

# **MONOPOLISTIC COMPETITION: EQUILIBRIUM AND OPTIMALITY**

**L.C. Corchón**

## **ABSTRACT**

In this paper, we study the relationship between efficient and optimal allocations in a Chamberlinian framework.

# MONOPOLISTIC COMPETITION: EQUILIBRIUM AND OPTIMALITY<sup>1</sup>

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## 1.- INTRODUCTION.

Since the publication of Chamberlin's book (*The Theory of Monopolistic Competition* (1933)), there has been some controversy about what is implied by adopting the Monopolistic Competition point of view (see for example Stigler (1968), p. 320 vs. Samuelson (1958)).

The first formal model of Monopolistic Competition is due to Spence (1976). He showed that if the utility function of the sole consumer is of the form  $u = v(\sum_{i=1}^n x_i^a) + l$ , being  $x_i$  the consumption of good  $i$  and  $l$  leisure, optimal and equilibrium output coincide and the optimal number of active firms is larger than the equilibrium number. Dixit-Stiglitz (1977) working with a somewhat different model, obtained the same conclusion. Also they showed that if the utility function is of the form  $(\sum_{i=1}^n v(x_i))^b \cdot l^{1-b}$ , equilibrium output may be greater, equal or smaller than the optimal output (see also Pettengill (1979), Dixit-Stiglitz (1979), and Koenker-Perry (1980)). Lancaster (1979) and Salop (1979) considered the characteristic approach and the Hotelling model respectively. Finally Sattinger (1984), Perloff-Salop (1985) and Hart (1985a) (1985b) studied models in which there are many consumers with preferences distributed as random variables, and Anderson-De Palma-Thisse (1987) have shown that under some conditions this model is equivalent to the representative consumer model.

In this paper I analyze the relationship between Monopolistically Competitive Equilibrium and Optimal Allocations in a General Equilibrium model with an infinite number of potentially produced outputs and an outside good.

The choice of framework is dictated by the fact that some of our results do not need a quasi-linear utility function. The results obtained here extend the analysis of Spence, Dixit and Stiglitz (S-D-S in the sequel) in three different respects.

First, I prove some new Propositions concerning the location of optimal output on the average cost curve (Proposition 1), the effect on welfare of an increase in the equilibrium number of firms (Proposition 2) and the equilibrium output (Proposition 3), and the relationship between "excess diversity" and "excess capacity" (Propositions 4 and 6). It is shown that if the representative consumer likes variety in a sense explained below, average costs must be declining in the optimum. Also an increase either in the equilibrium output or the number of firms in equilibrium always increases welfare. Moreover excess diversity implies excess capacity but not vice versa. In order to prove these results I only need to assume smoothness of the relevant functions, symmetry of goods in tastes and technology and that the number of firms can be treated as a continuous variable (Proposition 4 and 6 need some extra assumptions).

Second, I incorporate (in a limited way) quality choice distinguishing between the name of the product and its specification. One interpretation of this is that quality has two dimensions: one is fixed for each firm, but varies from firm to firm (as in S-D-S and all other models of a unique consumer) and the other is a decision variable for each firm (as in the characteristics model). The latter can also be interpreted as advertising. Then it is proved that under additional assumptions, optimal and equilibrium

qualities coincide (Proposition 5). However we remark that under alternative specifications of the cost function, (see Yarrow (1985) and Ireland (1987)) qualities are not optimal.

Third, I prove similar results to those of S-D-S about the relationship between optimal and equilibrium output, but allowing for more general preferences and technology (Propositions 7, 8 and 9). Also I provide a graphical setting which shows why these results happen. Finally Proposition 10 studies the effect on welfare of an increase in output and a reduction in the number of firms in equilibrium.

The rest of the paper is organized as follows. The next Section discusses the model and the general results. Section 3 considers a more specific model. Finally Section 4 presents our final comments.

## 2.- THE MODEL AND SOME GENERAL RESULTS.

There are two types of goods. Good 0 is labor which can be used either as an input or as a consumption good and whose initial endowment is denoted by  $w$ . The set of potentially produced goods (these goods are sometimes referred to as the differentiated commodity) is the set of natural numbers. Let us denote by  $n$  the number of goods which are effectively produced.

Firms each produce a unique output. The set of potential firms is also the set of natural numbers. We will assume that firm  $i$  produces good  $i$ , i.e. no good is produced by two firms. Thus  $n$  is also the number of active firms.

There is a sole consumer with preferences representable by a  $C^2$  utility function which is strictly quasi-concave and increasing in all its components. Let  $k$  be the vector of qualities (or advertisement) for a typical firm, and  $x$  be the output of this firm. If the  $n-1$  remaining firms produce their goods with an identical quality<sup>(2)</sup>  $q$  and at an output level  $y$ , this utility function can be written as  $u = u(k, q, n, x, y, l)$  where  $l$  is leisure. This function is assumed to be symmetrical such that if  $x = y$  and  $k = q$  then  $(n-1) \cdot \partial u / \partial x = \partial u / \partial y$  and  $(n-1) \cdot \nabla u / \nabla k = \nabla u / \nabla q$  (where  $\nabla$  denotes vector differentiation). Since all the allocations we will consider are symmetrical there is no loss of generality in representing the choice of all active firms except one by a common number (or a vector in the case of qualities).

2) Notice that if two firms produce the same quality it does not imply that these goods are perfect substitutes since they are intrinsically different. One may think of wine as the differentiated commodity and quality as years in storage. Different firms are located on different lands and their outputs are different despite of being of the same quality (i.e. the same vintage).



A special case of this utility function is a generalized version of the one used by S-D-S namely  $u = u(v(k).\phi(x) + (n-1)v(q).\phi(y), l)$ . In the next Section we will assume that the utility function has such a form but it is linear on  $l$ . In this case, differences with S-D-S are the consideration of qualities (since  $v(\cdot)$  affects the marginal rate of substitution between the differentiated commodity and labor) and that the form of  $u(\cdot)$  is more general.

Firms produce output from labor (which is assumed to be the numeraire). The technology of the representative firm is represented by a  $C^1$  cost function  $c = c(x, k)$ . Similarly for the rest of the firms the cost function is  $c(y, q)$ , i.e. we assume that the cost function is identical for all potential firms.

Finally we will assume that  $n$  is a continuous variable, i.e. the integer problem is neglected. This may be justified by assuming that optimal and equilibrium values of  $n$  are large, i.e. Chamberlin's large group assumption

Let us now introduce two pieces of notation. If the variables  $x$  and  $y$  (resp.  $k$  and  $q$ ) are bound to vary together so that  $x = y$  (resp.  $k = q$ ) we will denote them by  $z$  (resp.  $a$ ). In other words  $z$  (resp.  $a$ ) is the common output (resp. quality). When no confusion can arise we will use  $z$  and  $a$  to denote symmetrical allocations, i.e. those in which  $x = y$  and  $k = q$ . Also for an arbitrary function  $y=f(x)$  we will write  $\epsilon_x^y$  as the elasticity of  $y$  with respect to  $x$  and  $\epsilon_x^y(x)$  as the elasticity of  $y$  with respect to  $x$  as a function of  $x$ .

**Definition 1.-**  $(a^0, n^0, z^0, l^0,)$  is said to be a symmetrical optimum if it maximizes  $u(a, a, n, z, z, l)$  subject to  $l + n.c(z, a) = w$ .

Notice that such an allocation is symmetrical because active firms produce the same quantity of output. In some cases it can be shown (see Dixit-Stiglitz (1977) pp. 300-1) that our symmetry assumptions imply that the full optimum -i.e. the allocation which maximize utility over the feasible set- is symmetric. In other cases it may be understood as a kind of restricted optimum, useful as long as it simplifies the analysis.

Defining  $\varepsilon_z^u \equiv (\partial u / \partial x + \partial u / \partial y) \cdot z / u$  as the elasticity of utility with respect to common output, and  $\varepsilon_a^u \equiv (\nabla u / \nabla k + \nabla u / \nabla q) \cdot a / u$  as the elasticity of utility with respect to a common vector of qualities (notice that  $\varepsilon_a^u$  is a vector), both evaluated at a symmetrical allocation, we have that the first order conditions of a symmetrical optimum (assuming this is interior) are

$$\partial u / \partial x + \partial u / \partial y = n \partial c / \partial x \partial u / \partial l$$

$$\nabla u / \nabla k + \nabla u / \nabla q = n \partial u / \partial l \nabla c / \nabla k$$

$$\partial u / \partial n = c \partial u / \partial l$$

And dividing the first two equations by the third we get

$$\varepsilon_a^u = \varepsilon_k^c \cdot \varepsilon_n^u \quad (1)$$

$$\varepsilon_z^u = \varepsilon_x^c \cdot \varepsilon_n^u \quad (2)$$

If the utility function is a generalized S-D-S (as explained above) then equations (1) and (2) reduce to  $\varepsilon_k^c = \varepsilon_k^v$  and  $\varepsilon_x^c = \varepsilon_x^\phi$ .

If we have that  $\varepsilon_n^u > \varepsilon_z^u$  when elasticities are evaluated at some particular allocation we will say that people like variety (at this allocation) in the sense that utility increases faster with the number of brands, holding  $z$  as a constant, than with output, holding  $n$  as a constant. If the inequality is reversed we will say that people do not like variety (at

this allocation). Finally people are indifferent to variety (at some allocation) if the above equation holds with equality. It is easy to show that if the utility function is a generalized S-D-S, people like variety if and only if (iff in the sequel)  $\epsilon_x^\phi < 1$ . Under Monopolistic Competition it seems natural to assume that people like variety at each possible allocation. Indeed, at equilibrium, people must like variety (see the proof of Proposition 2 below). Now we have our first result.

**Proposition 1.-** *Average cost are increasing, constant or decreasing in the optimum iff respectively people do not like, are indifferent or like variety at the optimum.*

Proof/  $\partial(c(x,k)/x)/\partial x = (\partial c(\cdot)/\partial x - c/x) / x = (\epsilon_x^c - 1) c / x^2$ . And using equation (2) above we get the result.■

Let us now turn to the definition of an equilibrium. The consumer chooses the quantities of goods 1, ..., n and leisure in order to maximize utility at given qualities and prices for a given set of available products, i.e. if firm  $j$  is not active the consumer is not allowed to demand this brand (in other words the price of  $j$  is infinity). The inverse demand function for the representative firm is constructed as follows. Let  $p(k,q,n,x,y)$  be the marginal rate of substitution between  $x$  and  $l$  evaluated at  $l = w - c(k,x) - (n-1).c(q,y)$ , i.e. we assume that the labor market always balance (see Hart (1979) pp. 9-10). Then for the representative firm  $p = p(k,q,n,x,y)$  will be the inverse demand function ( $p$  being the market price of its product) and profits are  $p(k,q,n,x,y).x - c(k,x)$ .

**Definition 2.-**  $(a^e, n^e, z^e, l^e)$  is a Symmetrical Monopolistically Competitive Equilibrium (or in short an equilibrium) if

- a)  $a^e, z^e = \arg. \max_{k, x} p(k, a^e, n^e, x, z^e) \cdot x - c(k, x)$  and  
b)  $p(a^e, a^e, n^e, z^e, z^e) \cdot z^e - c(a^e, z^e) = 0$ .

where equilibrium in the labor market is implied by Walras law. If  $u(\cdot)$  is assumed to be linear on  $l$  and the allocation is interior, first order conditions of an equilibrium (part a) above) and the zero profit assumption (part b) above) can be written as follows

$$x \partial^2 u / \partial x^2 + \partial u / \partial x = \partial c / \partial x$$

$$x \nabla^2 u / \nabla x k = \nabla c / \nabla k$$

$$x \partial u / \partial x = c$$

And dividing the first two equations by the third we get

$$\varepsilon_x^{u'} + 1 = \varepsilon_x^c \quad (3)$$

$$\varepsilon_k^{u'} = \varepsilon_k^c \quad (4)$$

Where  $\varepsilon_x^{u'}$  (resp.  $\varepsilon_k^{u'}$ ) is the elasticity of  $\partial u / \partial x$  with respect to  $x$  (resp.  $k$ ). Conditions a) and b) in Definition 2 above, imply that if  $p(\cdot)$  is decreasing on  $x$ ,  $x^e$  will be located in the decreasing part of the average cost curve. Thus Proposition 1 implies that if the average cost curve is u-shaped with a minimum which does not depend on  $k$  and that in the optimum people do not like or are indifferent to variety, then  $z^o > z^e$ . Next we investigate the effects on utility of small variations on output or the number of firms if, starting from an equilibrium, we move to a (very close) feasible allocation.

**Proposition 2.-** An increase in the equilibrium number of firms, holding  $a^e$  and  $z^e$  as constant will never decrease welfare.

Proof/ First, let us compute  $\partial u/\partial n$  evaluated at equilibrium with  $l = w - n^e$ .  $c(z^e, a^e)$ . First order conditions of utility maximization and  $\partial u/\partial z = n \cdot \partial u/\partial x$  imply that  $\partial u/\partial n = u \cdot (\epsilon_n^u - \epsilon_z^u)/n^e$ . We will show that in equilibrium the consumer must like variety and therefore  $\epsilon_n^u \geq \epsilon_z^u$ . In order to see this, notice that the optimization performed by the consumer over goods implies that  $\partial u/\partial x + \partial u/\partial y = p \cdot n \cdot \partial u/\partial l$ . Since the consumer can reject an existent variety it must be that  $\partial u/\partial n \geq p \cdot z \cdot \partial u/\partial l$ . And the last two equations imply the result. ■

Now let us study the effect on welfare of an increase in output.

**Proposition 3.-** Let us assume that the inverse demand function  $p(\cdot)$  is strictly decreasing on  $x$ . Then an increase in  $z^e$  holding  $a^e$  and  $n^e$  as constant, always increases welfare.

Proof/ Computing  $\partial u/\partial z$  evaluated under the same conditions than in Proposition 2 we have that  $\partial u/\partial z = n^e \cdot \partial u/\partial x \cdot (1 - (\partial c/\partial x)/p)$  and since price exceeds marginal cost the Proposition is proved. ■

Proposition 2 and 3 imply that the effect on welfare of a simultaneous increase of  $z^e$  and a reduction of  $n^e$  is ambiguous. We will see in Section 3 that under additional assumptions a full characterization is available (see Proposition 10). Finally, we investigate the relationship between excess variety and excess capacity. We will assume either that qualities are fixed or that optimal and equilibrium qualities coincide (see Proposition 5 below).

**Proposition 4.-** Let us assume that average costs are non increasing on  $[z^0, z^e]$  and that qualities are fixed. Then  $n^e > n^0 \implies z^0 > z^e$ .

Proof/ First notice that the optimal bundle can be written as an  $n^e$ -dimensional vector  $(z^o, \dots, z^o, 0, \dots, 0)$  in which  $z^o$  is repeated  $n^o$  times. Equilibrium prices are  $(p^e, \dots, p^e)$ . Then by revealed preference we have  $n^o \cdot p^e \cdot z^o + 0 + l^o > n^e \cdot p^e \cdot z^e + l^e$ . Using the zero profit condition and that  $w = l + n \cdot c(x, a)$  we get  $p^e = c(z^e, a)/z^e > c(z^o, a)/z^o$  where  $a$  is a fixed vector of qualities. Then the result follows from our assumption on the average cost curve.■

Several comments are in order. First notice that only a revealed preference argument was used. Hence if several consumers are posited but their aggregate demand satisfies this property, Proposition 4 holds. Second the condition on the average cost curve is true not only when this curve is decreasing everywhere. For instance if this curve were u-shaped, inverse demand function were decreasing on  $x$  and people like variety at each possible allocation, this assumption holds since in the optimum and in equilibrium average cost is decreasing with respect to output. Third, a corollary of Proposition 4 is that  $z^e \geq z^o \implies n^o > n^e$ . This Corollary was hinted at by S-D-S in the special case of  $z^e = z^o$  and for the special class of utility functions described before. Finally we notice that the converse to Proposition 4 is not true, i.e. excess capacity does not imply excess diversity. For instance under economies of scale demand and cost curves may be such that no entry is profitable by any firm but total surplus associated with a single firm producing at price equals marginal cost is positive. Therefore, at the optimum we have more firms producing more output each than in equilibrium. This shows that we should not infer from the excess capacity theorem the excess diversity theorem, the reverse implication being correct.

### 3.- SPECIAL RESULTS.

In this Section, we assume that utility and cost functions are of the form  $u = u(v(k). \phi(x) + (n-1).v(q). \phi(y)) + l$  and  $c = F(k).f(x)$  (see Yarrow (1985) and Ireland (1987) for an alternative specification of  $c(\cdot)$ ). Notice that the form of  $c(\cdot)$  implies that  $\varepsilon_k^c = \varepsilon_k^F$  and  $\varepsilon_x^c = \varepsilon_x^f$ . We will also assume some kind of large group assumption. Defining  $s \equiv n. \phi(z).v(a)$  ( $s$  is an aggregate measure of the consumption of the differentiated commodity), the inverse demand function reads  $p = \partial u / \partial s . v(k). \partial \phi(x) / \partial x$ . We assume that firms regard  $s$  as constant with respect to  $x$  and  $k$  (see Spence (1976 p. 227 equation 52 and footnote 11) and Dixit-Stiglitz (1977) p. 299 equations 8-9). See Tirole (1989) p.288) and Costrell (1989) for alternative motivation for this assumption). Then, it is easy to show that  $\varepsilon_x^{u'} = \varepsilon_x^{\phi'}$ ,  $\varepsilon_k^{u'} = \varepsilon_k^v$ ,  $\varepsilon_a^u = \varepsilon_k^v . \varepsilon_n^u$  and  $\varepsilon_z^u = \varepsilon_x^{\phi} . \varepsilon_n^u$  where  $\varepsilon_x^{\phi'}$  is the elasticity of  $\partial \phi / \partial x$  with respect to  $x$ . Now, we have

**Proposition 5.-** *Under the above specification, if optimal and equilibrium qualities are unique, then  $a^o = a^e$ .*

Proof/ In our case, optimal qualities satisfy  $\varepsilon_k^v(a^o) = \varepsilon_k^F(a^o)$  where we have made explicit the dependence of elasticities with respect to their arguments. Also equilibrium qualities solve  $\varepsilon_k^v(a^e) = \varepsilon_k^F(a^e)$ . ■

For the rest of the paper we will assume, w.l.o.g., that  $v(a^e) = F(a^e) = 1^{(3)}$ . Next we investigate the relationship between  $s^o$  and  $s^e$ . This

3) The fact that under imperfect competition qualities may be optimal was first discovered by Swan under stronger conditions to ours (see Tirole (1989) p. 102 for references on that). Another interesting case in which Proposition 5 holds is when  $u = v(x,y,n) + x.r(k) + (n-1).y.r(q)+l$  and  $c = x.f(k)+t(x)$  as simple computations can show. This case generalizes the one considered by Riordan (1986).

relationship can also be used to obtain a result similar to Proposition 4 but under different assumptions.

**Proposition 6.-** Let us assume that  $\phi$  is concave,  $\varepsilon_x^\phi$  is non decreasing on  $x$  and that  $\varepsilon_s^u$  is non increasing on  $s$  but less than one. Then a)  $s^\circ > s^e$  and b) if equilibrium is not optimal,  $z^e \geq z^\circ \implies n^e < n^\circ$ .

Proof/ Let us prove part a). From the definition of an optimum we get  $u(s^\circ) - u(s^e) \geq n^\circ \cdot f(z^\circ) - n^e \cdot f(z^e)$ . Also we have  $f(z^\circ) \cdot n^\circ = s^\circ \cdot \partial u(s^\circ) / \partial s$  (from the first order condition of welfare maximization with respect to  $n$ ) and  $f(z^e) \cdot n^e = s^e \cdot \varepsilon_x^\phi \cdot \partial u(s^e) / \partial s$  (from the zero profit condition). Therefore we obtain  $u(s^\circ) - u(s^e) \geq s^\circ \cdot \partial u(s^\circ) / \partial s - \varepsilon_x^\phi(z^e) \cdot s^e \cdot \partial u(s^e) / \partial s$ . Now it is easy to show that if  $\varepsilon_x^\phi(\cdot)$  is concave and non decreasing  $\varepsilon_x^\phi(z) \leq 1$ . Then rearranging we obtain  $u(s^\circ) \cdot (1 - \varepsilon_s^u(s^\circ)) \geq u(s^e) \cdot (1 - \varepsilon_s^u(s^e))$ . Therefore if  $s^e > s^\circ$  we reach a contradiction. Part b) is now easily obtained from part a). ■

Notice that our assumption on  $u(s)$  covers the case of a generalized C.E.S. function  $u = (n \cdot \phi(z))^a$ ,  $a < 1$ . Assumptions on  $\varepsilon_x^\phi(\cdot)$  will be discussed later on. Next, we study the relationship between  $z^\circ$  and  $z^e$ . We will assume that the slope of  $\varepsilon_x^f(x)$  is greater than the slope of  $\varepsilon_x^\phi(x)$  for all  $x$ . This implies that the optimum is unique and that second order condition of welfare maximization with respect to output hold. With respect to  $\phi(x)$  we will assume either that (a)  $\varepsilon_x^\phi(\cdot)$  is decreasing on  $x$  or (b)  $\varepsilon_x^\phi(\cdot)$  is increasing on  $x$  or (c)  $\varepsilon_x^\phi(\cdot)$  is constant on  $x$ . It is easy to show that if  $\phi(\cdot)$  is concave cases b) and c) above imply that  $\varepsilon_x^\phi \leq 1$ , i.e. people like or are indifferent to variety. Therefore  $\varepsilon_x^\phi > 1$  (i.e. people do not like variety) implies case a).



The equations we will use are  $\varepsilon_x^\phi(z^o) = \varepsilon_x^f(z^o)$ ,  $1 + \varepsilon_x^{\phi'}(z^e) = \varepsilon_x^f(z^e)$  (they follow from (2) and (3) in our case) and that  $\partial \varepsilon_x^\phi(z)/\partial x > (\text{resp. } = \text{ or } <) 0 \iff 1 + \varepsilon_x^\phi(z) > (\text{resp. } = \text{ or } <) \varepsilon_x^\phi(z)$  where the last equation is found by simple differentiation. These equations can be used in Figures 1, 2 and 3 to locate both the optimal and the equilibrium output. Each case (a), b) and c) above) yields a different relationship between optimal and equilibrium output. Formal proofs of Propositions 7-9 are obtainable under request from the author

**Proposition 7.-** *If case a) above holds, then  $z^o > z^e$  (see Figure 1).*

**Proposition 8.-** *If case b) above holds, then  $z^e > z^o$  (see Figure 2).*

**Proposition 9.-** *If case c) above holds then,  $z^o = z^e$ . (see Figure 3).*

Proposition 7 and 8 were proved by Dixit-Stiglitz ((1977) equation 50, p. 304) assuming constant elasticity of  $u(\cdot)$  on  $s$  and constant marginal costs. Proposition 9 was proved by Spence ((1976) p. 231, Proposition 6) and Dixit-Stiglitz (pp. 298-302). Finally we study the effect on welfare of a simultaneous variation of  $z^e$  and  $n^e$ .

**Proposition 10.-** *An increase in  $z^e$ , holding  $l^e$  as constant a) increases (resp. decreases) welfare iff  $\varepsilon_x^\phi(\cdot)$  is decreasing (resp. increasing) in equilibrium and b) does not affect welfare iff  $\varepsilon_x^\phi(\cdot)$  is constant when evaluated at equilibrium.*

Proof/ We easily compute  $\partial u(z^e, z^e, (w - l^e) / f(z^e), l^e) / \partial z = (\varepsilon_x^\phi - \varepsilon_x^f) \cdot u / z^e \cdot \varepsilon_n^u = (\varepsilon_x^\phi - \varepsilon_x^{\phi'} - 1) \cdot u / z \cdot \varepsilon_n^u$  and hence the result. ■

In the case a) above  $z^{\circ} > z^e$  and welfare increases with a (small) increase of  $z^e$  coupled with a reduction of  $n^e$ . Conversely in the case b) above  $z^e > z^{\circ}$  (and if the assumptions quoted in Corollary 1 or Proposition 6 hold, also  $n^e > n^{\circ}$ ) and welfare increases with a reduction in equilibrium output and an increase in variety. Finally if c) holds then  $z^{\circ} = z^e$  and a small increase in  $z^e$  has no effect on welfare (envelope theorem).

#### 4.- FINAL COMMENTS.

1). We first notice that the methods developed in the previous Section can be applied to prove Propositions 5 and 7-9 in more general cases. For instance several consumers may be allowed if each of them consumes a particular bundle of the differentiated commodity and this bundle is consumed only by this consumer. Also many classes of differentiated commodities can be assumed.

2). An implication of our analysis is that under Monopolistic Competition there is voluntary unemployment in the following sense: welfare can be increased by increasing either the number of firms (Proposition 2) or output (Proposition 3). In both cases leisure must decrease in order to keep balance in the labor market. Therefore a decrease in leisure increases welfare and hence the result.

3). Finally our analysis can be summarized as follows. Under rather general assumptions, 1) Optimal output is located on the decreasing part of the average cost curve if and only if the representative consumer likes variety, 2) welfare is increasing with respect to output and the number of firms in equilibrium and 3) excess variety implies excess capacity but not vice versa. In a more specific framework, which however generalizes S-D-S analysis, 4) equilibrium and optimal qualities coincide, 5) optimal output can be greater, equal or less than equilibrium output and 6) welfare can be increased by reducing the difference between optimal and equilibrium output.

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FIGURE 1

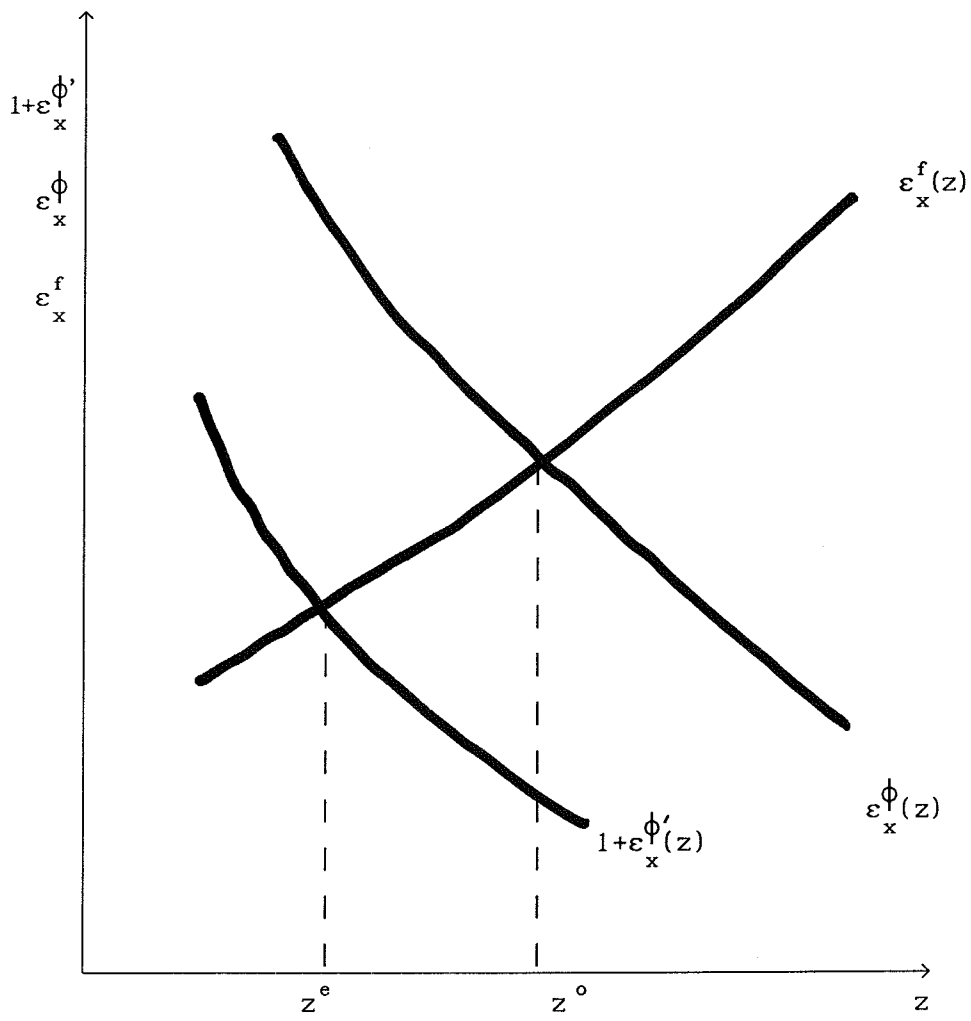


FIGURE 2

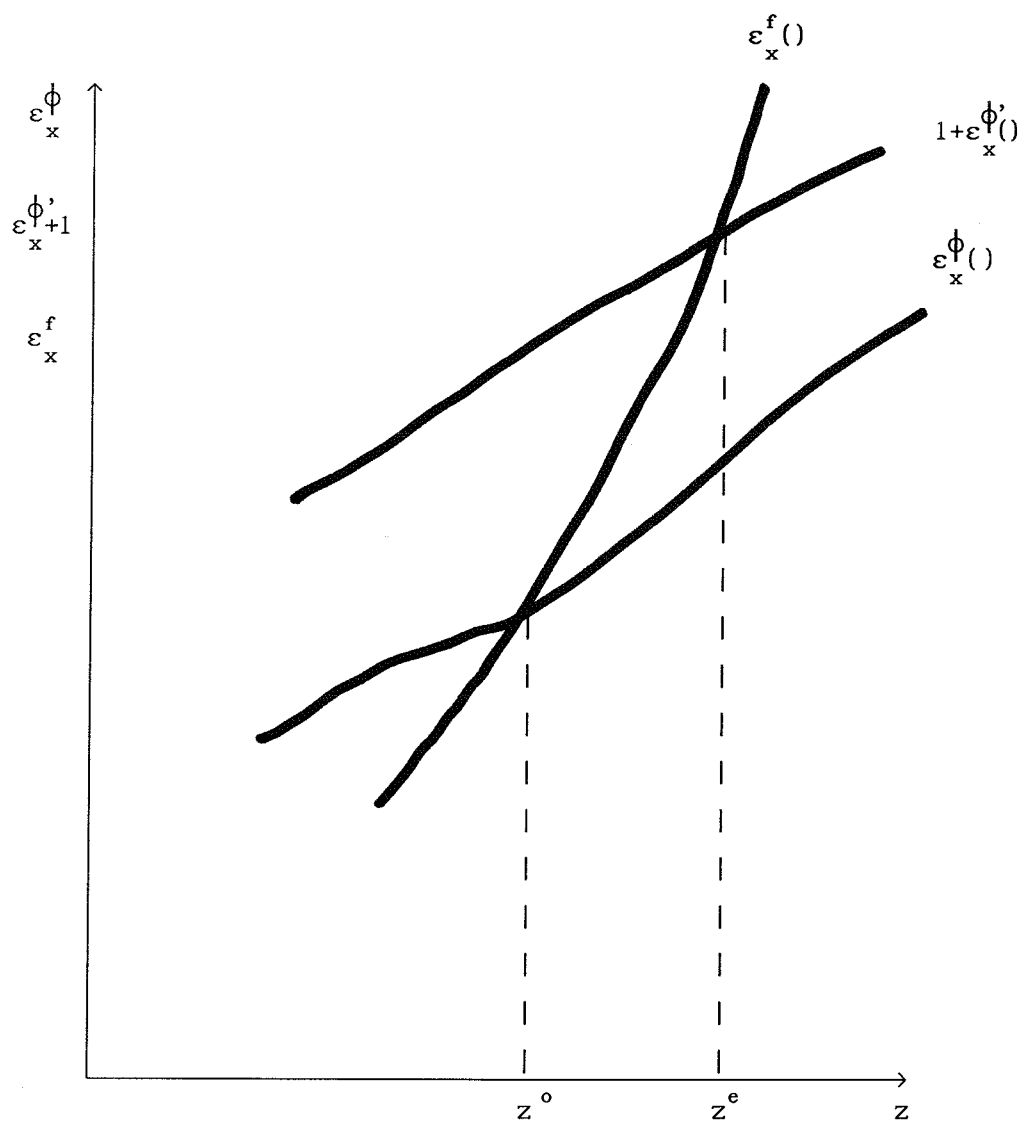
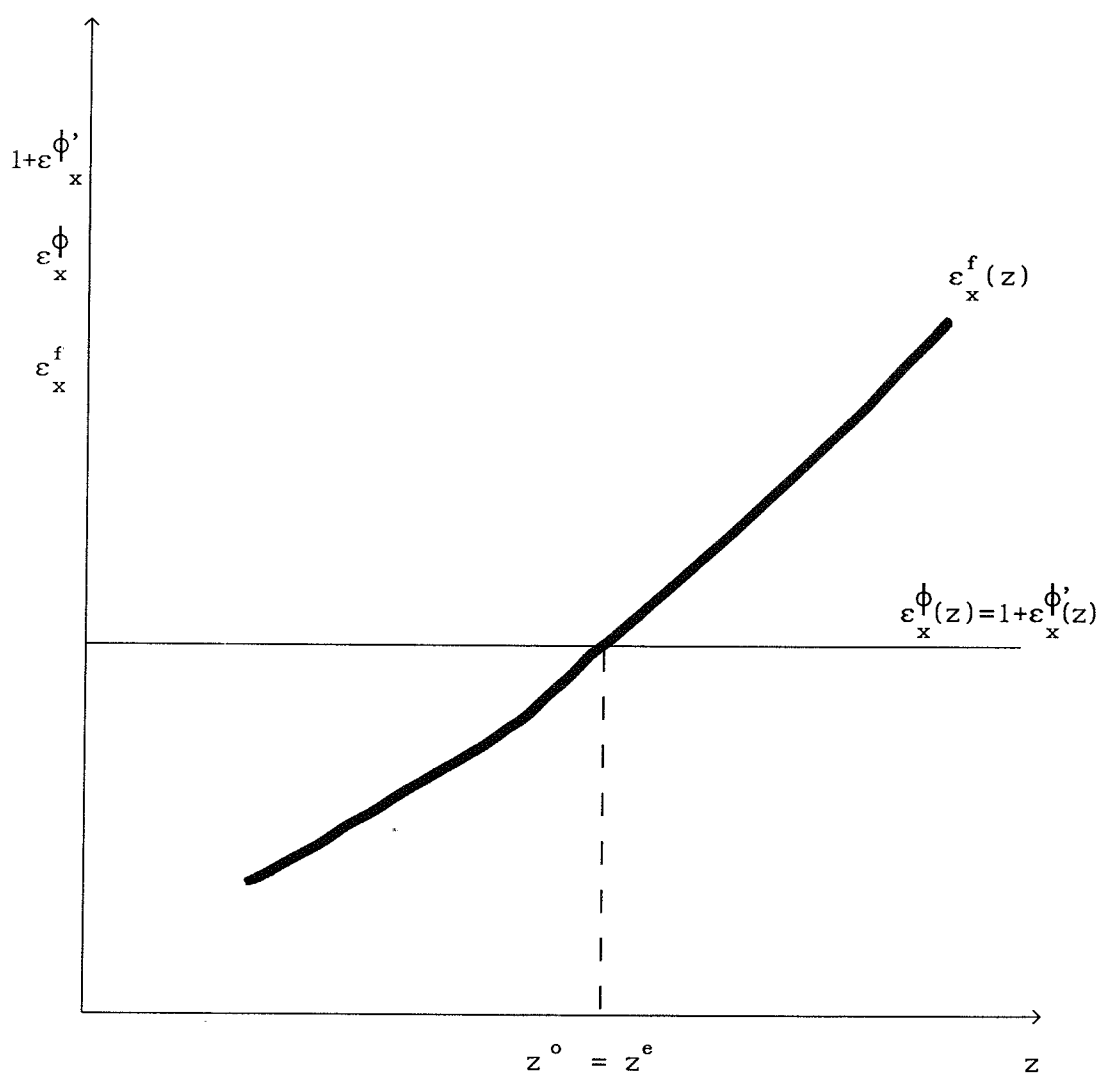




FIGURE 3





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