \mathbb{R}^I whose first z components are equal to one and the remaining components equal to zero. By (A.1), the maximum carrying capacity of the market \bar{z} is defined as follows:

$$\bar{z} = \max \left\{ z \in \mathbb{N} : \varphi_1(e_z) > v \right\} \quad (36)$$

The class $[\hat{\omega}]$ that shall be used in the argument is described as follows. There are \bar{z} incumbent firms, i.e., for any $\hat{\omega} \in [\hat{\omega}]$, $|Z(\hat{\omega})| = \bar{z}$. All firms $i \in P(\hat{\omega})$ (including the potential entrant) have the same technological choice sets $\hat{\Theta}_i = \{\hat{\theta}\}$, where $\hat{\theta}$ is also the status-quo technology of incumbent firms. Thus, $\hat{\omega}$ can be identified with some initial conditions, as described in Section 2.3, where the industry starts at "full capacity".

I provide the argument in detail for scenario (B). The reader should find no difficulty in adapting the line of proof to the other two simpler scenarios (L) and (R). Let $\omega(t_0)$ be any state in Ω^* prevailing at some t_0 . A chain of events of positive probability will be specified which leads to some state in $[\hat{\omega}]$ in finite time.

For any $t \in \mathbb{N}$, denote by

$$\Theta(t) \equiv \bigcup_{i \in P(t)} \Theta_i(t) \quad (37)$$

the set of globally available technologies. This set can be divided into a collection of (not necessarily disjoint) subsets $C(t) = \{c_1(t), c_2(t), \dots, c_n(t)\}$ where each $c_k(t)$ include those technologies in $\Theta(t)$ which belong to the same "technological line". This concept can be formalized as follows:

$$\forall \theta, \theta' \in \Theta(t), \{\theta, \theta'\} \subseteq c_k(t) \in C(t) \Leftrightarrow \theta \beta \theta' \text{ or } \theta' \beta \theta \quad (38)$$

It will be useful below to write the preceding expression in the following counter-reciprocal equivalent fashion:

$$\forall \theta, \theta' \in \Theta(t), \neg (\{\theta, \theta'\} \subseteq c_k(t)) \Leftrightarrow \exists q \in \mathbb{N}: \gamma(\theta, \theta') \geq q, \gamma(\theta', \theta) \geq q. \quad (39)$$

Throughout the contemplated chain of events, it will be supposed that, starting from t_0 , no future realization of the invention process "opens a new technological line". To formalize this idea, let $\hat{\beta}$ stand for the strict (non-reflexive) part of β and, for any $\theta \in \Theta$ and any subset $\Xi \subseteq \Theta$, let the notation $\theta \beta \Xi$ (or $\theta \hat{\beta} \Xi$) indicate that θ β -precedes (or $\hat{\beta}$ -precedes) some element of Ξ . Suppose that for all $t > t_0$, the following holds:

$$(i \in Z(t), \theta_{i(t-1)} \hat{\beta} \Theta(t-1)) \Rightarrow \forall \theta \in N_i(t), \theta \beta \Theta(t-1). \quad (40)$$

Given $B > 0$, let $m(B)$ be determined as required by Condition (+). If B is chosen large enough and the hypothesis of this condition is satisfied at some $t_1 > t_0$, then q in (39) can be chosen at t_1 arbitrarily large for any two available technologies belonging to different technological lines in $C(t_1)$. Or, more formally, $\forall q \in \mathbb{N}, \exists B > 0$ such that:

$$\begin{aligned} & \left(\sum_{\tau=t_0}^{t_1} n^*(\tau) \geq m(B), \theta \in c_k(t_1), \theta' \in c_{k'}(t_1), k \neq k' \right) \\ & \Rightarrow \gamma(\theta, \theta') \geq q, \gamma(\theta', \theta) \geq q \end{aligned} \quad (41)$$

The preceding statement is clearly true for current technological choices; to see that it must also hold for any of the available technologies, recall the argument used in the first part of the proof of Theorem 3.3.1 where it was shown that the gap of past but still available technologies to those currently used is bounded.

Thus, choose q large enough so that, by (A.2), it can be ensured that no shift across different technological lines will be performed at t_1 by any of the incumbents. Let

$$\hat{\rho} \equiv \max \{ \rho(\theta) : \theta \in \bigcup_{i \in Z(t_1)} \Theta_i(t_1) \},$$

and assume that for some t_2 and all $t \in \mathbb{N}$, $t_1 \leq t \leq t_2$, the following holds:

$$[i \in Z(t), \rho(\theta_i(t)) = \hat{\rho}] \Rightarrow N_i(t+1) = \{\circ\}; \quad (42a)$$

$$[i \in Z(t), \rho(\theta_i(t)) < \hat{\rho}] \Rightarrow N_i(t+1) \neq \{\circ\}. \quad (42b)$$

That is, only those firms below the technological frontier prevailing at t_1 obtain successful invention draws. By choosing t_2 sufficiently large, (40) implies that it must be that for all $\theta \in \Theta(t_2)$, $\rho(\theta) = \hat{\rho}$. Otherwise, Condition (*) would be clearly violated. If it happens that there exists a unique technological line in $C(t_2)$, then the essential part of the argument will be complete (see below). Otherwise, choose any of the technological lines, say $c_1(t_2)$, and assume that for some sufficiently large t_3 and all $t \in \mathbb{N}$, $t_2 \leq t \leq t_3$, the following holds:

$$[i \in Z(t), \theta_i(t) \in c_1(t)] \Rightarrow N_i(t+1) = \{\circ\}; \quad (43a)$$

$$[i \in Z(t), \theta_i(t) \notin c_1(t)] \Rightarrow N_i(t+1) \neq \{\circ\}, \quad (43b)$$

where $c_1(t)$, $t > t_2$, is defined in the obvious fashion, composed of the technological successors of the (unique) technology belonging to $c_1(t_2)$. By choosing t_3 large enough, it is clear that only incumbents using technologies

in $c_1(t_3)$ will remain. From there on, a chain of events analogous to that described in (42) can be contemplated such that for some finite t_4 ,

- (i) $|Z(t_4)| = \bar{z}$,
- (ii) $\theta_i(t_4) = \tilde{\theta}$,
- (iii) $\Theta_i(t_4) = \{\tilde{\theta}\}$.

Thus $\omega(t_3) \in [\hat{\omega}]$, as desired. Point (i) obtains, if the entry cost is small enough, by the rule of entry postulated in Subsection 2.5. Point (ii) follows from Condition (*), as above. Finally, Point (iii) is simply a consequence of the process of technological dissipation postulated in Subsection 2.3.

It just remains to show that the above described chain of events has positive probability and can be completed within a pre-specified duration which bounds $t_4 - t_0$. But this immediately follows from the assumptions of the model, completing the proof of the Theorem for scenario (B). As indicated above, the proof for the other two scenarios (L) and (R) is an easy adaptation of the preceding argument. ■

Proof of proposition 3.3.6:

Consider first the case where $\delta = 0$. Starting from an initial state ω_0 with the features described in Subsection 2.3, it is clear that for all time t and every $i, j \in Z(t)$, it must be the case that

$$\frac{\rho_i(t)}{\rho_j(t)} \leq \xi^{S_1}, \quad (44)$$

since it is a strict best response in every SMPE (which, under $\delta = 0$, is a Nash equilibrium of the one-shot game arising in each t) to adopt any available technology which is an immediate successor of the preceding one (c.f. (A.2)). This implies, by (A.1), that, provided that ξ_0 is chosen small enough, no incumbent firm will be forced to exit the industry. Thus, $Z(t) \subseteq Z(t')$, $\forall t' > t$, as desired, for this case. Since such conclusion is based on strict best responses by firms, it still holds if $\delta_0 > 0$ is selected sufficiently small and $\delta \leq \delta_0$. This shows the first part of the Proposition. For its second part note that, if entry costs are small, there is probability

1 that, at some t , the cardinality of $Z(t)$ equals \bar{z} . Since thereafter $Z(t') = Z(t)$ for all $t > t'$, the limit distribution $\lambda(\underline{\sigma}^*, \zeta_z, \cdot)$ must be concentrated at \bar{z} , as claimed. This completes the proof. ■

Proof of Proposition 3.3.7:

To establish the first part of the Proposition, it is enough to show that, at any given t_0 , every firm $i \in Z(t_0)$ has positive probability of exiting the market within some maximum number of periods. This ensures that set of incumbent firms at two sufficiently distant points in time will be disjoint with an arbitrarily high probability.

Consider any $\omega(t_0) \in \Omega^*$. By Proposition 3.3.3, $\exists \delta_0 > 0$ such that, if $\delta \leq \delta_0$, Condition (\dagger) is satisfied for any SMPE $\underline{\sigma}^*$. If, furthermore, the entry cost is small, the line of argument used in the proof of Theorem 3.3.4 shows that from any state $\omega \in \Omega^*$, there is some minimum positive probability p_0 that the process reaches a state in the equivalence class $[\hat{\omega}] \in \Omega^{**}$, as described above, within some pre-established number of periods, say r . Recall that in any such a state there are \bar{z} incumbent firms and all of them have the same technological choice sets $\hat{\Theta}_i = \{\hat{\theta}\}$, where $\hat{\theta}$ is also the status-quo technology of incumbent firms.

Let $\omega(t_1) \in [\hat{\omega}]$, $t_1 = t_0 + r$. From any such a state, choose one particular $i_0 \in Z(t_1)$ and consider the following chain of events. For all $t \in \{t_1+1, \dots, t_2\}$,

$$\forall j \in Z(t_1), N_j(t) \neq \{\emptyset\}, |N_j(t)| = 1; \quad (45a)$$

$$\forall j, j' \in Z(t_1), j \neq i_0 \neq j', N_j(t) = N_{j'}(t); \quad (45b)$$

$$\forall j \in Z(t_1), j \neq i_0, N_i(t_1+1) \cap N_j(t_1+1) = \emptyset. \quad (45c)$$

If t_2 is chosen large enough, (A.3) implies that both firm i_0 and the rest of incumbents are, with respect to each other, "technologically isolated". That is:

$$\forall j \in Z(t_1), j \neq i_0, \gamma(\theta_j(t_2), \theta_{i_0}(t_2)) \geq q^*, \gamma(\theta_{i_0}(t_2), \theta_j(t_2)) \geq q^*, \quad (46)$$

where q^* is large enough that it deters imitation in any direction across firm i_0 and the rest of incumbents in $Z(t_1)$.

Consider now, for all $t \in \{t_2+1, \dots, t_3\}$, the following further series of events:

$$\forall j \in Z(t), j \neq i_0, N_j(t) = \{\circ\}, \quad (47a)$$

$$N_{i_0}(t) \neq \{\circ\}. \quad (47b)$$

If t_3 is chosen large enough, (A.1) implies that there must be some t' , $t_2 < t' \leq t_3$ such that $Z(t') = \{i_0\}$. The preceding chain of events from t_1 to t_3 described above has, given our assumptions, some positive probability, say p_1 . Thus, from any state $\omega(t_0)$ prevailing at t_0 , from which the argument started, there is positive probability $\bar{p} = p_0 \cdot p_1$ that the eventual situation referred at t' obtains. Since the firm i_0 in (45) was chosen arbitrarily among the incumbents at t_1 , this implies that, from any state $\omega \in \Omega^*$, there is positive probability no smaller than \bar{p} that any current incumbent will exit the industry in the next $t_3 - t_1$ periods. This establishes the first part of the Proposition. As for its second part, note that the preceding argument shows that both $z = 1$ and $z = \bar{z}$ must have positive weight in the the ergodic distribution $\lambda(\underline{\sigma}^*, \zeta_z, \cdot)$ for the SMPE $\underline{\sigma}^*$. Since entry is gradual (c.f. (18)), this implies that $\lambda(\underline{\sigma}^*, \zeta_z, \cdot)(z) > 0$ for all $z \in \{1, 2, \dots, \bar{z}\}$. The proof of the Proposition is complete. ■

Proof of Proposition 3.3.8:

Since the proof of Parts (i) and (ii) parallels very closely that of Propositions 3.3.6 and 3.3.7 respectively, I will just sketch the argument.

To show Part (i), consider only one dimension and $r_1 = 1$. If switching costs are gradual enough, it is easy to see that, provided δ and $\hat{C}(2)$ are small, the inequality (44) derived above for scenario (L) is now replaced by:

$$\frac{\rho_i(t)}{\rho_j(t)} \leq \xi^{S_1+1}. \quad (48)$$

If ξ_0 is small, the desired conclusions follow for the case considered. This yields the upper bound claimed in the general statement.

As for Part (ii), it follows from the fact that, given $k \in \mathbb{N}^m$, if $\sum_{j=1}^m r_j$ is large enough, the chain of events described in (45)-(47) for scenario (B) can still be chosen so as to induce the sole survival of firm i_0 at some corresponding time $t' \leq t_3$. To see why, suppose, for simplicity, that there is just one dimension and postulate the series of events (45) and (47) with the following additional requirement in (45). For all t , $t_1 < t \leq t_2$,

$$\theta \in N_{i_0}(t) \Rightarrow d_1(\theta) = \min \{ d_1(\theta_{i_0}(t-1) + 1, r_1 \} \quad (49a)$$

$$\forall j \in Z(t_1), j \neq i_0, \theta \in N_j(t) \Rightarrow d_1(\theta) = \max \{ d_1(\theta_{i_0}(t-1) - 1, -r_1 \}. \quad (49a)$$

Even with these additional requirements, the contemplated series of events has positive probability, bounded above zero. If the "dimension range" r_1 is large enough, they will eventually produce the exit of all firms in $Z(t_1)$ except i_0 since, beyond t_2 , the technological gap between the former firms and the latter can be made arbitrarily large. Hereafter, the argument proceeds as in the proof of Proposition 3.3.7. ■

REFERENCES

Arthur, B. (1989): "Competing Technologies, Increasing Returns, and Lock-in by Historical Events", Economic Journal, 99, 116-31.

David, P. (1988): "Path-Dependence: Putting the Past into the Future of Economics", Technical Report no. 533, Institute for Mathematical Studies in the Social Sciences, Stanford University.

Dosi, G, K. Pavitt, & Luc Soete (1990): The Economics of Technological Change, Harvester Wheatsheaf.

Ericson, R. & A. Pakes (1989): "An Alternative Theory of Firm and Industry Dynamics", mimeo, Columbia and Yale Universities.

Foray, D. & A. Grübler (1991): "Morphological Analysis, Diffusion and Patterns of Technological Evolution: Ferrous Casting in France and the FRG", in Diffusion of Technologies and Social Behavior, edited by N. Nakícenović & A. Grübler, Springer Verlag.

Fudenberg, D. & D. Levine (1983): "Subgame Perfect Equilibria of Finite and Infinite Horizon Games", Journal of Economic Theory, 31, 227-256.

Fudenberg, D. & J. Tirole (1991): Game Theory, MIT Press.

Grossman, G. & E. Helpman (1991): "Quality Ladders and Product Cycles", Quarterly Journal of Economics, v. 106, 557-86.

Hopenhayn, H. (1992): "Entry, Exit, and Firm Dynamics in Long Run Equilibrium", Econometrica, 60, 1127-1150.

Iwai, K. (1984): "Schumpeterian Dynamics: An Evolutionary of Innovation and Imitation", Journal of Economic Behavior and Organization, 5, 159-90.

Jovanovic, B. (1982): "Selection and the Evolution of Industry", Econometrica, vol. 50 (3), 649-70.

Jovanovic, B. & G.M. McDonald (1988): "Competitive Diffusion", mimeo, New York University and University of Western Ontario.

Karlin, S. & H.M. Taylor (1975): A First Course in Stochastic Processes, Academic Press.

Nelson, R. & S. Winter (1982): The Evolutionary Theory of Economic Development, Harvard University Press.

Reinganum, J. (1991): "The Timing of Research, Development, and Diffusion", Handbook of Industrial Organization, vol. I, edited by R. Schmalensee & R.D. Willig, Elsevier Science Publishers B.V.

Swan, P. (1991): "Rapid Technological Change, 'Visions of the Future', Corporate Structure and Market Structure", International Symposium on "New Frontiers in Science and Engineering, Paris, May.

Vega-Redondo, F. (1991): "Technological Change and Market Structure: An Evolutionary Approach", Working Paper A Discusión, WP-AD 91-10, Universidad de Alicante.

PUBLISHED ISSUES

FIRST PERIOD

- 1 "A Metatheorem on the Uniqueness of a Solution"
T. Fujimoto, C. Herrero. 1984.
- 2 "Comparing Solution of Equation Systems Involving Semipositive Operators"
T. Fujimoto, C. Herrero, A. Villar. February 1985.
- 3 "Static and Dynamic Implementation of Lindahl Equilibrium"
F. Vega-Redondo. December 1984.
- 4 "Efficiency and Non-linear Pricing in Nonconvex Environments with Externalities"
F. Vega-Redondo. December 1984.
- 5 "A Locally Stable Auctioneer Mechanism with Implications for the Stability of General Equilibrium Concepts"
F. Vega-Redondo. February 1985.
- 6 "Quantity Constraints as a Potential Source of Market Inestability: A General Model of Market Dynamics"
F. Vega-Redondo. March 1985.
- 7 "Increasing Returns to Scale and External Economies in Input-Output Analysis"
T. Fujimoto, A. Villar. 1985.
- 8 "Irregular Leontief-Straffa Systems and Price-Vector Behaviour"
I. Jimenez-Raneda / J.A. Silva. 1985.
- 9 "Equivalence Between Solvability and Strictly Semimonotonicity for Some Systems Involving Z-Functions"
C. Herrero, J.A. Silva. 1985.
- 10 "Equilibrium in a Non-Linear Leontief Model"
C. Herrero, A. Villar. 1985.
- 11 "Models of Unemployment, Persistent, Fair and Efficient Schemes for its Rationing"
F. Vega-Redondo. 1986.
- 12 "Non-Linear Models without the Monotonicity of Input Functions"
T. Fujimoto, A. Villar. 1986.
- 13 "The Perron-Frobenius Theorem for Set Valued Mappings"
T. Fujimoto, C. Herrero. 1986.
- 14 "The Consumption of Food in Time: Hall's Life Cycle Permanent Income Assumptions and Other Models"
F. Antoñazas. 1986.
- 15 "General Leontief Models in Abstract Spaces"
T. Fujimoto, C. Herrero, A. Villar. 1986.

- 16 "Equivalent Conditions on Solvability for Non-Linear Leontief Systems"
J.A. Silva. 1986.
- 17 "A Weak Generalization of the Frobenius Theorem"
J.A. Silva. 1986
- 18 "On the Fair Distribution of a Cake in Presence of Externalities"
A. Villar. 1987.
- 19 "Reasonable Conjectures and the Kinked Demand Curve"
L.C. Corchón. 1987.
- 20 "A Proof of the Frobenius Theorem by Using Game Theory"
B. Subiza. 1987.
- 21 "On Distributing a Bundle of Goods Fairly"
A. Villar. 1987.
- 22 "On the Solvability of Complementarity Problems Involving Vo-Mappings and its Applications to Some Economic Models"
C. Herrero, A. Villar. 1987.
- 23 "Semipositive Inverse Matrices"
J.E. Peris. 1987.
- 24 "Complementary Problems and Economic Analysis: Three Applications"
C. Herrero, A. Villar. 1987.
- 25 "On the Solvability of Joint-Production Leontief Models"
J.E. Peris, A. Villar. 1987.
- 26 "A Characterization of Weak-Monotone Matrices"
J.E. Peris, B. Subiza. 1988.
- 27 "Intertemporal Rules with Variable Speed of Adjustment: An Application to U.K. Manufacturing Employment"
M. Burgess, J. Dolado. 1988.
- 28 "Orthogonality Test with De-Trended Data's Interpreting Monte Carlo Results using Nager Expansions"
A. Banerjee, J. Dolado, J.W. Galbraigh. 1988.
- 29 "On Lindhal Equilibria and Incentive Compatibility"
L.C. Corchón. 1988.
- 30 "Exploiting some Properties of Continuous Mappings: Lindahl Equilibria and Welfare Egalitaria Allocations in Presence of Externalities"
C. Herrero, A. Villar. 1988.
- 31 "Smoothness of Policy Function in Growth Models with Recursive Preferences"
A.M. Gallego. 1990.
- 32 "On Natural Selection in Oligopolistic Markets"
L.C. Corchón. 1990.

- 33 "Consequences of the Manipulation of Lindahl Correspondence: An Example"
C. Bevia, J.V. LLinares, V. Romero, T. Rubio. 1990.
- 34 "Egalitarian Allocations in the Presence of Consumption Externalities"
C. Herrero, A. Villar. 1990.

SECOND PERIOD

- WP-AD 90-01 "Vector Mappings with Diagonal Images"
C. Herrero, A.Villar. December 1990.
- WP-AD 90-02 "Langrangean Conditions for General Optimization Problems with Applications to Consumer Problems"
J.M. Gutierrez, C. Herrero. December 1990.
- WP-AD 90-03 "Doubly Implementing the Ratio Correspondence with a 'Natural' Mechanism"
L.C. Corchón, S. Wilkie. December 1990.
- WP-AD 90-04 "Monopoly Experimentation"
L. Samuelson, L.S. Mirman, A. Urbano. December 1990.
- WP-AD 90-05 "Monopolistic Competition : Equilibrium and Optimality"
L.C. Corchón. December 1990.
- WP-AD 91-01 "A Characterization of Acyclic Preferences on Countable Sets"
C. Herrero, B. Subiza. May 1991.
- WP-AD 91-02 "First-Best, Second-Best and Principal-Agent Problems"
J. Lopez-Cuñat, J.A. Silva. May 1991.
- WP-AD 91-03 "Market Equilibrium with Nonconvex Technologies"
A. Villar. May 1991.
- WP-AD 91-04 "A Note on Tax Evasion"
L.C. Corchón. June 1991.
- WP-AD 91-05 "Oligopolistic Competition Among Groups"
L.C. Corchón. June 1991.
- WP-AD 91-06 "Mixed Pricing in Oligopoly with Consumer Switching Costs"
A.J. Padilla. June 1991.
- WP-AD 91-07 "Duopoly Experimentation: Cournot and Bertrand Competition"
M.D. Alepuz, A. Urbano. December 1991.
- WP-AD 91-08 "Competition and Culture in the Evolution of Economic Behavior: A Simple Example"
F. Vega-Redondo. December 1991.
- WP-AD 91-09 "Fixed Price and Quality Signals"
L.C. Corchón. December 1991.

- WP-AD 91-10 "Technological Change and Market Structure: An Evolutionary Approach"
F. Vega-Redondo. December 1991.
- WP-AD 91-11 "A 'Classical' General Equilibrium Model"
A. Villar. December 1991.
- WP-AD 91-12 "Robust Implementation under Alternative Information Structures"
L.C. Corchón, I. Ortuño. December 1991.
- WP-AD 92-01 "Inspections in Models of Adverse Selection"
I. Ortuño. May 1992.
- WP-AD 92-02 "A Note on the Equal-Loss Principle for Bargaining Problems"
C. Herrero, M.C. Marco. May 1992.
- WP-AD 92-03 "Numerical Representation of Partial Orderings"
C. Herrero, B. Subiza. July 1992.
- WP-AD 92-04 "Differentiability of the Value Function in Stochastic Models"
A.M. Gallego. July 1992.
- WP-AD 92-05 "Individually Rational Equal Loss Principle for Bargaining Problems"
C. Herrero, M.C. Marco. November 1992.
- WP-AD 92-06 "On the Non-Cooperative Foundations of Cooperative Bargaining"
L.C. Corchón, K. Ritzberger. November 1992.
- WP-AD 92-07 "Maximal Elements of Non Necessarily Acyclic Binary Relations"
J.E. Peris, B. Subiza. December 1992.
- WP-AD 92-08 "Non-Bayesian Learning Under Imprecise Perceptions"
F. Vega-Redondo. December 1992.
- WP-AD 92-09 "Distribution of Income and Aggregation of Demand"
F. Marhuenda. December 1992.
- WP-AD 92-10 "Multilevel Evolution in Games"
J. Canals, F. Vega-Redondo. December 1992.
- WP-AD 93-01 "Introspection and Equilibrium Selection in 2x2 Matrix Games"
G. Olcina, A. Urbano. May 1993.
- WP-AD 93-02 "Credible Implementation"
B. Chakravorti, L. Corchón, S. Wilkie. May 1993.
- WP-AD 93-03 "A Characterization of the Extended Claim-Egalitarian Solution"
M.C. Marco. May 1993.
- WP-AD 93-04 "Industrial Dynamics, Path-Dependence and Technological Change"
F. Vega-Redondo. July 1993.
- WP-AD 93-05 "Shaping Long-Run Expectations in Problems of Coordination"
F. Vega-Redondo. July 1993.

- WP-AD 93-06 "On the Generic Impossibility of Truthful Behavior: A Simple Approach"
C. Beviá, L.C. Corchón. July 1993.
- WP-AD 93-07 "Cournot Oligopoly with 'Almost' Identical Convex Costs"
N.S. Kukushkin. July 1993.
- WP-AD 93-08 "Comparative Statics for Market Games: The Strong Concavity Case"
L.C. Corchón. July 1993.

