TECHNOLOGICAL CHANGE AND MARKET STRUCTURE: AN EVOLUTIONARY APPROACH*

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WP-AD 91-10

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* This work was completed while the author was a Research Fellow at the Institute for Advanced Studies of the Hebrew University of Jerusalem. I thank helpful conversations with J. Friedman, as well as financial support from the Spanish Ministry of Education, CICYT project n° PB90-0613.

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ABSTRACT

The paper studies an inter-temporal market context in which firms innovate, imitate, and compete in quantities and technological choices each period. Potential entrants enter if there are profitable opportunities; incumbent firms exit when they go bankrupt. The key aspect of the model is that technological change evolves along a directed graph. This graph reflects both the direction of technological change and the magnitude of costs involved in switching technologies. In this set-up, our main concern is to explore the implications of different technological structures on entry/exit dynamics and on the evolution of market characteristics (market concentration, profitability variance, etc.)

"Time is invention, or it is nothing at all".

Bergson [1911, p.361][1]

1.- INTRODUCCION

Following Schumpeter, the effect of an industry's market structure on its rate of technological change has long been a central concern of the Theory of Industrial Organization. In this paper, I shall focus on a reciprocal issue which, some important exceptions notwithstanding,[2] has attracted much less attention in recent literature. Namely, the role played by technological change in shaping some of the industry's "structural" characteristics; in particular, of course, its degree of concentration, but also some of its more dynamic features such as the industry's pattern of exit and entry.

We shall also be concerned with providing some insight on what the recent handbook survey of Cohen & Levin (1989) has viewed as an important gap in existing literature, viz., the lack of theoretical work addressing the persistence of major inter-firm differences in profitability performance. As reported by Schmalensee (1987), "orthodox" approaches for explaining these differences (like linking them to market concentration or some measure of industry-wide differential efficiency) simply do not work. A reason suggested for this failure is that, as Cubbin & Geroski (1987) conclude in another empirical paper in the same volume, "...considerable

1 Quoted in Prigogine & Stengers (1987, p. 92)

2 See Phillips (1966), Nelson & Winter (1974), Levin (1981), or Mansfield (1983).

heterogeneities exist within most industries.[3] That is, most firms' profitability experience differs considerably from those of their closest rivals." (Emphasis added.)

Our approach to studying the above set of issues will be to propose a model of market dynamics in which both strategic and path-dependence considerations are jointly taken into account. To incorporate the latter, the model will exhibit a somewhat "biological" flavor. In particular, the set of technological options will be endowed with a directed-graph structure, each of the firms that are currently active advancing along possibly different edges of it through the accumulation of short-run strategic choices. The graph structure will formalize two key features of the model. On the one hand, the idea that innovation is a gradual accumulation of know-how along a certain technological direction; on the other hand, that imitation may involve switching costs, possibly substantial.

Market exit and entry will be modeled explicitly as follows. A potential entrant enters if it can profitably do so given its technological possibilities. On the other side of the coin, an incumbent firm remains in the market only if it is able to meet a certain viability constraint. (For simplicity, we shall require non-negative profits every period.). As we shall see, this leads, under certain technological scenarios, to a continuous turnover in the set of active firms. As Schumpeter would phrase it, technological change in such contexts becomes a force of "creative destruction".

There exists a rather small literature which shares with this paper a similar evolutionary approach and concern. Noted representatives of it are Futia (1980), Nelson and Winter (1982), or Iwai (1984a&b). Among those pursuing a more standard approach but with a similar concern (to model the process of industrial innovation and diffusion), that of Jovanovic [1982]

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3 The authors study a sample of 217 UK firms during the period 1951-77.

could be singled out from a much more abundant literature. None of the papers mentioned, however, exhibits the blend of acute path-dependence and strategic considerations which is key to our present approach.

The paper is organized as follows. The next section describes the different components of the model. In Section 3, we conduct the analysis and discussion. The paper concludes with a summary in Section 4. The proofs of the results are contained in an Appendix.

2.- THE MODEL

I divide the presentation of the model into the following parts: (2.1) the firms; (2.2) the market; (2.3) technology; (2.4) technological change; (2.5) exit and entrance; (2.6) strategic game.

2.1. The Firms

There is a set $I \subset \mathbb{N}$ of potential firms in the market. As we shall explain below, only a fraction of them will be generally active at any given point in time. Time is measured discretely. At each $t = 0,1,2,\ldots,$ the behavior of any firm $i \in I$ is characterized by the pair $(\vartheta_i(t), x_i(t)) \in \Theta \times \mathbb{R}_+$,[4] where $\vartheta_i(t)$ denotes the "product variety" produced by firm i at t, $x_i(t)$ its ouput, and Θ stands for the set of all possible product varieties. Implicitly, therefore, we assume that each firm produces only a single variety at each point in time. If $\vartheta_i(t) = \emptyset$, we interpret it to mean that firm i is not active at t. This, in particular, means that its output $x_i(t)$ must equal zero.

[4] $\mathbb{R}_+$ will denote the non-negative reals; $\mathbb{R}_{++}$ the positive ones.

2.2. The Market

In the Chamberlin's monopolistic-competition tradition we shall assume that each firm i confronts an i-specific inverse-demand function

$$f_i:\ \Theta^I \times \mathbb{R}^I_+ \longrightarrow \mathbb{R}_+, \tag{1}$$

which, for each variety and output profiles $\underline{\vartheta} \in \Theta^I$ and $\underline{x} \in \mathbb{R}^I_+$, determines the market-clearing price for the product sold by firm i, $f_i(\underline{\vartheta},\underline{x})$. The dependence of $f_i(.)$ on the market pattern of product varieties is contained in the following assumption:

(A.1) There is a real function $\rho:\ \Theta \longrightarrow \mathbb{R}_+$ ($\rho(\emptyset) = 0$) *such that* $\forall i\in\mathbb{N}$, *the i-specific demand function* $f_i(\cdot)$ *has, for any* $(\underline{\vartheta},\underline{x}) \in \Theta \times \mathbb{R}_+$, *the following representation:*

$$f_i(\underline{\vartheta},\underline{x}) = P_i(\rho(\vartheta_i)_{i\in I}\ ,\ \underline{x})$$

for some function

$$P_i:\ \mathbb{R}^I_+ \times \mathbb{R}^I_+ \longrightarrow \mathbb{R}_+\ ,$$

which is homogeneous of degree zero in $\rho(\vartheta_i)_{i\in I}$ *and satisfies*

$P_i(\cdot) \longrightarrow 0$ *as* $\sum x_i \longrightarrow \infty$. *Moreover,* $\forall \varepsilon > 0$, $\exists \nu > 0$ *such that*

if $\dfrac{\rho(\vartheta_i)}{\rho(\vartheta_j)} \leq \nu$ *for some* $j \neq i$, *then* $P_i(\rho(\vartheta_i)_{i\in I};\underline{x}) \leq \varepsilon$ *for all* $\underline{x}\in\mathbb{R}^I_+$.

The precedent assumption has the following interpretation. The effect of product diversity on market demand is fully summarized by the value of some real function $\rho(\cdot)$, which may be viewed as measuring the market value (for short, we shall speak of the "quality") of each variety. For technical reasons, we normalize matters and assume that only relative qualities matter (i.e., each P_i is homogeneous of degree zero in quality levels). We shall further suppose that if the quality of the variety produced by some firm i deteriorates sufficiently relative to that of a competitor, so do its "market conditions", as reflected by its corresponding demand function

$P_i(\cdot)$. Such market conditions are assumed bounded in the sense that no firm can sell at some positive price if an arbitrarily large overall quantity floods the market. A simple example which satisfies (A.1) is given by a demand function of the form:

$$P_i(\vartheta,x)=\frac{\rho(\vartheta_i)}{\hat{\rho}(\vartheta)}\,\phi(\textstyle\sum_{i\in I} x_i^{\alpha}), \tag{2}$$

where $\alpha > 0$ and $\hat{\rho}(\underline{\vartheta})$ stands for the average quality offered by the active firms in the market, and $\phi(\cdot)$ is some appropriately decreasing real function.

2.3. Technology

For the sake of focus, diversity across firms will be centered on the market value of the variety it produces, not on its production technology. In this latter respect, we shall conveniently assume that all varieties are produced under the same simple cost structure: a fixed cost M and a constant marginal cost c, both positive.

Firms will be diverse because, in general, they will confront different decision problems over existing product varieties. Such asymmetries among firms will be the result of the following two phenomena. On the one hand, different innovation experiences. On the other hand, the existence of switching costs in implementing imitation choices.

In order to formalize these matters, the set Θ of possible product varieties will be endowed with the structure of a directed graph (a "digraph"), the direction reflecting technological precedence.(5) When two different varieties, ϑ, ϑ', are adjacent consecutive vertices of Θ (i.e. there is an "arrow" in the graph going from ϑ to ϑ'), we write $\vartheta\ \mu\ \vartheta'$ and say that ϑ <u>directly</u> <u>precedes</u> (technologically) ϑ'. Compositions of μ give

5 The elements of Θ are usually known as "vertices", the graph specifying which vertices are connected by "edges" and in what direction. See Berge (1985) for a classical reference on Graph Theory.

rise to the relation of general (as opposed to immediate or direct) technological precedence as follows. When ϑ and ϑ' are joined by a μ-chain starting at ϑ and ending at ϑ', we simply say that ϑ technologically precedes ϑ' and write $\vartheta \ \beta \ \vartheta'$. (For the sake of formal convenience, we make $\vartheta \ \beta \ \vartheta$, i.e., ϑ is joined to itself by a μ-chain of length zero). Motivated by our interpretation of β as a binary relation expressing technological precedence, we assume that ϑ is an ordering -- in general, a partial one. Or, in the language of Graph Theory, (Θ, μ) is an acyclic digraph.

We now propose a notion of technological distance on (Θ, μ). If ϑ β-precedes ϑ', the technological distance $d(\vartheta, \vartheta') \in \mathbb{N}$ is taken to be the usual one considered in Graph Theory, namely, the length of the shortest μ-chain joining ϑ and ϑ'. When ϑ does not β-precede ϑ', we generalize the precedent notion by proposing a concept of technological distance reminiscent of biological contexts. For all $\vartheta', \vartheta'' \in \Theta$ denote

$$\mathcal{P}(\vartheta', \vartheta'') = \{\vartheta \in \Theta: \vartheta \ \beta \ \vartheta' \ \& \ \vartheta \ \beta \ \vartheta''\}, \qquad (3)$$

i.e., the set of common predecessors of both ϑ' and ϑ''. We define

$$d(\vartheta', \vartheta'') = \min \{d(\vartheta, \vartheta''): \vartheta \in \mathcal{P}(\vartheta', \vartheta'')\}, \qquad (4)$$

where we adopt the convention that $d(\vartheta,\vartheta) = 0$ and, if $\mathcal{P}(\vartheta',\vartheta'') = \emptyset$, $d(\vartheta',\vartheta'') = \infty$. Note that d(.) is not a distance function in the usual mathematical sense. In particular, it is not symmetric. If, for example, $\vartheta \ \beta \ \vartheta'$, $\vartheta \neq \vartheta'$, we have $d(\vartheta,\vartheta') > 0$ whereas $d(\vartheta',\vartheta) = 0$.

Consider a firm currently producing the variety ϑ and considering whether to change its production to some other variety ϑ'. We shall assume that if this shift is performed, the firm will have to incur in some switching costs. Switching costs may be given different interpretations, not necessarily exclusive.[(6)] One possible interpretation is that the

6 For empirical evidence and a good discussion on the importance of switching costs in the process of technological change, the reader may refer, for example, to David (1985a & b), who discusses specific cases, or Basalla (1988) who provides a general perspective on these issues.

firm's plant (or any other type of sunk investment) is geared towards producing variety ϑ and needs to be adapted in order to produce the different variety ϑ'. A related explanation has to do with the existence of learning costs in training the firm's workers to produce a different variety. In any event, switching costs will be linked to the technological distance between the former and final varieties, as reflected by a function[7]

$$\tilde{\gamma}: \mathbb{N} \cup \{0\} \longrightarrow \mathbb{R}_{++} \tag{5}$$

assumed increasing, with $\tilde{\gamma}(1) = \tilde{\gamma}(0) = 0$ (i.e., "gradual" adjustment and no adjustment is costless),[8] and the gradient $\tilde{\gamma}(n+1)-\tilde{\gamma}(n)$ bounded above zero for all $n \in \mathbb{N}$. Thus, for any contemplated shift from ϑ to ϑ' the switching cost is given by

$$\gamma(\vartheta,\vartheta') = \tilde{\gamma}(d(\vartheta,\vartheta')), \tag{6}$$

where $d(\cdot)$ is as defined above.

2.4. Technological Change

(i) Innovation

Since our concern in this paper is not to explain the rate of innovation but its induced consequences, I shall assume that each firm's innovation is the result of an stochastic process of "inventing by doing". Each period, every firm that remains active in the market obtains an invention draw from the set of varieties that are technological successors of the one it previously produced. Such an invention becomes then currently available for adoption if the firm so decides.

7 $\mathbb{N}$ denotes the natural numbers.

8 We assume that $\tilde{\gamma}(1)=0$ in order to avoid that the innovation process may reach a standstill (see below).

By assumption (A.1), the "market value" of a given product variety ϑ is fully captured by its associated $\rho(\vartheta)$. Let $S(\vartheta) \equiv \{\vartheta' \in \Theta: \vartheta \; \mu \; \vartheta'\}$ denote the set of <u>direct</u> successors of ϑ, assumed finite for simplicity. We postulate:

(A.2)(i) At each t, every firm $i \in I$ with $x_i(t-1) > 0$ obtains an innovation draw $\mathcal{I}_i(t)$ from the set $S(\vartheta_i(t-1))$ of technological successors of the quality previously produced. These draws are obtained according to the (discrete) density function $\lambda_{\vartheta_i(t-1)}$ with $\lambda_{\vartheta_i(t-1)}(\vartheta) \geq \omega$ for all $\vartheta \in S(\vartheta_i(t-1))$ and a common $\omega > 0$. If $x_i(t-1) = 0$, then $\mathcal{I}_i(t) = \emptyset$.

(ii) $\forall \vartheta \in \Theta$, $\exists \; \hat{\vartheta} \in S(\vartheta)$ such that $\rho(\hat{\vartheta}) < \rho(\vartheta)$.

(iii) There exists some $\xi > 0$ such that $\forall \vartheta \in \Theta$, $\exists \; \tilde{\vartheta} \in S(\vartheta)$ such that $\rho(\tilde{\vartheta}) = \xi \; \rho(\vartheta) \geq \rho(\vartheta')$, $\forall \vartheta' \in S(\vartheta)$

By Part (i) of the precedent assumption, every firm obtains one[9] innovation draw to be used next period if, and only if, it is currently active. The fact that $\lambda_{\vartheta_i(t-1)}$ is assumed with full support implies, in view of part (ii), that every active firm has always positive probability of obtaining an innovation draw which is not" successful", i.e., yields a variety of lower quality than the one currently produced. By Part (iii), on the other hand, there is always positive probability of obtaining a worthwhile invention with a quality increase ratio of at least ξ. We shall also need that any such increase ratio be bounded above. For simplicity, we assume that ξ itself is the upper bound.

<u>(ii) Technological Diffusion</u>

Technological know-how on the arising product varieties will be assumed to filter gradually through the industry with some lag. To model this, denote by K(t) the set of state-of-the-art product varieties at t in

9 The fact that the number of draws in one rather than a finite number is inessential for our purposes.

the sense that this set is currently available to <u>every</u> firm in the industry (either if it is currently active or a potential entrant). As time proceeds, this set is enlarged by past inventions as they all become progressively available with some finite lag $s \in \mathbb{N}$ (an arbitrary[10] parameter of the model). The parameter s may be interpreted as reflecting some unavoidable gradualness in the process of diffusion or some legal limits to it induced by, say, a patent system. Formally, we postulate:

$$K(t) = K(t-1) \bigcup_{i \in I} \mathcal{T}_i(t-s), \tag{7}$$

for all $t \in \mathbb{N}$, where $K(0) = \emptyset$, and we use the convention $\mathcal{T}_i(t-s) = \emptyset$ if $t < s$. (See Section 3 for a description of the initial conditions on $\mathcal{T}_i(0)$.)

2.5. Exit and Entrance

Let $F(t) \subset I$ denote the set of <u>potentially</u> active firms in the market at time t. Only those firms in F(t) may participate in the market and produce a positive output. The set F(t) is partitioned into two subsets, N(t) and E(t). The set N(t) includes those incumbent firms that entered the market in the past and still remain in it. On the other hand, E(t) includes those firms which are currently considering entry at t. The set F(t) changes through time as a result of the processes of exit and entrance in the industry.

Exit is modeled in a straightforward way. If an incumbent firm $i \in N(t)$ is not able to make non-negative profits (net of any switching costs), then we assume that it goes bankrupt and is forced to exit the market. For simplicity, we assume it may never participate in the market again. That is, $i \notin F(\tau)$ for all $\tau > t$.

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10 Any value for s is consistent with our qualitative results. In fact, what matters for them is the ratio ξ/s of maximum quality innovation per "lag period".

Entry, on the other hand, is modeled gradually as follows. At each $t \in \mathbb{N}$, the set E(t) is composed of only one firm, that indexed e(t), which is the current potential entrant. Assuming firms are listed in the order of potential entrance, we choose e(t) as the firm with lowest index which has never yet entered. If given the game which is described in the next subsection, the firm e(t) decides to enter the market, $e(t) \in N(t+1)$ -i.e., it becomes an incumbent next period- and $e(t+1) = e(t) + 1$, as long as e(t) was not the last firm in I. If it instead decides to remain away from the market, we shall assume that it still remains the potential entrant at t+1. Thus, $e(t+1) = e(t)$.

2.6. Strategic Game

At each t, the players involved in the game are those in the set F(t). We shall consider a two-stage game, as follows.

In the first stage, each firm $i \in F(t)$ chooses simultaneously a product variety in

$$\Theta_i(t) \equiv K(t) \cup [\bigcup_{\tau \leq t} \mathcal{T}_i(\tau)] \quad , \tag{8}$$

which is its action space in the first stage of the game. Note that for the entrant e(t),

$$\Theta_{e(t)}(t) = K(t),$$

since $\mathcal{T}_{e(t)}(\tau) = \varnothing$ for all $\tau \leq t$. Also notice from (7) that $\varnothing \in K(t)$ for all t. If the entrant chooses $\vartheta_{e(t)} = \varnothing$, we interpret it to mean that the potential entrant decides to stay out and wait for the next period.

Given their choices in the first stage, firms compete as Cournot oligopolists in the second stage, choosing (simultaneously) how much output to produce of the selected variety. Let $(\underline{\vartheta}(t), \underline{x}(t))$ be the variety and output profiles prevailing at any chosen t. Given $\underline{\vartheta}(t-1)$, the variety profile prevailing in the precedent period, payoffs are as follows. For the incumbent firms $i \in N(t)$,

$$\pi_i(\underline{\vartheta}(t),\ \underline{x}(t),\underline{\vartheta}(t-1)) = \psi_i(\underline{\vartheta}(t),\underline{x}(t)) - \gamma(\vartheta_i(t-1),\vartheta_i(t)) \qquad (9)$$

where

$$\psi_i(\underline{\vartheta}(t),\underline{x}(t)) \equiv P_i(\underline{\vartheta}(t),\underline{x}(t))\ x_i(t) - M - cx_i(t)$$

denotes current profits gross of switching costs. And for the potential entrant, e = e(t),

$$\pi_e(\underline{\vartheta}(t),\ \underline{x}(t),\underline{\vartheta}(t-1)) = \psi_e(\underline{\vartheta}(t),\underline{x}(t)) \qquad \text{if } \vartheta_e(t)\neq\varnothing, \qquad (10a)$$

$$\pi_e(\underline{\vartheta}(t),\ \underline{x}(t),\underline{\vartheta}(t-1)) = 0 \qquad \text{if } \vartheta_e(t)=\varnothing. \qquad (10b)$$

Only the first contingency of (10) requires explanation. It expresses the idea that if the potential entrant indeed enters ($\vartheta_e(t)\neq\varnothing$), it is subject to no switching costs. The underlying assumption here is that, since the entrant has no previous technological base to switch from, it may acquire the state-of-the art technology of K(t) at no adjustment cost. We could assume, however, that the entrant has to pay a fixed entry cost for adopting any such technology without affecting our analysis. For the sake of implicitly, we take such fixed cost to equal zero.

Under our assumption that, within each stage, the choice of actions is made simultaneously by all the incumbent firms and the potential entrant, we have a standard two-stage game in which strategies for each player i are of the form (ϑ_i, $x_i(.)$). First, they include a choice of product variety among those feasible for the particular agent. Second, they specify a function expressing produced output contingent on the variety choices made by all agents in the first stage.

Denote by $A(\underline{\vartheta}) \equiv \{i \in I:\ \vartheta_i \neq \varnothing\}$, i.e., the set of active firms induced by $\underline{\vartheta}$. We postulate:

(A.3)(i) $\forall\ \underline{\vartheta} \in \Theta^I$, there exists a Cournot-Nash equilibrium (CNE) for the second stage of the game $x^*(\underline{\vartheta})$ such that, for all $i \in A(\underline{\vartheta})$, $\psi_i^*(\underline{\vartheta}) \equiv \psi_i(\underline{\vartheta},\ x^*(\underline{\vartheta}))$ is continuous in $[\rho(\vartheta_j)]_{j\in I}$, monotonically increasing in

$\rho(\vartheta_i)$ *if* $x^*_i(\underline{\vartheta}) > 0$, *and non-increasing in* $\rho(\vartheta_j)$, $j \neq i$.
(ii) $\exists \gamma > 0$ *such that* $\forall \underline{\vartheta} \in \Theta^I$, *if the cardinality* $|A(\underline{\vartheta})| = 2$ *and* $\rho(\vartheta_i) = \rho_o > 0$ *for each* $i \in A(\underline{\vartheta})$, *the CNE profits* $\psi^*_i(\underline{\vartheta}) \geq \gamma$.

By (A.3(i)), a symmetric Cournot-Nash equilibrium exists in the second stage of the game. (For simplicity, we take it to be in pure strategies.) Conditions on demand and costs that guarantee such existence are standard in the literature.[11] This then ensures equilibrium existence for the whole game, since the action space in the first stage of the game is finite. We also postulate in part (i) of (A.3) that the equilibrium profits of every active firm i are monotonically increasing in the quality of its respective ϑ_i and non-increasing in that of its competitors. This natural requirement may be easily verified for the specification proposed in (2) if ϕ is of a standard type (linear, with constant elasticity, etc.). Finally, part (ii) of (A.3) simply establishes that the "size" of the market is large enough to allow for the coexistence of at least two symmetric firms.

3.- ANALYSIS

At $t = 0$, we assume $N(0) = \{1\}$ with $\mathcal{T}_1(0) = \{\vartheta^0\}$ and $\rho(\vartheta^0) > 0$. Starting from such initial conditions, we shall study the ensuing evolution of the industry under different assumptions on both entry conditions and technological characteristics.

Entry conditions will be captured by the cardinality of the set I of potentially active firms. If this set is finite, we interpret it to represent a context of limited entry. If we assume instead that the set I has infinite cardinality, market entry is interpreted unlimited (i.e., open to an infinite number of potential firms).

11 Essentially, they hinge upon the strict quasi-concavity of the profit functions which are induced by the demand and cost functions.

As for technological conditions, we shall focus on two polar and stylized contexts which highlight best the type of issues involved. In one of these alternative contexts we shall postulate that, with some finite (although arbitrarily long) maximum recurrence, the technological structure includes "branching and seminal innovations" (for short, we shall call them *basic innovations*). A certain variety qualifies as such if:

(i) it embodies essential technological know-how incorporated in all its successors (i.e., it represents for them a sort of indispensable "building block"),

(ii) it is not the unique quality-enhancing innovation, as viewed from their immediate technological predecessors.

To define (i) and (ii) formally, I introduce two additional pieces of notation. First, for any $\vartheta \in \Theta$, we shall denote by $\Xi(\vartheta)$ the set of technological successors of ϑ, i.e., $\Xi(\vartheta) \equiv \{\vartheta' \in \Theta: \vartheta \; \beta \; \vartheta'\}$. Second, for any ϑ, $\vartheta' \in \Theta$, we shall write as $\chi(\vartheta',\vartheta'') = (\vartheta_1,\vartheta_2,\ldots,\vartheta_m)$ a typical μ-chain of technologically consecutive varieties joining ϑ and ϑ'. That is, $\vartheta_1 = \vartheta'$, $\vartheta_i \; \mu \; \vartheta_{i+1}$ $(i=1,2,\ldots,m)$, $\vartheta_m = \vartheta''$.

Denote by Θ_b the set of all basic innovations and let $\tilde{\vartheta} \in \Theta_b$. Corresponding to (i) and (ii) above, such a variety must meet the following two formal requirements:

(i)' $\forall \vartheta' \notin \Xi(\tilde{\vartheta})$, $\forall \vartheta'' \in \Xi(\tilde{\vartheta})$, $\forall \; \chi(\vartheta',\vartheta'') = (\vartheta_1,\vartheta_2,\ldots,\vartheta_m)$, $\vartheta_i = \tilde{\vartheta}$ for some $i = 2,\ldots,m$.

(ii)' $\forall \vartheta \in \Theta$ s.t. $\tilde{\vartheta} \in S(\vartheta)$, $\rho(\tilde{\vartheta}) > \rho(\vartheta)$, and $\exists \; \vartheta' \neq \tilde{\vartheta}$ s.t. $\vartheta' \in S(\vartheta)$ & $\rho(\vartheta') > \rho(\vartheta)$.

Condition (i)' formalizes (i) by requiring that every μ-path joining any variety $\vartheta' \notin \chi(\tilde{\vartheta})$ and another variety $\vartheta'' \in \chi(\tilde{\vartheta})$ must necessarily include $\tilde{\vartheta}$. It is in this sense that we say that $\tilde{\vartheta}$ is an essential building block for its technological successors. Condition (ii)' is a straightforward formalization of (ii). The first of the scenarios we shall consider is characterized by the following condition:

(B) There exists some $q \in \mathbb{N}$ such that:

$$[\vartheta',\vartheta'' \in \Theta,\ \vartheta'\ \beta\ \vartheta'',\ d(\vartheta',\vartheta'') \geq q] \Rightarrow [\exists \hat{\vartheta} \in \Theta:\ \vartheta'\ \beta\ \hat{\vartheta}\ \beta\ \vartheta''\ \&\ S(\hat{\vartheta}) \cap \Theta_b \neq \emptyset]$$

The precedent condition states that basic innovations always become reachable (i.e., are among the possible successors of prevailing varieties) within some pre-specified maximum number of "μ-steps". The motivation for this condition derives from the work of a number of scholars in the field of technological change who have stressed that, in technologically dynamic contexts, such "branching" inventions tend to arise quite recurrently.(12) Along the new technological line they open, further innovations build upon a specific type of know-how which brings them apart from alternative technological courses, equally viable ex-ante. Admittedly, the technological implications of (A.2) are rather special. Its role, however, should not be evaluated literally but in terms of its usefulness as a theoretical benchmark. A particularly simple case which meets (B) is that where the ordering β induces on Θ a tree-like structure.

Assumption (B) will be contrasted with a polar alternative scenario in which no innovation ever stands out as "basic". Since under these circumstances the technological structure must be to some extent interwoven, we shall focus on the stylized case where such structure is, in fact, linearly β-ordered. That is, a context in which no distinct technological paths ever arise since all varieties belong to the same one. Formally,

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12 See, for example, Rosenberg (1986, pp. 23-27) or Basalla (1988, pp.189-204) for a good discussion of these issues and a number of illustrative examples. Just to note a few of them we can mention the cases of vacuum tubes/semiconductors, natural/artificial fibers, gasoline/diesel engines for automobiles, or piston/jet engines for planes. All of these pairs of alternative technological courses were originally developed along separate lines. In some of these cases, not both of them survived in the long run.

(L) Θ is totally β-ordered (i.e., β defines a total order on Θ).

Substantial generalizations of (L) would suffice for our purposes. We focus on it, however, as a specially clear-cut alternative to (B) above.

3.1. Scenario (B)

We first establish that if there is limited entry (i.e., the cardinality of I is finite) and (B) applies, then the market structure will eventually evolve into a monopoly with only one firm remaining in the market. All other firms, sooner or later, will be forced out of it by bankruptcy. Formally:

Proposition 1: Assume (B) and $|I| < \infty$. There exists with probability one a firm $i \in I$ and $t_o \in \mathbb{N}$ such that $F(t) = \{i\}$ for all $t \geq t_o$.

The essential intuition of this result is as follows. Under (B), the technological process of innovation and imitation within the industry will eventually lead to a situation where each firm follows a separate "technological line". When this occurs, the independent technological evolution of each firm will lead to a point where one of them so dominates the others that only the former will survive.

If instead of limited entry, there is an infinite set of potential entrants, we establish next that, under (B), there will be a continuous process of firm turnover. The intuition here springs from that of the former result. Specifically, no firm will survive forever; eventually, every firm will be forced out of the market with full probability. A straightforward implication of this result is that every firm in I (no matter how high its entry index) will eventually enter the market. This will compare drastically with our conclusion in this respect for scenario (L) - see Proposition 5 below. A formal statement of these conclusions now follows.

Proposition 2: Assume (B) and $|I| = \infty$. $\forall i \in I$, there exists with probability one some $\tau \in \mathbb{N}$ such that $i \in A(\tau)$ and $i \notin A(t)$ for all $t > \tau$.[13]

The dynamics established by the precedent proposition may be seen as a stylized formalization of Schumpeter's "merciless" and never-ending process of creative destruction. In view of Proposition 1 (which established that, under limited entry, an eventual monopoly will obtain) the question now arises as to whether this conclusion will be maintained if entry is unlimited. The following straightforward result addresses this question.

Proposition 3: Assume $|I| = \infty$. There exists some $\tilde{\xi} > 1$ such that, if $\xi \leq \tilde{\xi}$, then $|A(t)| \geq 2$ for all $t \in \mathbb{N}$.

Note that the precedent proposition does not depend on the technological scenario considered, (B) or (L). In both of them, unlimited entry precludes monopoly. This will be achieved, however, with drastically different underlying dynamics in each scenario: under (B), with continuous firm turnover and occasionally large profitability differences across active firms (see Proposition 4 below); under (L), with neither firm turnover nor large differences in firms' performance (see the next subsection).

The pattern of firm bankruptcy underlying the conclusions of Propositions 1 and 2 implies the occurrence of significant profitability differences among active firms (in particular, of course, among those that survive till next period and those that do not). In general, these differences have to be linked (if ξ is small) to large technological differences across firms (that is, to large technological distances between

13 Abusing notation, we write $A(t)$ for $A(\underline{\vartheta}(t))$. We will adopt similar conventions in other analogous cases when there is no risk of misunderstanding.

the varieties they produce). That these differences will recurrently become arbitrarily large is established by the following proposition.

Proposition 4: Assume $|I| = \infty$. There is some $\tilde{\xi} > 1$ such that, if $\xi \leq \tilde{\xi}$, then $\forall\ R > 0$, $\forall\ t \in \mathbb{N}$, there is probability one (conditional at t) that $d(\vartheta_i(t'),\vartheta_j(t')) \geq R$ for some $t' \geq t$ and $i,j \in A(t')$.

We now compare the previous conclusions with those of scenario (L).

3.2. Scenario (L)

The following proposition stands in stark contrast with Propositions 1 and 2.

Proposition 5: Assume (L). There is some $\tilde{\xi} > 1$ such that if $\xi \leq \tilde{\xi}$, then $A(t) \subseteq A(t+1)$ for all $t \in \mathbb{N}$.

The above proposition establishes that, in scenario (L), no firm will ever be forced out of the market by bankruptcy if innovation is sufficiently gradual ($\tilde{\xi}$ is not too large). Note that this conclusion is independent of the cardinality (finite or infinite) of the set I. To obtain a more ready comparison of this result and those derived in the previous subsection for scenario (B), we state next an immediate corollary of it. At each $t \in \mathbb{N}$, denote by m(t) the highest index included in the current A(t). We have:

Corollary: Assume (L). There exists some $\tilde{\xi} > 1$, m_1, $m_2 \in \mathbb{N}$, such that if $\xi \leq \tilde{\xi}$, then $m_1+1 \leq m(t) \leq m_2$ for all t.

In combination with Proposition 5, the precedent corollary establishes clear-cut differences between the dynamics of each of our considered scenarios. In contrast with (B), scenario (L) neither leads to the eventual establishment of a monopoly under limited entry nor, under unlimited entry, to a never-ending process of firm turnover (c.f. Propositions 1 and 2).

As for the emergence of inter-firm differences in technology and performance, we shall also be led to marked contrasts. Denote

$$\pi^*(t) \equiv \max \{ \pi\ (t),\ i \in A(t) \},$$

and

$$\pi\ (t) \equiv \max \{ 0\ ,\ \min \{ \pi\ (t),\ i \in A(t) \} \}\ ,$$

the maximum and minimum profit levels (losses are considered null profits) prevailing at any time t.

<u>Proposition 6:</u> Assume (L). For all $\delta > 0$, there exists some $\tilde{\tilde{\xi}} > 1$ such that if $\xi \leq \tilde{\tilde{\xi}}$, then $(\pi^*(t)/\ \pi_*(t)) \leq 1+\delta$ for all $t \in \mathbb{N}$.

And with respect to inter-firm technological differences, we have:

<u>Proposition 7:</u> Assume (L). There exists some $\tilde{\tilde{\xi}} > 1$ such that if $\xi \leq \tilde{\tilde{\xi}}$, $\max \left\{ d(\vartheta_i(t),\vartheta_j(t)),\ i,j \in A(t) \right\} \leq s$ for all $t \in \mathbb{N}$, where s is the diffusion lag.

The precedent analysis may be summarized by the following table. (The numbers in parentheses refer to the propositions which establish the result in question. The different assumptions on the cardinality of the set I which are arranged vertically ($|I|$ finite or infinite) only apply to scenario (B) since our results for scenario (L) are independent of this cardinality.)

	(B)	(L)
$\lvert I \rvert < \infty$	(1) For some $i \in I$, $\tau \in \mathbb{N}$ $F(t) = \{i\} \quad \forall t \geq \tau$	(5) $A(t) \subset A(t+1)$ $\forall t \in \mathbb{N}$
$\lvert I \rvert = \infty$	(2) $m(t) \longrightarrow \infty$	(5) $\forall t \in \mathbb{N},\ m(t) \leq \bar{m}$
	(3) $\forall t \in \mathbb{N},\ \lvert A(t) \rvert \geq 2$	(5) $\forall t \in \mathbb{N},\ \lvert A(t) \rvert \geq 2$
	(2) $\forall t \in \mathbb{N},\ \exists t' \geq t:$ $\dfrac{\pi^*(t')}{\pi_*(t')} = \infty$	(6) $\forall \delta > 1,\ \exists \tilde{\xi} > 1: \forall t \in \mathbb{N}$ $\dfrac{\pi^*(t)}{\pi_*(t)} \leq 1+\delta$
	(4) $\forall R > 0,\ \forall t \in \mathbb{N},\ \exists t' \geq t:$ $d(\vartheta_i(t'), \vartheta_j(t')) \geq R$	(7) $\forall i, j \in A(t)$, $d(\vartheta_i(t), \vartheta_j(t)) \leq s$

4.- CONCLUSION

The model of market technological processes proposed in this paper has stressed the following points:

(1) Technological change is a highly path dependent process.

(2) Firms' technological choices are subject to switching costs.

(3) Firms' survival requires meeting some bankruptcy constraint.

(4) Market entry is dependent on technological availability.

In addressing the above points, we have abstracted from important considerations. Among others, we may list:

(a) Firms' decisions and their viability (bankruptcy) constraints are really inter-temporal, i.e., neither "myopic" nor "instantaneous".

(b) Innovation is the outcome of an economic decision, not merely the result of learning-by-doing activities; So is, for that matter, diffusion and imitation.

(c) The activity of firms is subject to both considerations of scale and essential domains of uncertainty.

Existing literature in Industrial Organization has addressed the precedent issues with a variety of approaches and emphases (see Tirole (1988) for a comprehensive survey). For our present purposes, it was thought best to avoid them in order to focus on our primary concern. Namely, to explore the implications of alternative technological structures and their associated processes of technological change on some of the industry's structural properties: market concentration, entry/exit dynamics, and inter-firm variability in technologies and performance.

Our main conclusions in this respect can be summarized as follows. If the technological structure exhibits "branching" (i.e., occasionally divergent technological lines), the evolution of the market will be, either towards monopoly if entry is limited, or to a process of continuous turnover if the set of potential entrants is unlimited. In either case, the differences among active firms in both the technological and profitability spheres will become large.

Such conclusions contrast with those obtaining within a technological scenario in which, loosely speaking, all product varieties belong to the same technological line. Under these circumstances, no incumbent firm is ever forced out of the market. Thus, in particular, the number of active firms in it is a never decreasing set which reaches the upper bound which the market can support. Moreover, at any point in the process, the technological and profitability profiles of active firms never becomes too heterogeneous.

APPENDIX

Proof of Proposition 1:

Suppose that, for all $t \in \mathbb{N}$, either $|A(t)| \geq 2$ or/and $E(t) \neq \emptyset$. The proof builds a contradiction upon this hypothesis in two steps. First we show that, for all t and any two firms in $[A(t) \cup E(t)]$, there is (conditional at t) probability one that either one of them goes bankrupt or that the technological distance between the varieties produced by them will become arbitrarily large. Second, we prove that the latter possibility implies that (again with full probability) one of those firms will go bankrupt sooner or later.

For simplicity of exposition, we carry out the argument for the case $I = \{1,2\}$. Its extension to an arbitrary (finite) I is straightforward. Denote by $B(\vartheta) \equiv \{\vartheta \in \Theta_b : \vartheta \ \beta \ \vartheta\}$, i.e., the set of basic inventions β-preceding any given $\vartheta \in \Theta$. We shall need the following lemma.

Lemma: Choose any given time $\hat{t}$, and assume that $[A(t) \cup E(t)] = \{1,2\}$ for all $t \geq \hat{t}$. $\forall r \in \mathbb{N}$, there exist some $\delta > 0$ and $v \in \mathbb{N}$ such that $\forall t \geq \hat{t}$ there is probability (conditional at t) of at least δ that for some $i,j = 1,2$, $i \neq j$:

(i) $B(\vartheta_i(t+v)) \setminus B(\vartheta_j(t+v)) \equiv D_{ij}(t+v) \neq \emptyset$;

(ii) $\exists \ \tilde{\vartheta} \in D_{ij}(t+v)$ such that $\rho(\vartheta_k(t+v)) > \rho(\tilde{\vartheta}) \ \xi^r$ for each $k = 1,2$.

Proof: Given t ($\geq \hat{t}$) and its corresponding $\vartheta_1(t)$ and $\vartheta_2(t)$ prevailing at it, consider the following sequence of events associated to some given v_1, v_2, $v_3 \in \mathbb{N}$. [We suppose w.l.o.g. that $\rho(\vartheta_1(t)) \geq \rho(\vartheta_2(t))$, where recall that $\rho(\vartheta_2(t))=0$ if $\vartheta_2(t)=\emptyset$, i.e., firm 2 is not active at t.]

(1) In the time interval $[t+1 \ , \ t+v_1]$:

(a) For $\tau = t+1,\dots,t+v_1$, $\rho(\vartheta_2(\tau)) = \xi\ \rho(\vartheta_2(\tau-1))$ if $2 \in A(\tau-1)$.

(b) For $\tau = t+1,\dots,t+v_1$, $\rho(\vartheta_1(\tau)) = \rho(\vartheta_1(\tau-1))$.

(c) $|\ \rho(\vartheta_1(t+v_1)) - \rho(\vartheta_2(t+v_1))\ | \le \xi$.

(2) In the time interval $[t+v_1+1 \ , \ t+v_2]$:

(a) For $\tau = t+v_1+1,\dots,t+v_2-2$, $i=1,2$, $\rho(\vartheta_i(\tau)) = \xi\ \rho(\ \vartheta_i(\tau-1))$.

(b) For some $i,j = 1,2$, $i\neq j$, $\vartheta_i(t+v_2) \in \Theta_b$, $\vartheta_j(t+v_2) \notin \Xi(\vartheta_i(t+v_2))$.

(c) $|\ \rho(\vartheta_1(t+v_2)) - \rho(\vartheta_2(t+v_2))\ | \le \xi$

(3) In the time interval $[t+v_2+1 \ , \ t+v_3]$:

$$\rho(\vartheta_i(\tau)) = \xi\ \rho(\vartheta_i(\tau-1)) \text{ for each } i = 1,2.$$

We now argue that there is some $\delta > 0$ and some pre-specified bounds on v_1, v_2, and v_3 such that the string of events described in (1) through (3) has probability no smaller than δ for all time t. Starting by the events in (1a) and (1b), these have, by (A.2), positive probability for any given v_1. The fact that v_1 (which is chosen to satisfy (3c)) may be bounded above independently of t derives from the fact that either:

(i) both firms belong to A(t) and thus, by virtue of (A.1), $\rho(\vartheta_2(t)) \ge \tilde{\nu}\ \rho(\vartheta_1(\tau-1))$ for some pre-specified $\tilde{\nu} > 0$, or

(ii) firm 2 does not belong to A(t) and, by (A.3(ii)), it can be ensured that $v_1 \le s$, where s is the diffusion lag.

With respect to (2), assumption (A.2) induces positive probability to (2a) for any v_2. By this same assumption and condition (B), (2b) and (2c) obtain with positive probability for some $v_2 \le v_1 + q + 2$, where q is the parameter contemplated in the latter condition.

As in the case of (2a), the event described in (3) has positive probability for any choice of v_3. If we make $v = v_3$, part (i) of the lemma follows. By choosing $v_3 \geq v_2 + r + 1$, we also have part (ii), completing the proof of the lemma.

Since the choice of δ and v in the precedent lemma can be made independently of t, it follows that (i) and (ii) will indeed obtain for some $\tilde{t} \geq \hat{t}$ with probability one. By (ii) and (B) we shall have:

$$d(\vartheta_i(\tilde{t}+v),\vartheta_j(\tilde{t}+v)) \geq r \tag{11}$$

for each i,j = 1,2, i≠j. By (A.1), profits for any individual firm are bounded above by some $\bar{\pi}$. Choose r in the lemma such that $\tilde{\gamma}(r) > \bar{\pi}$. Then (11) implies that from $\tilde{t}+v$ onwards, each firm will proceed along independent technological lines, never imitating the other firm. Thus, (i), (ii), and (11) will continue to hold for all $t' \geq \tilde{t}+v$.

Once shown that with probability one the two firms will eventually become technological isolates, it is immediate to see that, given this state of affairs, one of them will, again with probability one, go bankrupt within finite time. For, choose ε in (A.1) less than the marginal cost c. By virtue of this assumption, if at some $t \in \mathbb{N}$ we have:

$$\frac{\rho(\vartheta_j(t))}{\rho(\vartheta_i(t))} \leq \nu, \tag{12}$$

then $P_j(.) \leq \varepsilon$ and, therefore, firm j will necessarily obtain negative equilibrium profits at t. From t+1 onwards $j \notin F(t)$. Since, by (A.2), (12) will obtain with probability one at some $t \geq \tilde{t}$, the desired contradiction follows, completing the proof of the proposition. ∎

Proof of Proposition 2:

Given $t \in \mathbb{N}$, choose any $i \in A(t)$. Denote

$$F_{-i}(t) \equiv \{ j \in A(t) \cup E(t), j \neq i\}, \tag{13}$$

and

$$M_{-i}(t) \equiv \{ k \in F_{-i}(t) : \pi_k(t) \geq 0 \ \ \& \ \ \psi_k(t) \geq \psi_j(t), \ \forall j \in F_{-i}(t) \}. \tag{14}$$

Let $j \in M_{-i}(t)$. Applying a logic analogous to that of Proposition 1, we may conclude that, for any $\nu > 0$, there exists some $\delta > 0$, $v \in \mathbb{N}$, such that:

$$\frac{\rho(\vartheta_i(t+v))}{\rho(\vartheta_j(t+v))} \leq \nu, \tag{15}$$

with probability no smaller than δ. By choosing ν small enough, (15) and (A.1) imply that $\psi_i(t+v) < 0$. Thus, firm i goes bankrupt at t+v and, therefore, $i \notin F(t+v+1)$. Since the choice of δ and v can be made independent of t, we conclude that firm i must go bankrupt, with probability one, at some $t' \geq t$. This then implies, in view of (A.3), that, again with probability one, every firm in I will eventually enter the market. ▮

<u>Proof of Proposition 3:</u>

By the monotonicity of each $\psi_i^*(\cdot)$ postulated by (A.3(i)), $|N(t)| \geq 1$ for every $t \in \mathbb{N}$. (The firm producing the highest quality always survives till next period.) Suppose that, at some given t, $N(t) = \{i\}$ for some $i \in I$. If ξ were equal to one, (A.3(ii)) would imply that $\pi^*_{e(t)}(t) > 0$ and, therefore, $|A(t)| = |N(t) \cup E(t)| = 2$. Thus, by the continuity assumed in (A.3(i)), there exists some $\tilde{\xi} > 1$ such that if $\xi \leq \tilde{\xi}$, the precedent conclusion must still hold. This completes the proof. ▮

<u>Proof of Proposition 4:</u>

Consider any time t, and choose an arbitrary $R > 0$. Denote

$$M(t) \equiv \{ k \in F(t) : \pi_k(t) \geq 0 \ \ \& \ \ \psi_k(t) \geq \psi_j(t), \ \forall j \in F(t) \}, \tag{16}$$

and choose some $i \in M(t)$ and $j \in M_{-i}(t)$, as defined in (14). By a line of argument repeatedly used, we may choose $\delta > 0$ and $v \in \mathbb{N}$ such that the probability (conditional at t) of the event

$$d(\vartheta_i(t+v),\vartheta_j(t+v)) \geq R \qquad (17)$$

is bounded below by δ. Thus, since δ and v may be selected independently of t, the conclusion of the proposition follows. ∎

Proof of Proposition 5:

In context (L), all firms $i,j \in A(t)$ will satisfy:

$$\left|\ell n[\rho(\vartheta_i(t))] - \ell n[\rho(\vartheta_j(t))]\right| \leq s \,\ell n\, \xi \qquad (18)$$

for any $t \in \mathbb{N}$, where s is the diffusion lag. (Recall that we have assumed that gradual adjustment is cost-less, i.e., $\tilde{\gamma}(1) = 0$.) Thus, if ξ is small, so will be the maximum quality ratio among active firms.

Denote $|I| \equiv m$. For each $i \in I$, consider the function

$$\tilde{\psi}_i : \mathbb{R}^m_+ \longrightarrow \mathbb{R},$$

defined by:

$$\tilde{\psi}_i(\underline{\rho}) = \psi^*_i(\underline{\vartheta}), \qquad (19)$$

where $\rho_i = \rho(\vartheta_i)$ and $\underline{\rho} = (\rho_i)_{i\in\mathbb{N}}$. For each $n \in \mathbb{N}$, $n \leq m$, let ℓ_n stand for the m-dimensional vector $(1,1,\ldots,1,0,0,\ldots)$ with 1's in its n first coordinates and 0's in the remaining m-n. Let

$$\bar{n} = \max \{ n \in \mathbb{N}: \tilde{\psi}_i(\ell_n) > 0,\ i = 1,2,\ldots,n \}. \qquad (20)$$

By (A.1), n is finite. Moreover, the monotonicity of each $\psi^*_i(\cdot)$ postulated by (A.3(i)) implies that $\bar{n}$ is the maximum number of firms which the market can possibly accommodate. Choose $z > 0$ such that $\underline{\rho}^o = \ell_{\bar{n}} - (z,0,0,\ldots)$ and

$\tilde{\psi}_1(\underline{\rho}^{\circ}) > 0$. If $\tilde{\tilde{\xi}}$ is chosen such that:

$$s \, \ell n \, \tilde{\tilde{\xi}} \leq -\ell n \, (1-z), \tag{21}$$

we may ensure, in view of (16), that no firm ever active will later on become bankrupt. This completes the proof of the Proposition. ▮

Proof of Corollary to Proposition 5:

By the precedent proposition, we can choose $m_2 = \bar{n}$. From Proposition 3, we can make $m_1 = 1$. ▮

Proof of Proposition 6:

A direct consequence of (16). ▮

Proof of Proposition 7:

By (7), any variety $\vartheta \in \Theta_i(t)$ available to firm i at t becomes available to all other firms in the market at most s periods after, i.e., $\vartheta \in K(t')$ for all $t' \geq t+s$. Since we have assumed that $\tilde{\gamma}(1) = 0$, this implies the desired conclusion. ▮

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